# Analyzing the Impact of Non-Homogeneity on the Vibrational Dynamics of Orthotropic Visco-Elastic Rectangular Plates with Linearly Varying Thickness

**Dr. Neerie Agarwal**

PhD Mathematics, Accurate Group of Institutions, Greater Noida

**Abstract:** ***This study delves into the vibrational behavior of orthotropic visco-elastic rectangular plates with linearly varying thickness, particularly under the influence of non-homogeneity. This aspect has gained significance due to its implications in high-temperature scenarios, such as those encountered by hypersonic vehicles. The research employs Galerkins technique to solve the fourth-order partial differential equation of motion, considering small deflection and linear visco-elastic properties. The study focuses on the effect of non-homogeneity parameters on the deflection, time period, and logarithmic decrement of the plates vibration in various modes. It reveals a correlation between the non-homogeneity parameters, taper constant, and aspect ratio with the vibrational characteristics of the plate, presenting a comprehensive understanding of these dynamics.***

**Keywords:** Orthotropic Visco-Elastic Plates, Non-Homogeneity, Vibrational Dynamics, Galerkins Technique, Linearly Varying Thickness

## 1. Introduction

In recent year, the exploration of dynamical behavior of plates because of non-homogeneity became important due to high temperatures reached on external skin panel of hypersonic vehicles.

Sobotka [196] has investigated the vibration of rectangular orthotropic visco-elastic plates. Leissa [108,111] has given the solution for rectangular plate of variable thickness. Kishor and Rao [75] have discussed non linear vibration of rectangular plate on visco-elastic foundation. Saito and Yamaguchi [176] solved the problem of free vibration of a rectangular plate with visco-elastic stiffness. Jongwon Seok, Tiersten and Scarton [67, 68] have solved the problem of free vibrations of rectangular cantilever plates.

Many real bodies can posses an initial non-homogeneity due to an inclusion of a material or imperfections. Therefore in elastic bodies and the material properties are not constant but vary with position in a random manner. It is well known, that in the presence of a constant non homogeneity parameter, the elastic coefficient of homogeneous materials become functions of the space variables.

Cheung and Zhou [21] have discussed the free vibration of tapered rectangular plate using the Rayliegh-Ritz method. The problem of vibration of non-uniform orthotropic rectangular plate has been solved by Tomar, Sharma and Gupta [210]. Zhou and Cheung [231] have solved the problem of free vibration of line supported rectangularplate using a set of static beam functions. Laura, Avalos and Larrondo [83] have solved the problem of forced vibrations of simple supported anisotropic rectangular plate. Dickinson [26] has discussed the flexural vibration of rectangular orthotropic plates. Bambil, Laura and Rossi [8] have discussed the vibration of an orthotropic rectangular plate of linearly varying thickness.

The object of work presented in this chapter is to study the non homogeneity effect on the vibration of orthotropic visco-elastic rectangular plate of linearly varying thickness. The assumption of small deflection and linear visco-elastic properties are made. Assuming that the visco-elastic properties of the plate are of the 'Kelvin type'. Galerkin's technique has been applied to solved the forth order partial differential equation of motion. Deflection, Time period and Logarithmic decrement at different points for the first two modes of vibration are obtained for various values of aspect ratio, taper constant and three non-homogeneous parameters.

## 2. Equation of Motion & Analysis

The equation of motion of a viscoelastic orthotropic rectangular plate of variable thickness may be written in the form, as by Sobotka [197]

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \rho h \frac{\partial^2 w}{\partial t^2} \qquad \textbf{(1.1)}$$

Here

$M_x, M_y$ and $M_{xy}$ are moments per unit length of plate, $\rho$ is mass per unit volume, $h$ is thickness of plate and w is displacement at time t.

The expression for $M_x, M_y, M_{xy}$ are given by

$$M_x = -\tilde{D}\left[D_x \frac{\partial^2 w}{\partial x^2} + D_1' \frac{\partial^2 w}{\partial y^2}\right]$$
$$M_y = -\tilde{D}\left[D_1' \frac{\partial^2 w}{\partial x^2} + D_y \frac{\partial^2 w}{\partial y^2}\right] \qquad \textbf{(1.2)}$$
$$M_{xy} = -2\tilde{D} D_{xy} \frac{\partial^2 w}{\partial x \partial y}$$

where


Licensed Under Creative Commons Attribution CC BY

$$D_x = \frac{E_x h^3}{12(1-v_x v_y)}$$

is called the flexural rigidity of the plate in x- direction, and

$$D_y = \frac{E_y h^3}{12(1-v_x v_y)}$$

is called the flexural rigidity of the plate in y- direction, and

$$D_{xy} = \frac{G_{xy} h^3}{12}$$

is called the torsional rigidity,
and

$$D_1^{'} = v_x D_y \ (= v_y D_x\ )$$

Here $E_x$ & $E_y$ are the moduli of elasticity in x- and y-direction respectively, $v_x$ and $v_y$ are the possion ratio & $G_{xy}$ is the shear modulus.

On substituting equation (1.2) in (1.1), we obtain

$$\tilde{D}[D_x \frac{\partial^4 w}{\partial x^4} + D_y \frac{\partial^4 w}{\partial y^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + 2\frac{\partial H}{\partial x}\frac{\partial^3 w}{\partial x \partial y^2} + 2\frac{\partial H}{\partial y}\frac{\partial^3 w}{\partial x^2 \partial y} + 2\frac{\partial D_x}{\partial x}\frac{\partial^3 w}{\partial x^3} + 2\frac{\partial D_y}{\partial y}\frac{\partial^3 w}{\partial y^3} + \frac{\partial^2 D_x}{\partial x^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 D_y}{\partial y^2}\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 D_1^{'}}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 D_1^{'}}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + 4\frac{\partial^2 D_{xy}}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y}] + \rho h \frac{\partial^2 w}{\partial t^2} = 0 \quad \textbf{(1.3)}$$

Where

$$H = D_1^{'} + 2D_{xy}$$

The solution of equation (1.3) can be taken in the form of product of two functions as follows:

$$w = w(x,y,t) = W(x,y)T(t) \quad \textbf{(1.4)}$$

where $W(x,y)$ is the deflection and $T(t)$ is the time function.

