Impact of Non-Homogeneity on Vibration Dynamics of Orthotropic Visco-Elastic Plates with Exponential Thickness Variation

Dr. Neerie Agarwal

PhD Mathematics, Accurate Group of Institutions, Greater Noida

Abstract: ***This study investigates the vibrational behavior of orthotropic visco-elastic rectangular plates with exponential varying thickness under non-homogeneous conditions. Utilizing Galerkins technique, the research evaluates the influence of non-homogeneity parameters, taper constant, and aspect ratio on the plate's vibration characteristics. This analysis is crucial for applications in high-temperature environments, such as in aerospace engineering, offering valuable insights for the design and analysis of structures in dynamic stress conditions.***

Keywords: Orthotropic Visco-Elastic Plates, Non-Homogeneity, Vibrational Dynamics, Galerkins Technique, Exponential Varying Thickness

1. Introduction

This chapter aims to study the effect of exponential non-homogeneity on the vibration of orthotropic viscoelastic rectangular plates with exponentially varying thickness. The equation of motion derived in chapter-1 for visco-elastic rectangular plate of variable thickness has been used. Hewitt and Mazumdar [55] have considered vibration of triangular viscoelastic plates. Huffington and Hoppmann [60] have solved the problem of the transverse vibrations of rectangular orthotropic plates.

In this chapter, the thickness of plate is assumed to vary exponentially in one direction (along x-axis). Galerkin's technique has been applied to determine the frequency equation of the plate. Deflection, Time period and Logarithmic decrement at different points for the first two modes of vibration are obtained for various values of aspect ratio, taper constant and three non-homogeneous parameters.

2. Equation of Motion & Analysis

The governing differential equation of transverse motion and differential equation of time function of a visco-elastic orthotropic rectangular plate of variable thickness in Cartesian coordinate are the governing differential equations, as detailed in my previous research titled Impact of Non-Homogeneity on the Vibrational Dynamics of Orthotropic Visco-Elastic Rectangular Plates with Linearly Varying Thickness, are presented in sections 1.6 and 1.7.

$$[D_x \frac{\partial^4 W}{\partial x^4} + D_y \frac{\partial^4 W}{\partial y^4} + 2H \frac{\partial^4 W}{\partial x^2 \partial y^2} + 2\frac{\partial H}{\partial x}\frac{\partial^3 W}{\partial x \partial y^2} + 2\frac{\partial H}{\partial y}\frac{\partial^3 W}{\partial x^2 \partial y} + 2\frac{\partial D_x}{\partial x}\frac{\partial^3 W}{\partial x^3} + 2\frac{\partial D_y}{\partial y}\frac{\partial^3 W}{\partial y^3} + \frac{\partial^2 D_x}{\partial x^2}\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 D_y}{\partial y^2}\frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 D_1^{'}}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 D_1^{'}}{\partial y^2}\frac{\partial^2 W}{\partial x^2} + 4\frac{\partial^2 D_{xy}}{\partial x \partial y}\frac{\partial^2 W}{\partial x \partial y}] - \rho h p^2 W = 0 \quad (2.1)$$

And

$$\ddot{T} + p^2 \tilde{D} T = 0 \quad (2.2)$$

where $W(x,y)$ the transverse displacement, h is the plate thickness.

Equation (2.1) is a differential equation of motion of transverse motion for orthotropic plate of variable thickness and equation (2.2) is differential equation of time function of free vibration of visco-elastic orthotropic rectangular plate.

If the thickness and non-homogeneity varies exponentially in x-direction only. Consequently, the thickness h, flexural rigidity D_x, D_y and torsional rigidity D_{xy} of plate become function of x only. Further let the two opposite edges, y=0 and y=b of the plate be simply supported so that the free transverse vibrations of the plate can be expressed as

$$W(x,y) = W_1(x) Sin(\frac{\pi y}{b}) \quad (2.3)$$

Using equation (2.3) in (2.1) and simplifying, we have

$$D_x \frac{\partial^4 W_1}{\partial x^4} + 2\frac{\partial D_x}{\partial x}\frac{\partial^3 W_1}{\partial x^3} + [\frac{\partial^2 D_x}{\partial x^2} - 2H\frac{\pi^2}{b^2}]\frac{\partial^2 W_1}{\partial x^2} - 2[v_y \frac{\partial D_x}{\partial x} + 2\frac{\partial D_{xy}}{\partial x}]\frac{\pi^2}{b^2}\frac{\partial W_1}{\partial x} +$$
$$[D_y \frac{\pi^4}{b^4} - \frac{\partial^2 D_1'}{\partial x^2}\frac{\pi^2}{b^2}]W_1 - \rho h p^2 W_1 = 0 \qquad (2.4)$$

