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73 Is the Only Largest Prime Power Number and Composite Power Numbers

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Abstract: ***In natural numbers we don't have largest one till now. The X-power set of a prime number which has only prime numbers. In this we are introducing the largest one among such prime numbers and also, we are introducing the composite X-power set and composite power number. Along with this we are presenting the program for getting X -power set and order of X -power set of a positive integer. In this paper we have presented "There is no other prime power number which is greater than 73 and composite power numbers "or" 73 is the only largest prime power number and composite power numbers".***

Keywords: Numbers, sets, X-power set, prime X-power numbers, composite X-power numbers and Algorithm

1. Introduction

Number sets are collections of numbers classified as groups according to the values of their elements. One might say that set theory was born in late 1873, when Jonson, Philip (1972) made the amazing discovery that the linear continuum, that is, the real line, is not countable meaning that its points cannot be counted using the natural numbers read in the history of the set Theory by Tom M.Apostol [10]. Set theory is one of the greatest achievements of modern mathematics .The set theory , however , was founded by a single paper in 1874 by Georg Cantor "On a Property of the Collection of Real Algebraic Numbers " .A set wrote by Cantor, is a collection of definite and distinguishable objects perception or thought conceived as a whole. The objects are called elements or members or numbers of the set that is regarded as being a single object. To indicates that object x_1 is a number of a set X and we write $x_1 \in X$. A set may be defined by a membership rule or formula or by listing its numbers within braces. The order of a set means the number of elements or objects in the set. With the help of the set we can find the number of elements or numbers in the set .Order of set is also called cardinal number of the set . The notion of cardinality, as now understood which was formulated Georg Cantor the originator of set theory in 1874-1884.

A. Sudhakaraiah, A. Madhankumar, P. Gopi introduced in 2022 the newly concept " P_X- power set of Even And Odd Number of a Number 'X' " [1] . Set theory is the branch of mathematical logic that studies sets which can be informally described as collections of objects. Although objects of any kind can be collected into a set. The set theory as a branch of mathematics and the modern study of set theory was initiated by the German mathematicians Richard Dedekind and Georg Cantor in the 1970s. In particular, Georg Cantor is commonly considered the founder of the set theory. The positive integers are actually part of a large group of numbers called integers. Boyer's." Fundamental Steps in the Development of Numeration [3], all the Integers are the whole numbers both positive and negative.

As Set theory is the branch of mathematical logic that studies sets, the modern study of set theory was initiated by the German mathematicians Richard Dedekind and Georg Cantor in the 1870s. Set theory is the mathematical branch that studies the sets and their properties, the operations on sets. The beginning of the modern set theory was around 1870. Set theory is a fundamental branch for the entire mathematics as it is the base for many fields of Mathematics like Algebra, Topology, Probability and other branches of Sciences like biological, Chemical sciences. It gives also an essential foundation to the development of many concepts like infinity and many other sophisticated mathematical concepts. Set theory is used beyond that, it has numerous applications in other sciences like computer science and philosophy etc. In mathematics, the power set of a set S is the set of all subsets of S, including the empty set and S itself. The power set of S is variously denoted as P(S) or (S). The X-power set was introduced by Dr.A.Sudhakaraiah , A.Madhankumar,P.Gopi [2], also introducing composite power numbers as a part of continuation of the work .

2. Preliminaries

Perhaps, the main thing a power set is good to provide a universe for other things to take place in. For example, a lot of areas of math being by singling out certain subsets of a set X as special. Topology deals with open sets, measure theory deals with measurable sets. Also the new concept of two digits, three digits and so on the natural numbers equal to "two" focused by Dr.A.Sudhakaraiah , A. Madhan Kumar , 2021 [2]. As per the study of set theory set is the mathematical theory of well determined collections called objects by Jech , Thomas's set theory, Springer Monographs in Mathematics 2003. Pure set theory deals exclusively with set. So, only the sets under consideration are those whose members are also sets. In this connection, we introduce the newly definition by the cardinality of X-power set which will be very useful to find the number of numbers of the X-power set. Also, we have discussed some preliminary definitions like digits , numbers , whole numbers , even numbers , even digits and odd numbers, odd digits , prime numbers , prime digits , factorial , permutations, combinations, sets, subsets, power sets. All these concepts we have used in our newly developed concept "the X – Power set" and as follows some necessary definitions.