Substituting equation (1.4) in (1.3), we obtain

$$[D_x \frac{\partial^4 W}{\partial x^4} + D_y \frac{\partial^4 W}{\partial y^4} + 2H \frac{\partial^4 W}{\partial x^2 \partial y^2} + 2\frac{\partial H}{\partial x}\frac{\partial^3 W}{\partial x \partial y^2} + 2\frac{\partial H}{\partial y}\frac{\partial^3 W}{\partial x^2 \partial y} + 2\frac{\partial D_x}{\partial x}\frac{\partial^3 W}{\partial x^3} + 2\frac{\partial D_y}{\partial y}\frac{\partial^3 W}{\partial y^3} + \frac{\partial^2 D_x}{\partial x^2}\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 D_y}{\partial y^2}\frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 D_1^{'}}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 D_1^{'}}{\partial y^2}\frac{\partial^2 W}{\partial x^2} + 4\frac{\partial^2 D_{xy}}{\partial x \partial y}\frac{\partial^2 W}{\partial x \partial y}] / \rho h W = -\frac{\ddot{T}}{\tilde{D} T} \quad \textbf{(1.5)}$$

Here dot denotes the differentiation with respect to t.

Equation (1.5) is satisfied if both its sides are equal to a constant. Denoting this constant by $p^2$, we get two equations

$$[D_x \frac{\partial^4 W}{\partial x^4} + D_y \frac{\partial^4 W}{\partial y^4} + 2H \frac{\partial^4 W}{\partial x^2 \partial y^2} + 2\frac{\partial H}{\partial x}\frac{\partial^3 W}{\partial x \partial y^2} + 2\frac{\partial H}{\partial y}\frac{\partial^3 W}{\partial x^2 \partial y} + 2\frac{\partial D_x}{\partial x}\frac{\partial^3 W}{\partial x^3} + 2\frac{\partial D_y}{\partial y}\frac{\partial^3 W}{\partial y^3} + \frac{\partial^2 D_x}{\partial x^2}\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 D_y}{\partial y^2}\frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 D_1^{'}}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 D_1^{'}}{\partial y^2}\frac{\partial^2 W}{\partial x^2} + 4\frac{\partial^2 D_{xy}}{\partial x \partial y}\frac{\partial^2 W}{\partial x \partial y}] - \rho h p^2 W = 0 \quad \textbf{(1.6)}$$

and

$$\ddot{T} + p^2 \tilde{D} T = 0 \quad \textbf{(1.7)}$$

Equation (1.6) is a differential equation of motion for orthotropic rectangular plate of variable thickness and (1.7) is a differential equation of time functions of free vibration of viscoelastic rectangular orthotropic plate.

Assuming that the thickness and non-homogeneity varies linearly in x-direction only. Consequently, the thickness h, flexural rigidity $D_x$ and $D_y$ & torsional rigidity $D_{xy}$ of plate become function of x only. Further let the two opposite edges, y=0 and y=b of the plate be simply supported so that the free transverse vibrations of the plate can be expressed as

$$W(x,y) = W_1(x) Sin(\frac{\pi y}{b}) \quad \textbf{(1.8)}$$

Using equation (1.8) in (1.6) and simplifying, we have

$$D_x \frac{\partial^4 W_1}{\partial x^4} + 2\frac{\partial D_x}{\partial x}\frac{\partial^3 W_1}{\partial x^3} + [\frac{\partial^2 D_x}{\partial x^2} - 2H\frac{\pi^2}{b^2}]\frac{\partial^2 W_1}{\partial x^2} - 2[v_y \frac{\partial D_x}{\partial x} + 2\frac{\partial D_{xy}}{\partial x}]\frac{\pi^2}{b^2}\frac{\partial W_1}{\partial x} + [D_y \frac{\pi^4}{b^4} - \frac{\partial^2 D_1^{'}}{\partial x^2}\frac{\pi^2}{b^2}]W_1 - \rho h p^2 W_1 = 0 \quad \textbf{(1.9)}$$


Licensed Under Creative Commons Attribution CC BY

Thus equation (1.9) reduces to a form independent of y and on introducing the non-dimensional variables

$$\bar{H} = \frac{h}{a}, \quad \bar{W} = \frac{W_1}{a}, \quad X = \frac{x}{a}, \quad D_X = \frac{D_x}{a^3}, \quad D_Y = \frac{D_y}{a^3}$$

into equation (1.9), it becomes in non-dimensional form

$$D_X \frac{\partial^4 \bar{W}}{\partial X^4} + 2\frac{\partial D_X}{\partial X}\frac{\partial^3 \bar{W}}{\partial X^3} + [\frac{\partial^2 D_X}{\partial X^2} - 2r^2\{v_y D_X + G_{XY}\frac{\bar{H}^3}{6}\}]\frac{\partial^2 \bar{W}}{\partial X^2}$$
$$- 2r^2[v_y \frac{\partial D_X}{\partial X} + \frac{G_{XY}}{6}\frac{\partial(\bar{H}^3)}{\partial X}]\frac{\partial \bar{W}}{\partial X} + r^2\,[r^2 D_Y - v_y \frac{\partial^2 D_X}{\partial X^2}]\bar{W} - \rho \bar{H} a^2 p^2 \bar{W} = 0 \qquad (1.10)$$