Thus equation (2.4) reduces to a form independent of y and on introducing the non-dimensional variables.

$$\bar{H} = \frac{h}{a}, \quad \bar{W} = \frac{W_1}{a}, \quad X = \frac{x}{a}, \quad D_X = \frac{D_x}{a^3}, \quad D_Y = \frac{D_y}{a^3}$$

into equation (2.4). It becomes in non-dimensional form.

$$D_X \frac{\partial^4 \bar{W}}{\partial X^4} + 2\frac{\partial D_X}{\partial X}\frac{\partial^3 \bar{W}}{\partial X^3} + [\frac{\partial^2 D_X}{\partial X^2} - 2r^2\{v_y D_X + G_{XY}\frac{H^3}{6}\}]\frac{\partial^2 \bar{W}}{\partial X^2} -$$
$$2r^2[v_y \frac{\partial D_X}{\partial X} + \frac{G_{XY}}{6}\frac{\partial (H^3)}{\partial X}]\frac{\partial \bar{W}}{\partial X} + r^2[r^2 D_Y - v_y \frac{\partial^2 D_X}{\partial X^2}]\bar{W} - \rho \bar{H} a^2 p^2 \bar{W} = 0 \qquad (2.5)$$

where $r^2 = \left(\frac{\pi a}{b}\right)^2$

Let the thickness variation of the plate is.

$$\bar{H}(X) = H_0 \, e^{\beta X} \qquad (2.6)$$

where β is taper constant and $H_0 = \bar{H}\Big|_{X=0}$ the modulus variations are

$$E_x(X) = E_1 e^{\alpha X}$$
$$E_y(X) = E_2 e^{\alpha X} \qquad (2.7)$$

and the density varies as

$$\rho = \rho_0 e^{\alpha X} \qquad (2.8)$$

where α is non-homogeneous parameter and

$$\rho_0 = \rho\Big|_{X=0}, E_1 = E_x\Big|_{X=0}, E_2 = E_y\Big|_{X=0}$$

Considering equation (2.6), (2.7) and (2.8), the expression for rigidities comes out as

$$\left.\begin{aligned} D_X &= D_0 e^{\alpha X} e^{3\beta X} \\ D_Y &= D_1 e^{\alpha X} e^{3\beta X} \end{aligned}\right] \qquad (2.9)$$

where

$$D_0 = \frac{E_1 H_0^3}{12(1 - v_x v_y)},$$

$$D_1 = \frac{E_2 H_0^3}{12(1 - v_x v_y)}$$

Substituting equations (2.6), (2.8) and (2.9) into (2.5) the differential equation takes the form.

$$e^{\alpha X} e^{2\beta X}\frac{\partial^4 \bar{W}}{\partial X^4} + 2[\alpha e^{\alpha X} e^{2\beta X} + 3\beta e^{\alpha X} e^{2\beta X}]\frac{\partial^3 \bar{W}}{\partial X^3} + [6\alpha\beta e^{\alpha X} e^{2\beta X} + 9\beta^2 e^{\alpha X} e^{2\beta X} + \alpha^2 e^{\alpha X} e^{2\beta X}$$
$$- 2r^2 v_y e^{\alpha X} e^{2\beta X} - \frac{4r^2 G_{XY}}{E_1}(1 - v_x v_y) e^{2\beta X}]\frac{\partial^2 \bar{W}}{\partial X^2} + [-r^2 v_y \{6\beta e^{\alpha X} e^{2\beta X} + 2\alpha e^{\alpha X} e^{2\beta X}\}$$
$$- \frac{4r^2 G_{XY}}{E_1}(1 - v_x v_y) 3\beta e^{2\beta X}]\frac{\partial \bar{W}}{\partial X} + [r^4 \frac{E_2}{E_1} e^{\alpha X} e^{2\beta X} - r^2 v_y \{6\alpha\beta e^{\alpha X} e^{2\beta X} + 9\beta^2 e^{\alpha X} e^{2\beta X}$$
$$+ \alpha^2 e^{\alpha X} e^{2\beta X}\}]\bar{W} - \lambda^2 p^2 e^{\alpha X} \bar{W} = 0$$