Where

$$r^2 = \left(\frac{\pi a}{b}\right)^2$$

Let the thickness variation of the plate is

$$\bar{H}(X) = H_0(1+\beta X) \qquad \textbf{(1.11)}$$

where $\beta$ is taper constant and $H_0 = \bar{H}\big|_{X=0}$

and the modulus variation are

$$\left.\begin{array}{l} E_x(X) = E_1(1+\alpha X) \\ E_y(X) = E_2(1+\alpha X) \end{array}\right] \qquad \textbf{(1.12)}$$

and the density varies as

$$\rho = \rho_0(1+\alpha X) \qquad \textbf{(1.13)}$$

where $\alpha$ is non-homogeneous parameter and

$$\rho_0 = \rho\big|_{X=0}, E_1 = E_x\big|_{X=0}, E_2 = E_y\big|_{X=0}$$

Considering equation (1.11), (1.12) and (1.13), the expression for rigidities comes out as

$$\left.\begin{array}{l} D_X = D_0(1+\alpha X)(1+\beta X)^3 \\ D_Y = D_1(1+\alpha X)(1+\beta X)^3 \end{array}\right] \qquad \textbf{(1.14)}$$

where

$$D_0 = \frac{E_1 H_0^3}{12(1-v_x v_y)},$$

$$D_1 = \frac{E_2 H_0^3}{12(1-v_x v_y)}$$

Substituting equation (1.11), (1.13) and (1.14) into (1.10), the differential equation takes the form

$$B_1 \frac{\partial^4 \bar{W}}{\partial X^4} + B_2 \frac{\partial^3 \bar{W}}{\partial X^3} + B_3 \frac{\partial^2 \bar{W}}{\partial X^2} + B_4 \frac{\partial \bar{W}}{\partial X} + [B_5 - p^2\lambda^2(1+\alpha X)]\bar{W} = 0 \quad \textbf{(1.15)}$$

where

$$B_1 = [1+\alpha X](1+\beta X)^2,$$

$$B_2 = 2[\alpha(1+\beta X)^2 + 3\beta(1+\beta X)(1+\alpha X)],$$

$$B_3 = 6\alpha\beta(1+\beta X) + 6\beta^2(1+\alpha X) - 2r^2 v_y(1+\beta X)^2(1+\alpha X) - 4r^2\frac{G_{XY}}{E_1}(1+\beta X)^2(1-v_x v_y),$$

$$B_4 = -2r^2[v_y\{\alpha(1+\beta X)^2 + 3\beta(1+\beta X)(1+\alpha X)\} + 6\frac{G_{XY}}{E_1}\beta(1+\beta X)(1-v_x v_y)],$$

$$B_5 = r^4\frac{E_2}{E_1}(1+\alpha X)(1+\beta X)^2 - 6r^2 v_y \alpha\beta(1+\beta X) - 6r^2 v_y \beta^2(1+\alpha X),$$

$$\lambda^2 = \frac{12(1-v_x v_y)\rho_0 a^4}{E_1 H_0^2}$$

and $p$ is a frequency parameter.

## 3. Solution of Free Vibration of Rectangular Plate

Let the deflection function $\bar{W}(X)$ of the plate be assumed to be a finite sum of characteristic functions $\bar{W}_k(X)$

$$\bar{W}(X) = \sum_{k=1}^{n} A_k \bar{W}_k(X) \quad \textbf{(1.16)}$$

where $A_k$'s are the undetermined coefficients and $\bar{W}_k(X)$ are the characteristic functions chosen to satisfy the boundary conditions of the plate.


Licensed Under Creative Commons Attribution CC BY

For a rectangular clamped plate at both the edges X = 0 and X = 1, boundary conditions are that the deflection and the slope of the plate must be zero i.e.

$$\left.\bar{W}\right|_{X=0} = \left.\frac{\partial \bar{W}}{\partial X}\right|_{X=0} = 0$$

$$\left.\bar{W}\right|_{X=1} = \left.\frac{\partial \bar{W}}{\partial X}\right|_{X=1} = 0$$

**(1.17)**

Using Galerkin's technique, we requires that

$$\int_R L[\bar{W}(X)]\bar{W}(X)\,dX = 0 \qquad \textbf{(1.18)}$$

where $L[\bar{W}(X)]$ is the left hand side of equation (1.15). Taking the first two terms of series (1.16) for the function $\bar{W}(X)$ as the solution of equation (1.15), one has

$$\bar{W}(X) = X^2(1-X^2)[A_1 + A_2 X(1-X)] \qquad \textbf{(1.19)}$$

where $A_1$ and $A_2$ are undetermined coefficients.
Using equation (1.15) and (1.19) in equation (1.18) and then eliminating $A_1$ and $A_2$, gives the frequency equation as

$$\begin{vmatrix} F_1 & F_2 \\ F_2 & F_3 \end{vmatrix} = 0 \qquad \textbf{(1.20)}$$

where

$$F_1 = [\frac{4}{5} + \frac{2}{5}\alpha + \frac{4}{5}\beta + \frac{2}{7}\beta^2 + \frac{22}{35}\alpha\beta + \frac{9}{35}\alpha\beta^2 + r^2 v_y(\frac{4}{105} + \frac{2}{105}\alpha + \frac{4}{105}\beta + \frac{1}{210}\beta^2 + \frac{11}{630}\alpha\beta + \frac{1}{630}\alpha\beta^2) + 4r^2\frac{G_{xy}}{E_1}(1 - v_x v_y)(\frac{2}{105} + \frac{2}{105}\beta + \frac{3}{420}\beta^2) + r^4\frac{E_2}{E_1}(\frac{1}{630} + \frac{1}{1260}\alpha + \frac{1}{630}\beta + \frac{1}{2310}\beta^2 + \frac{1}{1155}\alpha\beta + \frac{1}{3960}\alpha\beta^2) - (\frac{1}{630} + \frac{1}{1260}\alpha)p^2\lambda^2],$$

$$F_2 = [\frac{12}{35} + \frac{6}{35}\alpha + \frac{12}{35}\beta + \frac{11}{105}\beta^2 + \frac{5}{21}\alpha\beta + \frac{3}{35}\alpha\beta^2 + r^2 v_y(\frac{2}{105} + \frac{1}{105}\alpha + \frac{2}{105}\beta + \frac{17}{6930}\beta^2 + \frac{59}{6930}\alpha\beta + \frac{1}{1386}\alpha\beta^2) + 4r^2\frac{G_{xy}}{E_1}(1 - v_x v_y)(\frac{1}{105} + \frac{1}{105}\beta + \frac{47}{13860}\beta^2) + r^4\frac{E_2}{E_1}(\frac{1}{1386} + \frac{1}{2772}\alpha + \frac{1}{1386}\beta + \frac{1}{5148}\beta^2 + \frac{1}{2574}\alpha\beta + \frac{1}{9009}\alpha\beta^2) - (\frac{1}{1386} + \frac{1}{2772}\alpha)p^2\lambda^2],$$

$$F_3 = [\frac{2}{35} + \frac{1}{35}\alpha + \frac{2}{35}\beta + \frac{13}{770}\beta^2 + \frac{29}{770}\alpha\beta + \frac{1}{77}\alpha\beta^2 + r^2 v_y(\frac{1}{385} + \frac{1}{770}\alpha + \frac{1}{385}\beta + \frac{23}{60060}\beta^2 + \frac{71}{60060}\alpha\beta + \frac{2}{15015}\alpha\beta^2) + 4r^2\frac{G_{xy}}{E_1}(1 - v_x v_y)(\frac{1}{770} + \frac{1}{770}\beta + \frac{53}{120120}\beta^2) + r^4\frac{E_2}{E_1}(\frac{1}{12012} + \frac{1}{24024}\alpha + \frac{1}{12012}\beta + \frac{1}{45045}\beta^2 + \frac{2}{45045}\alpha\beta + \frac{1}{80080}\alpha\beta^2) - (\frac{1}{12012} + \frac{1}{24024}\alpha)p^2\lambda^2],$$

The frequency equation (1.20) is a quadratic equation in $p^2$ from which the two values of $p^2$ can be found.Hence deflection function $\bar{W}(X)$ can be obtained from equation (1.19) after determining constants $A_1$ and $A_2$. Choosing $A_1$ = 1, we obtains $A_2 = -\frac{F_1}{F_2}$ and then $\bar{W}(X)$ comes out as

$$\bar{W}(X) = X^2(1-X)^2[1 - \frac{F_1}{F_2}X(1-X)] \qquad \textbf{(1.21)}$$

## 4. Time Functions of Vibrations of Viscoelastic Orthotropic Plates

Time functions of free vibrations of viscoelastic orthotropic plates are defined by the general ordinary differential equation (1.7). Their form depends on the viscoelastic operator $\tilde{D}$.
We have taken Kelvin's model, for which

$$\tilde{D} \equiv (1 + \frac{\eta}{G}\frac{d}{dt}) \qquad \textbf{(1.22)}$$

where $G$ is shear modulus and $\eta$ is viscoelastic constant. Takes variation of G and $\eta$ as


Licensed Under Creative Commons Attribution CC BY

$$\left.\begin{aligned} G(X) &= G_0[1+\alpha_1 X] \\ \eta(X) &= \eta_0[1+\alpha_2 X] \end{aligned}\right] \quad \textbf{(1.23)}$$

where $\alpha_1$ and $\alpha_2$ are different non-homogeneous parameters and $G_0 = G|_{X=0}, \eta_0 = \eta|_{X=0}$.

Using equation (1.23) in (1.22), we get

$$\tilde{D} \equiv 1 + s\frac{d}{dt} \quad \textbf{(1.24)}$$

where

$$s = \frac{\eta_0[1+\alpha_2 X]}{G_0[1+\alpha_1 X]} \quad \textbf{(1.25)}$$

Using equation (1.24) in equation (1.7), we obtains

$$\ddot{T} + p^2 s\dot{T} + p^2 T = 0 \quad \textbf{(1.26)}$$

Equation (1.26) is a differential equation of second order for time function T.
On solving equation (1.26), its solution comes out as

$$T(t) = e^{(\frac{-p^2 st}{2})}[c_1 \cos at + c_2 \sin at] \quad \textbf{(1.27)}$$

$$\text{Where} \quad a^2 = p^2 - \frac{1}{4}p^4 s^2 \quad \textbf{(1.28)}$$

and $c_1, c_2$ are constants to be determined from initial conditions of the plate which assume as $T = 1$ and $\dot{T} = 0$ $at$ $t = 0$ **(1.29)**

Using condition (1.29) in equation (1.27), we obtains

$$T(t) = e^{(\frac{-p^2 st}{2})}[\cos at + \frac{p^2 s}{2a}\sin at] \quad \textbf{(1.30)}$$

Hence, deflection $w(x, y, t)$ may be expressed from equation (1.4), (1.8), (1.21) and (1.30), as

$$w(x, y, t) = W_1(x)\sin(\frac{\pi y}{b})e^{(\frac{-p^2 st}{2})}[\cos at + \frac{p^2 s}{2a}\sin at]$$

**(1.31)**

Time period of the vibration of the plate is given by

$$K = \frac{2\pi}{p} \quad \textbf{(1.32)}$$

where $p$ is the frequency given by equation (1.20).
Logarithmic decrement of the vibration is given by the standard formula

$$\wedge = \log_e(\frac{w_2}{w_1}) \quad \textbf{(1.33)}$$

where $w_1$ the deflection at any is point of the plate at a time period $K = K_1$ and $w_2$ is the deflection at the same point at the time period succeeding $K_1$.