implying that

$$B_1 \frac{\partial^4 \bar{W}}{\partial X^4} + B_2 \frac{\partial^3 \bar{W}}{\partial X^3} + B_3 \frac{\partial^2 \bar{W}}{\partial X^2} + B_4 \frac{\partial \bar{W}}{\partial X} + (B_5 - p^2 \lambda^2 e^{\alpha X})\bar{W} = 0 \qquad (2.10)$$

where

$$B_1 = e^{\alpha X} e^{2\beta X},$$

$$B_2 = 2[\alpha\, e^{\alpha X} e^{2\beta X} + 3\beta\, e^{\alpha X} e^{2\beta X}],$$

$$B_3 = [6\alpha\beta\, e^{\alpha X} e^{2\beta X} + 9\beta^2\, e^{\alpha X} e^{2\beta X} + \alpha^2 e^{\alpha X} e^{2\beta X} - 2r^2 v_y\, e^{\alpha X} e^{2\beta X} - \frac{4r^2 G_{XY}}{E_1}(1 - v_x v_y)\, e^{2\beta X}],$$

$$B_4 = [-r^2 v_y \{6\beta\, e^{\alpha X} e^{2\beta X} + 2\alpha\, e^{\alpha X} e^{2\beta X}\} - \frac{4r^2 G_{XY}}{E_1}(1 - v_x v_y) 3\beta\, e^{2\beta X}],$$

$$B_5 = [r^4 \frac{E_2}{E_1} e^{\alpha X} e^{2\beta X} - r^2 v_y \{6\alpha\beta\, e^{\alpha X} e^{2\beta X} + 9\beta^2\, e^{\alpha X} e^{2\beta X} + \alpha^2\, e^{\alpha X} e^{2\beta X}\}],$$

$$\lambda^2 = \frac{12(1 - v_x v_y)\rho_0 a^4}{E_1 H_0^2}$$

and p is a frequency parameter.

3. Solution of Free Vibration of Rectangular Plate

Let the deflection function $\bar{W}(X)$ of the plate be assumed to be a finite sum of characteristic functions $\bar{W}_k(X)$

$$\bar{W}(X) = \sum_{k=1}^{n} A_k \bar{W}_k(X) \qquad (2.11)$$

where A_k's are the undetermined coefficients and $\bar{W}_k(X)$ are the characteristic functions chosen to satisfy the boundary conditions of the plate.

For a rectangular clamped plate at both the edges X = 0 and X = 1, boundary conditions are that the deflection and the slope of the plate must be zero i.e.

$$\left.\begin{aligned} \bar{W}\Big|_{X=0} &= \frac{\partial \bar{W}}{\partial X}\Big|_{X=0} = 0 \\ \bar{W}\Big|_{X=1} &= \frac{\partial \bar{W}}{\partial X}\Big|_{X=1} = 0 \end{aligned}\right] \qquad (2.12)$$

Using Galerkin's technique, one requires that.

$$\int_R L[\bar{W}(X)]\bar{W}(X)\, dX = 0 \qquad (2.13)$$

where $L[\bar{W}(X)]$ is the left-hand side of equation (2.10). Taking the first two terms of sum (2.11) for the function $\bar{W}(X)$ as the solution of equation (2.10), one has

$$\bar{W}(X) = X^2(1-X)^2[A_1 + A_2 X(1-X)] \qquad (2.14)$$

where A_1 and A_2 are undetermined coefficients.

We have expanded $e^{\alpha X}$ and $e^{2\beta X}$ up to a term of order X^5.

Using equation (2.10) and (2.14) in equation (2.13) and then eliminating A_1 and A_2, gives the frequency equation as

$$\begin{vmatrix} F_1 & F_2 \\ F_2 & F_3 \end{vmatrix} = 0 \qquad (2.15)$$

where

$$F_1 = [\frac{4}{5} - \frac{4}{35}\alpha\beta - \frac{6}{35}\beta^2 - \frac{2}{105}\alpha^2 + r^2 v_y (\frac{4}{105} - \frac{1}{105}\alpha\beta - \frac{1}{70}\beta^2 - \frac{1}{630}\alpha^2)$$

$$+ 4r^2 \frac{G_{xy}}{E_1}(1 - v_x v_y)(\frac{2}{105} - \frac{1}{420}\alpha\beta - \frac{1}{105}\alpha + \frac{1}{840}\alpha^2\beta + \frac{1}{420}\alpha^2 - \frac{1}{3080}\alpha^3\beta - \frac{1}{2520}\alpha^3$$

$$+ \frac{1}{15840}\alpha^4\beta + \frac{1}{20790}\alpha^4 - \frac{1}{102960}\alpha^5\beta - \frac{1}{237600}\alpha^5) + r^4 \frac{E_2}{E_1}(\frac{1}{630})$$

$$-(\frac{1}{630} - \frac{1}{630}\beta + \frac{1}{1155}\beta^2 - \frac{1}{2970}\beta^3 + \frac{2}{19305}\beta^4 - \frac{2}{75075}\beta^5)p^2\lambda^2],$$

$$F_2 = [\frac{12}{35} - \frac{2}{35}\alpha\beta - \frac{3}{35}\beta^2 - \frac{1}{105}\alpha^2 + r^2 v_y(\frac{2}{105} - \frac{1}{231}\alpha\beta - \frac{1}{154}\beta^2 - \frac{1}{1386}\alpha^2)$$
$$+ 4r^2\frac{G_{xy}}{E_1}(1 - v_x v_y)(\frac{1}{105} - \frac{1}{924}\alpha\beta - \frac{1}{210}\alpha + \frac{1}{1848}\alpha^2\beta + \frac{4}{3465}\alpha^2 - \frac{1}{6864}\alpha^3\beta - \frac{1}{5544}\alpha^3$$
$$+ \frac{1}{36036}\alpha^4\beta + \frac{1}{51480}\alpha^4 - \frac{1}{240240}\alpha^5\beta - \frac{1}{772200}\alpha^5) + r^4\frac{E_2}{E_1}(\frac{1}{1386})$$
$$-(\frac{1}{1386} - \frac{1}{1386}\beta + \frac{1}{2574}\beta^2 - \frac{4}{27027}\beta^3 + \frac{2}{45045}\beta^4 - \frac{1}{90090}\beta^5)p^2\lambda^2],$$
$$F_3 = [\frac{2}{35} - \frac{3}{385}\alpha\beta - \frac{9}{770}\beta^2 - \frac{1}{770}\alpha^2 + r^2 v_y(\frac{1}{385} - \frac{1}{2002}\alpha\beta - \frac{3}{4004}\beta^2 - \frac{1}{12012}\alpha^2)$$
$$+ 4r^2\frac{G_{xy}}{E_1}(1 - v_x v_y)(\frac{1}{770} - \frac{1}{8008}\alpha\beta - \frac{1}{1540}\alpha + \frac{1}{16016}\alpha^2\beta + \frac{19}{120120}\alpha^2 - \frac{1}{60060}\alpha^3\beta - \frac{1}{40040}\alpha^3$$
$$+ \frac{1}{320320}\alpha^4\beta + \frac{1}{360360}\alpha^4 - \frac{1}{2178176}\alpha^5\beta - \frac{1}{4804800}\alpha^5) + r^4\frac{E_2}{E_1}(\frac{1}{12012})$$
$$-(\frac{1}{12012} - \frac{1}{12012}\beta + \frac{2}{45045}\beta^2 - \frac{1}{60060}\beta^3 + \frac{1}{204204}\beta^4 - \frac{1}{835380}\beta^5)p^2\lambda^2],$$

The frequency equation (2.15) is a quadratic equation in p^2 from which the two values of p^2 can be found.

Hence deflection function $\bar{W}(X)$ can be obtained from equation (2.14) after determining constants A_1 and A_2.

Choosing $A_1 = 1$, we obtain $A_2 = -\frac{F_1}{F_2}$ and then $\bar{W}(X)$ comes out as

$$\bar{W}(X) = X^2(1-X)^2[1 - \frac{F_1}{F_2}X(1-X)] \quad (2.16)$$

4. Time Functions of Vibrations of Viscoelastic Orthotropic Plates

Time functions of free vibrations of viscoelastic orthotropic plates are defined by the general ordinary differential equation (2.2). Their form depends on the viscoelastic operator $\tilde{D}$.

We have taken Kelvin's model, for which.

$$\tilde{D} \equiv (1 + \frac{\eta}{G}\frac{d}{dt}) \quad (2.17)$$

where G is shear modulus and η is viscoelastic constant.

Taking variation of G and η as linearly i.e.

$$\left.\begin{aligned} G(X) &= G_0[1+\alpha_1 X] \\ \eta(X) &= \eta_0[1+\alpha_2 X] \end{aligned}\right] \quad (2.18)$$

where α_1 and α_2 are different non-homogeneous parameters and $G_0 = G|_{X=0}, \eta_0 = \eta|_{X=0}$.