## 5. Results and Discussions

Time period *K*, Deflection *w* and Logarithmic decrement $\wedge$ are computed for a clamped viscoelastic orthotropic rectangular plate of linearly varying thickness for different values of non-homogeneous parameters α, $\alpha_1$, $\alpha_2$ and taper constant β and aspect ratio a/b at different points for first two modes of vibrations. All these results are presented in the tables 1.1 to 1.15 and graphically shown in figures from 1.1 to 1.15.

For the numerical computation, the following orthotropic material parameters are used:

$$\frac{E_2}{E_1} = .01, \quad \frac{G_{xy}(1-\nu_x\nu_y)}{E_1} = 0.0333$$

$$\nu_y = 0.3, \quad \frac{E_1}{(1-\nu_x\nu_y)\rho_0} = 3\times10^5$$

$$\frac{\eta_0}{G_0} = 0.000069, \ H_0 = 0.01\, meter$$

In tables 1.1–1.3: results of time period *K* for first two modes of vibrations for all *X*, *Y* and $\alpha_1$, $\alpha_2$ are given as follows:

Table 1.1: Different non-homogeneous parameter $\alpha$ and fixed aspect ratio $\frac{a}{b}$ =1.5 for two values of taper constant β i.e. β=0.0 and β=0.4

Table 1.2: Different taper constant β and fixed aspect ratio $\frac{a}{b}$ =1.5 for two values of non-homogeneous parameter $\alpha$ i.e. α = 0.0 and α = 0.4

Table 1.3: Different aspect ratio $\frac{a}{b}$ and four combination of non-homogeneous parameter $\alpha$ and taper constant β i.e. α =0.0, β =0.0; α = 0.0, β=0.4; α = 0.4, β=0.0 and α =0.4, β=0.4

Table 1.1: shows that as non-homogeneous parameter α increase time period *K* of vibration also increases for uniform plate i.e. for β=0.0 for both the modes of vibration and decreases for non-uniform plate i.e. β=0.4 for both the modes of vibration. Figure 1.1 shows the effect of non-homogeneous parameter on time period *K*. It is clearly observed in figure1.1 that there is a steady increase in time period K with increase of non-homogeneous parameter α for β=0.0.

Tables 1.2 and 1.3 shows that as taper constant β and aspect ratio a/b increase respectively, time period K decrease for the first two modes of vibration. It is clearly shown in figures 1.2 and 1.3 that there is a steady decrease in time period K with increase of taper constant β and aspect ratio a/b respectively.

In tables 1.4 – 1.11 results of deflection for the first two modes of vibrations for different X, Y and a fixed aspect


Licensed Under Creative Commons Attribution CC BY

ratio a/b = 1.5 for initial time 0.K and time 5.K are given for the following combination of α, $\beta\alpha_1$ and $\alpha_2$:

$$\alpha = 0.0\,,\ \beta = 0.0\,,\ \alpha_1 = 0.0\,,\ \alpha_2 = 0.0$$
$$\alpha = 0.4\,,\ \beta = 0.4\,,\ \alpha_1 = 0.0\,,\ \alpha_2 = 0.0$$
$$\alpha = 0.0\,,\ \beta = 0.0\,,\ \alpha_1 = 0.4\,,\ \alpha_2 = 0.2$$
$$\alpha = 0.4\,,\ \beta = 0.4\,,\ \alpha_1 = 0.4\,,\ \alpha_2 = 0.2$$

It can be seen from tables 1.4 – 1.11 that deflection $w$ starts from zero to increase and then decrease to zero for the first mode of vibration while for the second mode of vibration, deflection $w$ start from zero to increase then decrease then increase and finally come to zero, for fixed Y and different value of X for time 0.K and 5.K.

It is also note that for fixed X, deflection $w$ start from zero to increase and then decrease in both modes of vibration for both time 0.K and 5.K for different values of Y.

One can conclude also that deflection $w$ decrease for time increase for both the modes of vibration. These results are plotted in figures 1.4 to 1.11.

In tables 1.12 - 1.15 results of logarithmic decrement $\wedge$ for first two modes of vibration for different X, Y and constant aspect ratio a/b = 1.5 are given for the following four cases:

$$\alpha = 0.0\,,\ \beta = 0.0\,,\ \alpha_1 = 0.0\,,\ \alpha_2 = 0.0$$
$$\alpha = 0.4\,,\ \beta = 0.4\,,\ \alpha_1 = 0.0\,,\ \alpha_2 = 0.0$$
$$\alpha = 0.0\,,\ \beta = 0.0\,,\ \alpha_1 = 0.4\,,\ \alpha_2 = 0.2$$
$$\alpha = 0.4\,,\ \beta = 0.4\,,\ \alpha_1 = 0.4\,,\ \alpha_2 = 0.2$$

It is interesting observed that the logarithmic decrement $\wedge$ is constant across the plate for $\alpha = 0.0\,,\ \beta = 0.0\,,\ \alpha_1 = 0.0\,,\ \alpha_2 = 0.0$ and $\alpha = 0.4\,,\ \beta = 0.4\,,\ \alpha_1 = 0.0\,,\ \alpha_2 = 0.0$ while increase for $\alpha = 0.0\,,\ \beta = 0.0\,,\ \alpha_1 = 0.4\,,\ \alpha_2 = 0.2$ and $\alpha = 0.4\,,\ \beta = 0.4\,,\ \alpha_1 = 0.4\,,\ \alpha_2 = 0.2$ for different values of X and fixed value of Y. But it is same for fixed value of X and different value of Y. These results are plotted in figures 1.12 to 1.15.