Using equation (2.18) in (2.17), we get

$$\tilde{D} \equiv 1 + s\frac{d}{dt} \quad (2.19)$$

where

$$s = \frac{\eta_0[1+\alpha_2 X]}{G_0[1+\alpha_1 X]} \quad (2.20)$$

Using equation (2.19) in equation (2.2), we obtain

$$\ddot{T} + p^2 s\dot{T} + p^2 T = 0 \quad (2.21)$$

Equation (2.21) is a differential equation of second order for time function T.

On solving equation (2.21), its solution comes out as

$$T(t) = e^{(\frac{-p^2 st}{2})}[c_1 \cos at + c_2 \sin at] \quad (2.22)$$

where

$$a^2 = p^2 - \frac{1}{4}p^4 s^2 \quad (2.23)$$

and c_1, c_2 are constants to be determined from initial conditions of the plate which assume as

$$T = 1 \text{ and } \dot{T} = 0 \; at \; t = 0 \quad (2.24)$$

Using condition (2.24) in equation (2.22), we obtain

$$T(t) = e^{(\frac{-p^2 st}{2})}[\cos at + \frac{p^2 s}{2a}\sin at] \quad (2.25)$$

Hence, deflection $w(x, y, t)$ may be expressed from equation (2.3), (2.16) and (2.25), as

$$w(x, y, t) = W_1(x)\sin(\frac{\pi y}{b})e^{(\frac{-p^2 st}{2})}[\cos at + \frac{p^2 s}{2a}\sin at] \quad (2.26)$$

Time period of the vibration of the plate is given by.

$$K = \frac{2\pi}{p} \quad (2.27)$$

where p is the frequency given by equation (2.15).

Logarithmic decrement of the vibration is given by the standard formula.

$$\wedge = \log_e(\frac{w_2}{w_1}) \quad (2.28)$$

where w_1 is the deflection at any point of the plate at a time period $K= K_1$ and w_2 is the deflection at the same point at the time period succeeding K_1.

5. Results and Discussions

Time period K, Deflection w and Logarithmic decrement $\wedge$ are computed for a clamped visco-elastic orthotropic rectangular plate of exponentially varying thickness for different values of non-homogeneous parameters α , α_1, α_2 and taper constant β and aspect ratio a/b at different points for first two modes of vibrations. All these results are presented in tables 2.1 to 2.15 and graphically shown in figures from 2.1 to 2.15.

For the numerical computation, the following orthotropic material parameters are used:

$$\frac{E_2}{E_1} = .01, \quad \frac{G_{xy}(1 - v_x v_y)}{E_1} = 0.0333$$

$$v_y = 0.3, \quad \frac{E_1}{(1 - v_x v_y)\rho_0} = 3 \times 10^5$$

$$\frac{\eta_0}{G_0} = 0.000069, \, H_0 = 0.01\, meter$$

In tables 2.1 – 2.3 results of time period K for first two modes of vibrations for all X, Y and α_1, α_2 are given as follows:

Table 2.1: Different non-homogeneous parameter α and fixed aspect ratio $\frac{a}{b} = 1.5$ for two values of taper constant β i.e. β = 0.0 and β= 0.4.

Table 2.2: Different taper constant β and fixed aspect ratio $\frac{a}{b} = 1.5$ for two values of non-homogeneous parameter α i.e. α = 0.0 and α= 0.4.

Table 2.3: Different aspect ratio $\frac{a}{b}$ and four combination of non-homogeneous parameter α and taper constant β i.e. α =0.0, β =0.0; α = 0.0, β=0.4; α = 0.4, β=0.0 and α =0.4, β=0.4

Table 2.1 shows that as non-homogeneous parameter α increase time period K of vibration also increases. Figure 2.1 shows the effect of non-homogeneous parameter α on time period K. It is clearly observed in figure 2.1 that there is a steady increase in time period K with increase of non-homogeneous parameterα.

Tables 2.2 and 2.3 shows that as taper constant β and aspect ratio a/b increase, time period K decrease for the first two modes of vibration. It is clearly shown in figures 2.2 and 2.3 that there is a steady decrease in time period K with increase of taper constant β and aspect ratio a/b.

In tables 2.4 – 2.11 results of deflection for the first two modes of vibrations for different X, Y, and a fixed aspect ratio a/b = 1.5 for initial time 0.K and time 5.K are given for the following combination of α, β, α1 and α2:

$$\alpha = 0.0, \beta = 0.0, \alpha_1 = 0.0, \alpha_2 = 0.0$$
$$\alpha = 0.4, \beta = 0.4, \alpha_1 = 0.0, \alpha_2 = 0.0$$
$$\alpha = 0.0, \beta = 0.0, \alpha_1 = 0.4, \alpha_2 = 0.2$$
$$\alpha = 0.4, \beta = 0.4, \alpha_1 = 0.4, \alpha_2 = 0.2$$

It can be seen from tables 2.4 – 2.11 that deflection w starts from zero to increase and then decrease to zero for the first mode of vibration but for the second mode of vibration, deflection w starts from zero to increase then decrease then increase and finally come to zero for fixed Y and different value of X for time 0.K and 5.K.

It is also note that for fixed X, deflection w starts from zero to increase and then decreases in both modes of vibration for both time 0.K and 5.K for different values of Y.

One can conclude also that deflection w decreases for time increase for both the modes of vibration. These results are plotted in figures 2.4 to 2.11.

In tables 2.12 - 2.15 are given results of logarithmic decrement $\wedge$ for first two modes of vibration for different X, Y and constant aspect ratio a/b = 1.5 for the following four cases

$\alpha = 0.0$, $\beta = 0.0$, $\alpha_1 = 0.0$, $\alpha_2 = 0.0$

$\alpha = 0.4$, $\beta = 0.4$, $\alpha_1 = 0.0$, $\alpha_2 = 0.0$

$\alpha = 0.0$, $\beta = 0.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.2$

$\alpha = 0.4$, $\beta = 0.4$, $\alpha_1 = 0.4$, $\alpha_2 = 0.2$

It is interesting observed that the logarithmic decrement $\wedge$ is constant across the plate for $\alpha = 0.0$, $\beta = 0.0$, $\alpha_1 = 0.0$, $\alpha_2 = 0.0$ and $\alpha = 0.4$, $\beta = 0.4$, $\alpha_1 = 0.0$, $\alpha_2 = 0.0$ while increase for $\alpha = 0.0$, $\beta = 0.0$, $\alpha_1 = 0.4$, $\alpha_2 = 0.2$ and $\alpha = 0.4$, $\beta = 0.4$, $\alpha_1 = 0.4$, $\alpha_2 = 0.2$ for different values of X and fixed value of Y. But it is same for fixed value of X and different value of Y. These results are plotted in figures 2.12 to 2.15.

Table 2.1: Time period K (in seconds) for different non-homogeneous parameter (α) and a constant aspect ratio (a/b = 1.5) for all X, Y and α1, α2

α	β = 0.0		β = 0.4	
	First Mode	Second Mode	First Mode	Second Mode
0.0	0.149813	0.029783	0.125528	0.024786
0.2	0.150257	0.029808	0.126798	0.024841
0.4	0.150833	0.029837	0.128218	0.024900
0.6	0.151549	0.029871	0.129806	0.024963
0.8	0.152413	0.029910	0.131583	0.025031

Table 2.2: Time period K (in seconds) for different taper constant (β) and a constant aspect ratio (a/b = 1.5) for all X, Y and α1, α2

β	α = 0.0		α = 0.4	
	First Mode	Second Mode	First Mode	Second Mode
0.0	0.149813	0.029783	0.150833	0.029837
0.2	0.136334	0.027060	0.138209	0.027146
0.4	0.125528	0.024786	0.128218	0.024900
0.6	0.117035	0.022876	0.120598	0.023016
0.8	0.110629	0.021241	0.115249	0.021403

Table 2.3: Time period K (in seconds) for different aspect ratio (a/b) for all X, Y and α1, α2

a/b	α =0.0, β =0.0		α =0.0, β =0.4		α =0.4, β =0.0		α =0.4, β =0.4	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.5	0.173851	0.030987	0.145469	0.025792	0.174321	0.031007	0.147744	0.025880
1.0	0.163628	0.030520	0.137000	0.025401	0.164373	0.030553	0.139499	0.025500
1.5	0.149813	0.029783	0.125528	0.024786	0.150833	0.029837	0.128218	0.024900
2.0	0.135004	0.028830	0.1131960	0.023990	0.136204	0.028910	0.115953	0.024123
2.5	0.120795	0.027723	0.101331	0.023065	0.122067	0.027827	0.104028	0.023216

Table 2.4: Deflection w for different X, Y and α =0.0, β =0.0, α1 =0.0, α2 = 0.0 and a/b = 1.5 at initial time 0.K