**Table 1:** Time period K (in seconds) for different non-homogeneous parameter (α) and a constant aspect ratio (a/b = 1.5) for all X, Y and α1, α2

| α | β = 0.0 | | β = 0.4 | |
|---|---|---|---|---|
| | First Mode | Second Mode | First Mode | Second Mode |
| 0.0 | 0.149813 | 0.029783 | 0.124489 | 0.024648 |
| 0.2 | 0.150161 | 0.029804 | 0.124079 | 0.024472 |
| 0.4 | 0.150452 | 0.029822 | 0.123746 | 0.024328 |
| 0.6 | 0.150700 | 0.029837 | 0.123470 | 0.024208 |
| 0.8 | 0.150913 | 0.029849 | 0.123237 | 0.024107 |

**Table 1.2:** Time period K (in seconds) for different taper constant (β) and a constant aspect ratio (a/b = 1.5) for all X, Y and α1, α2

| β | α = 0.0 | | α = 0.4 | |
|---|---|---|---|---|
| | First Mode | Second Mode | First Mode | Second Mode |
| 0.0 | 0.149813 | 0.029783 | 0.150452 | 0.029822 |
| 0.2 | 0.136078 | 0.027020 | 0.135881 | 0.026840 |
| 0.4 | 0.124489 | 0.024648 | 0.123746 | 0.024328 |
| 0.6 | 0.114620 | 0.022611 | 0.113521 | 0.022202 |
| 0.8 | 0.106137 | 0.020852 | 0.104809 | 0.020388 |

**Table 1.3:** Time period K (in seconds) for different aspect ratio (a/b) for all X, Y and α1, α2

| a/b | α =0.0, β =0.0 | | α =0.0, β =0.4 | | α =0.4, β =0.0 | | α =0.4, β =0.4 | |
|---|---|---|---|---|---|---|---|---|
| | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode |
| 0.5 | 0.173851 | 0.030987 | 0.144168 | 0.025637 | 0.173963 | 0.030992 | 0.142311 | 0.025259 |
| 1.0 | 0.163628 | 0.030520 | 0.135814 | 0.025254 | 0.164000 | 0.030538 | 0.134483 | 0.024898 |
| 1.5 | 0.149813 | 0.029783 | 0.124489 | 0.024648 | 0.150452 | 0.029822 | 0.123746 | 0.024328 |
| 2.0 | 0.135004 | 0.028830 | 0.112305 | 0.023866 | 0.135828 | 0.028893 | 0.112045 | 0.023588 |
| 2.5 | 0.120795 | 0.027723 | 0.100575 | 0.022955 | 0.121708 | 0.027810 | 0.100648 | 0.022723 |

**Table 1.4:** Deflection w for different X, Y and α =0.0, β =0.0, α1 =0.0, α2 = 0.0 and a/b = 1.5 at initial time 0.K

| X | Y = 0.2 | | Y = 0.4 | | Y = 0.6 | | Y= 0.8 | |
|---|---|---|---|---|---|---|---|---|
| | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode |
| 0.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.2 | 0.015438 | 0.004461 | 0.024975 | 0.007217 | 0.024965 | 0.007214 | 0.015412 | 0.004454 |
| 0.4 | 0.035170 | -0.001877 | 0.056897 | -0.003037 | 0.056873 | -0.003036 | 0.035109 | -0.001874 |
| 0.6 | 0.035170 | -0.001877 | 0.056897 | -0.003037 | 0.056873 | -0.003036 | 0.035109 | -0.001874 |
| 0.8 | 0.015438 | 0.004461 | 0.024975 | 0.007217 | 0.024965 | 0.007214 | 0.015412 | 0.004454 |
| 1.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |


Licensed Under Creative Commons Attribution CC BY

**Table 1.5:** Deflection w for different X, Y and α =0.0, β =0.0, $\alpha_1$ =0.0, $\alpha_2$ = 0.0 and a/b = 1.5 at time 5.K

| X | Y = 0.2 | | Y = 0.4 | | Y = 0.6 | | Y= 0.8 | |
|---|---|---|---|---|---|---|---|---|
| | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode |
| 0.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.2 | 0.014751 | 0.003549 | 0.023863 | 0.005741 | 0.023853 | 0.005739 | 0.014725 | 0.003543 |
| 0.4 | 0.033604 | -0.001493 | 0.054363 | -0.002416 | 0.054341 | -0.002415 | 0.033546 | -0.001491 |
| 0.6 | 0.033604 | -0.001493 | 0.054363 | -0.002416 | 0.054341 | -0.002415 | 0.033546 | -0.001491 |
| 0.8 | 0.014751 | 0.003549 | 0.023863 | 0.005741 | 0.023853 | 0.005739 | 0.014725 | 0.003543 |
| 1.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |

**Table 1.6:** Deflection w for different X, Y and α =0.4, β =0.4, $\alpha_1$ =0.0, $\alpha_2$ = 0.0 and a/b = 1.5 at initial time 0.K

| X | Y = 0.2 | | Y = 0.4 | | Y = 0.6 | | Y= 0.8 | |
|---|---|---|---|---|---|---|---|---|
| | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode |
| 0.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.2 | 0.015847 | 0.004467 | 0.025636 | 0.007226 | 0.025625 | 0.007223 | 0.015819 | 0.004459 |
| 0.4 | 0.036548 | -0.001858 | 0.059126 | -0.003006 | 0.059102 | -0.003005 | 0.036485 | -0.001855 |
| 0.6 | 0.036548 | -0.001858 | 0.059126 | -0.003006 | 0.059102 | -0.003005 | 0.036485 | -0.001855 |
| 0.8 | 0.015847 | 0.004467 | 0.025636 | 0.007226 | 0.025625 | 0.007223 | 0.015819 | 0.004459 |
| 1.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |

**Table 1.7:** Deflection w for different X, Y and α =0.4, β =0.4, $\alpha_1$ =0.0, $\alpha_2$ = 0.0 and a/b = 1.5 at time 5.K

| X | Y = 0.2 | | Y = 0.4 | | Y = 0.6 | | Y= 0.8 | |
|---|---|---|---|---|---|---|---|---|
| | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode |
| 0.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.2 | 0.014997 | 0.003376 | 0.024261 | 0.005461 | 0.024251 | 0.005459 | 0.014971 | 0.003370 |
| 0.4 | 0.034588 | -0.001404 | 0.055954 | -0.002272 | 0.055931 | -0.002271 | 0.034528 | -0.001402 |
| 0.6 | 0.034588 | -0.001404 | 0.055954 | -0.002272 | 0.055931 | -0.002271 | 0.034528 | -0.001402 |
| 0.8 | 0.014997 | 0.003376 | 0.024261 | 0.005461 | 0.024251 | 0.005459 | 0.014971 | 0.003370 |
| 1.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |

**Table 1.8:** Deflection w for different X, Y and α =0.0, β =0.0, $\alpha_1$ =0.4, $\alpha_2$ = 0.2 and a/b = 1.5 at initial time 0.K

| X | Y = 0.2 | | Y = 0.4 | | Y = 0.6 | | Y= 0.8 | |
|---|---|---|---|---|---|---|---|---|
| | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode |
| 0.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.2 | 0.015438 | 0.004461 | 0.024975 | 0.007217 | 0.024965 | 0.007214 | 0.015412 | 0.004454 |
| 0.4 | 0.035170 | -0.001877 | 0.056897 | -0.003037 | 0.056873 | -0.003036 | 0.035109 | -0.001874 |
| 0.6 | 0.035170 | -0.001877 | 0.056897 | -0.003037 | 0.056873 | -0.003036 | 0.035109 | -0.001874 |
| 0.8 | 0.015438 | 0.004461 | 0.024975 | 0.007217 | 0.024965 | 0.007214 | 0.015412 | 0.004454 |
| 1.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |

**Table 1.9:** Deflection w for different X, Y and α =0.0, β =0.0, $\alpha_1$ =0.4, $\alpha_2$ = 0.2 and a/b = 1.5 at time 5.K

| X | Y = 0.2 | | Y = 0.4 | | Y = 0.6 | | Y= 0.8 | |
|---|---|---|---|---|---|---|---|---|
| | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode |
| 0.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.2 | 0.014776 | 0.003579 | 0.023903 | 0.005790 | 0.023893 | 0.005788 | 0.014750 | 0.003573 |
| 0.4 | 0.033710 | -0.001517 | 0.054534 | -0.002454 | 0.054511 | -0.002453 | 0.033651 | -0.001514 |
| 0.6 | 0.033752 | -0.001527 | 0.054603 | -0.002470 | 0.054580 | -0.002469 | 0.033694 | -0.001524 |
| 0.8 | 0.014832 | 0.003649 | 0.023995 | 0.005902 | 0.023985 | 0.005900 | 0.014807 | 0.003642 |
| 1.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |

**Table 1.10:** Deflection w for different X, Y and α =0.4, β =0.4, $\alpha_1$ =0.4, $\alpha_2$ = 0.2 and a/b = 1.5 at initial time 0.K

| X | Y = 0.2 | | Y = 0.4 | | Y = 0.6 | | Y= 0.8 | |
|---|---|---|---|---|---|---|---|---|
| | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode |
| 0.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.2 | 0.015847 | 0.004467 | 0.025636 | 0.007226 | 0.025625 | 0.007223 | 0.015819 | 0.004459 |
| 0.4 | 0.036548 | -0.001858 | 0.059126 | -0.003006 | 0.059102 | -0.003005 | 0.036485 | -0.001855 |
| 0.6 | 0.036548 | -0.001858 | 0.059126 | -0.003006 | 0.059102 | -0.003005 | 0.036485 | -0.001855 |
| 0.8 | 0.015847 | 0.004467 | 0.025636 | 0.007226 | 0.025625 | 0.007223 | 0.015819 | 0.004459 |
| 1.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |

**Table 1.11:** Deflection w for different X, Y and α =0.4, β =0.4, $\alpha_1$ =0.4, $\alpha_2$ = 0.2 and a/b = 1.5 at time 5.K

| X | Y = 0.2 | | Y = 0.4 | | Y = 0.6 | | Y= 0.8 | |
|---|---|---|---|---|---|---|---|---|
| | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode |
| 0.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 0.2 | 0.015027 | 0.003411 | 0.024310 | 0.005518 | 0.024300 | 0.005516 | 0.015001 | 0.003405 |


Licensed Under Creative Commons Attribution CC BY

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0.4 | 0.034720 | -0.001432 | 0.056167 | -0.002316 | 0.056144 | -0.002315 | 0.034659 | -0.001429 |
| 0.6 | 0.034773 | -0.001443 | 0.056253 | -0.002334 | 0.056230 | -0.002333 | 0.034712 | -0.001440 |
| 0.8 | 0.015097 | 0.003492 | 0.024423 | 0.005649 | 0.024413 | 0.005647 | 0.015071 | 0.003486 |
| 1.0 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |

**Table 1.12:** Logarithmic decrement Λ for different X, Y and α =0.0, β =0.0, $\alpha_1$ =0.0, $\alpha_2$ = 0.0 and a/b = 1.5

| X | Y = 0.2 | | Y = 0.4 | | Y = 0.6 | | Y= 0.8 | |
|---|---|---|---|---|---|---|---|---|
| | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode |
| 0.2 | -0.009098 | -0.045753 | -0.009098 | -0.045753 | -0.009098 | -0.045753 | -0.009098 | -0.045753 |
| 0.4 | -0.009098 | -0.045753 | -0.009098 | -0.045753 | -0.009098 | -0.045753 | -0.009098 | -0.045753 |
| 0.6 | -0.009098 | -0.045753 | -0.009098 | -0.045753 | -0.009098 | -0.045753 | -0.009098 | -0.045753 |
| 0.8 | -0.009098 | -0.045753 | -0.009098 | -0.045753 | -0.009098 | -0.045753 | -0.009098 | -0.045753 |