X	Y = 0.2		Y = 0.4		Y = 0.6		Y= 0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.015438	0.004461	0.024975	0.007217	0.024965	0.007214	0.015412	0.004454
0.4	0.035170	-0.001877	0.056897	-0.003037	0.056873	-0.003036	0.035109	-0.001874
0.6	0.035170	-0.001877	0.056897	-0.003037	0.056873	-0.003036	0.035109	-0.001874
0.8	0.015438	0.004461	0.024975	0.007217	0.024965	0.007214	0.015412	0.004454
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 2.5: Deflection w for different X, Y and α =0.0, β =0.0, α1 =0.0, α2 = 0.0 and a/b = 1.5 at time 5.K

X	Y = 0.2		Y = 0.4		Y = 0.6		Y= 0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.014751	0.003549	0.023863	0.005741	0.023853	0.005739	0.014725	0.003543
0.4	0.033604	-0.001493	0.054363	-0.002416	0.054341	-0.002415	0.033546	-0.001491
0.6	0.033604	-0.001493	0.054363	-0.002416	0.054341	-0.002415	0.033546	-0.001491
0.8	0.014751	0.003549	0.023863	0.005741	0.023853	0.005739	0.014725	0.003543
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 2.6: Deflection w for different X, Y and α =0.4, β =0.4, α1 =0.0, α2 = 0.0 and a/b = 1.5 at initial time 0.K

X	Y = 0.2		Y = 0.4		Y = 0.6		Y= 0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.004722	0.004460	0.007680	0.007215	0.007635	0.007212	0.004713	0.004452
0.4	-0.000998	-0.001881	-0.001615	-0.003043	-0.001615	-0.003042	-0.000997	-0.001878
0.6	-0.000998	-0.001881	-0.001615	-0.003043	-0.001615	-0.003042	-0.000997	-0.001878
0.8	0.004722	0.004460	0.007680	0.007215	0.007635	0.007212	0.004713	0.004452
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 2.7: Deflection w for different X, Y and α =0.4, β =0.4, α1 =0.0, α2 = 0.0 and a/b = 1.5 at time 5.K

X	Y = 0.2		Y = 0.4		Y = 0.6		Y= 0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.004477	0.003392	0.007243	0.005488	0.007240	0.005486	0.004469	0.003386
0.4	-0.000947	-0.001431	-0.001532	-0.002315	-0.001531	-0.002314	-0.000945	-0.001428
0.6	-0.000947	-0.001431	-0.001532	-0.002315	-0.001531	-0.002314	-0.000945	-0.001428
0.8	0.004477	0.003392	0.007243	0.005488	0.007240	0.005486	0.004469	0.003386
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 2.8: Deflection w for different X, Y and α =0.0, β =0.0, α1 =0.4, α2 = 0.2 and a/b = 1.5 at initial time 0.K

X	Y = 0.2		Y = 0.4		Y = 0.6		Y= 0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.015438	0.004461	0.024975	0.007217	0.024965	0.007214	0.015412	0.004454
0.4	0.035170	-0.001877	0.056897	-0.003037	0.056873	-0.003036	0.035109	-0.001874
0.6	0.035170	-0.001877	0.056897	-0.003037	0.056873	-0.003036	0.035109	-0.001874
0.8	0.015438	0.004461	0.024975	0.007217	0.024965	0.007214	0.015412	0.004454
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 2.9: Deflection w for different X, Y and α =0.0, β =0.0, α1 =0.4, α2 = 0.2 and a/b = 1.5 at time 5.K

X	Y = 0.2		Y = 0.4		Y = 0.6		Y= 0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.014776	0.003579	0.023903	0.005790	0.023893	0.005788	0.014750	0.003573
0.4	0.033710	-0.001517	0.054534	-0.002454	0.054511	-0.002453	0.033651	-0.001514
0.6	0.033752	-0.001527	0.054603	-0.002470	0.054580	-0.002469	0.033694	-0.001524
0.8	0.014832	0.003649	0.023995	0.005902	0.023985	0.005900	0.014807	0.003642
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 2.10: Deflection w for different X, Y and α =0.4, β =0.4, α1 =0.4, α2 = 0.2 and a/b = 1.5 at initial time 0.K

X	Y = 0.2		Y = 0.4		Y = 0.6		Y= 0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.004722	0.004460	0.007680	0.007215	0.007635	0.007212	0.004713	0.004452
0.4	-0.000998	-0.001881	-0.001615	-0.003043	-0.001615	-0.003042	-0.000997	-0.001878
0.6	-0.000998	-0.001881	-0.001615	-0.003043	-0.001615	-0.003042	-0.000997	-0.001878
0.8	0.004722	0.004460	0.007680	0.007215	0.007635	0.007212	0.004713	0.004452
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 2.11: Deflection w for different X, Y and α =0.4, β =0.4, α1 =0.4, α2 = 0.2 and a/b = 1.5 at time 5.K

X	Y = 0.2		Y = 0.4		Y = 0.6		Y= 0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.004486	0.003427	0.007257	0.005544	0.007254	0.005541	0.004478	0.003421
0.4	-0.000950	-0.001458	-0.001537	-0.002359	-0.001537	-0.002358	-0.000949	-0.001455
0.6	-0.000952	-0.001469	-0.001539	-0.002377	-0.001539	-0.002376	-0.000950	-0.001467
0.8	0.004506	0.003507	0.007289	0.005673	0.007286	0.005671	0.004498	0.003501
1.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Table 2.12: Logarithmic decrement Λ for different X, Y and α =0.0, β =0.0, α1 =0.0, α2 = 0.0 and a/b = 1.5

X	Y = 0.2		Y = 0.4		Y = 0.6		Y= 0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.2	-0.009098	-0.045753	-0.009098	-0.045753	-0.009098	-0.045753	-0.009098	-0.045753
0.4	-0.009098	-0.045753	-0.009098	-0.045753	-0.009098	-0.045753	-0.009098	-0.045753

0.6	-0.009098	-0.045753	-0.009098	-0.045753	-0.009098	-0.045753	-0.009098	-0.045753
0.8	-0.009098	-0.045753	-0.009098	-0.045753	-0.009098	-0.045753	-0.009098	-0.045753

Table 2.13: Logarithmic decrement Λ for different X, Y and α =0.4, β =0.4, α1 =0.0, α2 = 0.0 and a/b = 1.5

X	Y = 0.2		Y = 0.4		Y = 0.6		Y= 0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.2	-0.010630	-0.054726	-0.010630	-0.054726	-0.010630	-0.054726	-0.010630	-0.054726
0.4	-0.010630	-0.054726	-0.010630	-0.054726	-0.010630	-0.054726	-0.010630	-0.054726
0.6	-0.010630	-0.054726	-0.010630	-0.054726	-0.010630	-0.054726	-0.010630	-0.054726
0.8	-0.010630	-0.054726	-0.010630	-0.054726	-0.010630	-0.054726	-0.010630	-0.054726

Table 2.14: Logarithmic decrement Λ for different X, Y and α =0.0, β =0.0, α1 =0.4, α2 = 0.2 and a/b = 1.5

X	Y = 0.2		Y = 0.4		Y = 0.6		Y= 0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.2	-0.008761	-0.044059	-0.008761	-0.044059	-0.008761	-0.044059	-0.008761	-0.044059
0.4	-0.008471	-0.042598	-0.008471	-0.042598	-0.008471	-0.042598	-0.008471	-0.042598
0.6	-0.008218	-0.041326	-0.008218	-0.041326	-0.008218	-0.041326	-0.008218	-0.041326
0.8	-0.007996	-0.040208	-0.007996	-0.040208	-0.007996	-0.040208	-0.007996	-0.040208

Table 2.15: Logarithmic decrement Λ for different X, Y and α =0.4, β =0.4, α1 =0.4, α2 = 0.2 and a/b = 1.5

X	Y = 0.2		Y = 0.4		Y = 0.6		Y= 0.8	
	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.2	-0.010236	-0.052699	-0.010236	-0.052699	-0.010236	-0.052699	-0.010236	-0.052699
0.4	-0.009897	-0.050951	-0.009897	-0.050951	-0.009897	-0.050951	-0.009897	-0.050951
0.6	-0.009602	-0.049430	-0.009602	-0.049430	-0.009602	-0.049430	-0.009602	-0.049430
0.8	-0.009342	-0.048092	-0.009342	-0.048092	-0.009342	-0.048092	-0.009342	-0.048092

6. Conclusion

In conclusion, the research presents significant insights into the vibrational behavior of orthotropic visco-elastic rectangular plates with exponential varying thickness variation, under the influence of non-homogeneity. The study findings highlight the intricate relationship between the non-homogeneous parameters, taper constants, and aspect ratios, and their collective impact on the plate's vibration characteristics. It emphasizes the importance of considering these factors in the design and analysis of structures subjected to dynamic stress, particularly in high-temperature environments. This research paves the way for further exploration in the field of material science and structural engineering, especially in applications involving extreme conditions such as those encountered in aerospace engineering.

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