**Table 1.13:** Logarithmic decrement Λ for different X, Y and α =0.4, β =0.4, $\alpha_1$ =0.0, $\alpha_2$ = 0.0 and a/b = 1.5

| X | Y = 0.2 | | Y = 0.4 | | Y = 0.6 | | Y= 0.8 | |
|---|---|---|---|---|---|---|---|---|
| | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode |
| 0.2 | -0.011014 | -0.056012 | -0.011014 | -0.056012 | -0.011014 | -0.056012 | -0.011012 | -0.056012 |
| 0.4 | -0.011014 | -0.056012 | -0.011014 | -0.056012 | -0.011014 | -0.056012 | -0.011012 | -0.056012 |
| 0.6 | -0.011014 | -0.056012 | -0.011014 | -0.056012 | -0.011014 | -0.056012 | -0.011012 | -0.056012 |
| 0.8 | -0.011014 | -0.056012 | -0.011014 | -0.056012 | -0.011014 | -0.056012 | -0.011012 | -0.056012 |

**Table 1.14:** Logarithmic decrement Λ for different X, Y and α =0.0, β =0.0, $\alpha_1$ =0.4, $\alpha_2$ = 0.2 and a/b = 1.5

| X | Y = 0.2 | | Y = 0.4 | | Y = 0.6 | | Y= 0.8 | |
|---|---|---|---|---|---|---|---|---|
| | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode |
| 0.2 | -0.008761 | -0.044059 | -0.008761 | -0.044059 | -0.008761 | -0.044059 | -0.008761 | -0.044059 |
| 0.4 | -0.008471 | -0.042598 | -0.008471 | -0.042598 | -0.008471 | -0.042598 | -0.008471 | -0.042598 |
| 0.6 | -0.008218 | -0.041326 | -0.008218 | -0.041326 | -0.041326 | -0.008218 | -0.041326 | -0.008218 |
| 0.8 | -0.007996 | -0.040208 | -0.007996 | -0.040208 | -0.007996 | -0.040208 | -0.007996 | -0.040208 |

**Table 1.15:** Logarithmic decrement Λ for different X, Y and α =0.4, β =0.4, $\alpha_1$ =0.4, $\alpha_2$ = 0.2 and a/b = 1.5

| X | Y = 0.2 | | Y = 0.4 | | Y = 0.6 | | Y= 0.8 | |
|---|---|---|---|---|---|---|---|---|
| | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode | First Mode | Second Mode |
| 0.2 | -0.010606 | -0.053937 | -0.010606 | -0.053937 | -0.010606 | -0.053937 | -0.010606 | -0.053937 |
| 0.4 | -0.010255 | -0.052149 | -0.010255 | -0.052149 | -0.010255 | -0.052149 | -0.010255 | -0.052149 |
| 0.6 | -0.009949 | -0.050591 | -0.009949 | -0.050591 | -0.009949 | -0.050591 | -0.009949 | -0.050591 |
| 0.8 | -0.009679 | -0.049223 | -0.009679 | -0.049223 | -0.009679 | -0.049223 | -0.009679 | -0.049223 |

## 6. Conclusion

In conclusion, the research presents significant insights into the vibrational behavior of orthotropic visco-elastic rectangular plates with linear thickness variation, under the influence of non-homogeneity. The study findings highlight the intricate relationship between the non-homogeneous parameters, taper constants, and aspect ratios, and their collective impact on the plates vibration characteristics. It emphasizes the importance of considering these factors in the design and analysis of structures subjected to dynamic stress, particularly in high-temperature environments. This research paves the way for further exploration in the field of material science and structural engineering, especially in applications involving extreme conditions such as those encountered in aerospace engineering.

## References

[1] Abu, A.I., Turhan, D and Mengis, D. 'Two-Dimensional Transient Wave Propagation inViscoelastic Layered Media', J. Sound and Vibration, Vol. 244, No. 5, Pp. 837-858, 2001.

[2] Aksu, G'Dynamic Analysis of Orthotropic Plates Using A Finite Difference Formulation', Ph. D. Thesis, Loughorough Univ., 1974.

[3] Amabili, M. and Garziera, R. 'Transverse Vibrations of Circular, Annular Plates with Several Combinations of Boundary Conditions', J. Sound and Vibration, Vol. 228, No. 2, Pp. 443-446, 1999.

[4] Appl., F.C. and Byers, N.R. 'Fundamental Frequency of Simply – Supported Rectangular Plates with Linearly Varying Thickness', J.Appl. Mach., Trans.ASME, Vol. 32, No. 1 Pp. 163-167, 1965.

[5] Avalos, D.R., Larrondo, H.A. and Laura, P.A.A. 'Analysis of Vibrating Rectangular Anisotropic Plates with Free-Edge Holes', J. Sound and Vibration, Vol. 222, No. 4, Pp. 691-695, 1999.

[6] Avalos, D.R. and Laura, P.A.A. 'Transverse Vibrations of a Simply Supported Plate of Generalized Anisotropy with an Oblique Cut-Out', J. Sound and Vibration, Vol. 258, No. 4, Pp. 773-776, 2002.

[7] Bala Subrahmanyan, P. and Sujith R.I.'Exact Solution for Ax symmetric Vibration of Solid Circular and Annular Membranes with Continuously Varying Density', J. Sound and Vibration, Vol. 248, No. 2, Pp. 371-378, 2001.

[8] Bambill, D.V., C.A., Laura, P.A.A. and Rossi, R.E.'Transverse Vibration of An Orthotropic Rectangular Plate of Linearly Varying Thickness and With a Free Edge', J. Sound and Vibration, Vol. 235, No. 3, Pp. 530-538, 2000.


Licensed Under Creative Commons Attribution CC BY

[9] Bambill, D.V. and Laura, P.A.A. 'Fundamental Frequency of Transverse Vibration of a Clamped Rectangular Plate of Cylindrical Orthotropic', J. Sound and Vibration, Vol. 220, No. 3, Pp. 571-576, 1999.


Licensed Under Creative Commons Attribution CC BY