II (i): X – Power set

The X-Power set of a number X contains all numbers formed by the digits of X.

Example: X=15, PX ={1, 5, 15, 51}

II (ii): X - Power numbers

X -Power number is defined as the number which has X -power set.

X = 36, P_X = {3, 6, 36, 63}.

Note: X-power numbers are whole numbers.

II (i): Even X-power number

If X is a positive even number then the X-power set formed by the digits of that number. If the elements of that X-power set are all even numbers then X is called as even power number.

For example, X = 48 is even number then the X –Power set P_X = {4, 8, 48 , 84 } here all numbers of X –power set is even numbers , therefore X = 48 is an even X -power number . Suppose X = 34 is an even number then the X – Power set PX= {3 , 4 , 34 , 43 } .Here all numbers of P_X are not even numbers . Since 3 and 43 are not even so that X = 34 is not an even X -power number.

II(iv): Odd X-Power number

Consider X as a positive odd number then if the X-power set contains only odd numbers as its elements then X is called as an odd X -power number.

For example, X = 19 is an odd number then the X–Power set P_X = { 1 , 9 , 19 , 91 }. Here all numbers of X –power set is an odd numbers. Therefore X = 19 is an odd X - power number.

Considering another example say X = 43. It is an odd number and the X –Power set P_X = {3, 4, 34, 43}. Here all numbers of P_X are not odd numbers since 4 and 34 are even. So that we deduce X = 43 is not an odd X -power number.

II(v): Prime X-power number

If X is a positive prime number then the X-power set formed by the digits of that number. If the elements of that X-power set are all prime numbers then X is called as prime X -power number.

For example, X = 37 is a prime number then the X –Power set P_X = {3, 7, 37, 73}. We note that all numbers of X –power set is prime numbers and therefore X = 37 is a prime X -power number.

Suppose if X = 53, it is a prime number then the X – Power set P_X = {5, 3, 53, 35}. It is clearly can be observed that all numbers of P_X is not prime numbers. Since 35 is not a prime so that X = 53 is not a prime X -power number.

II (vi): Composite X-power number

If X is a power number, X is called a composite X -power number when all the elements of X-power set formed by the digits of X are composite numbers.

Example:

Let us consider X= x_1x_2 = 46 which is a composite number, where x_1 = 4 and x_2 = 6 (digits of 46) and let P_{46} be the X -power set of 46.

We have P_{46}= {4, 6, 46, 64}. Here 4, 6, 46, 64 are all composite numbers.

∴ X= 46 is a Composite X -power number.

Counter Example:

Let X = 98 is a composite number. Here x_1 = 9, x_2 = 8 (digits of 98) and let P_{98} be the X -power set of 98. We have P_{98} = {8, 9, 89, 98}. Here 8, 9, 98 are composite numbers but 89 is a prime number which means X= 98 is not a Composite X-power number.

3. Lemmas

Lemma1:

If X is a composite X -power number, P_x is a composite X -power set and S is the set of multiple of 5, then S ∩ P_x = ∅ where S ⊄ P_x .

Proof:

Let X be a composite X -power number and P_x be a composite X -power set.

Let us consider that S is the set of multiple of 5.

To prove that S ⊄ Px or S ∩ P_x = ∅,

Let us assume that x ∈ S

We know that by the divisibility rule of 5, x will have 0 or 5 in its one's place.

i.e. by the definition of X-power set, P_x contains 0 or 5

0 ∈ Px or 5 ∈ Px.

i.e. P_x is not composite X -power set

This is a contradiction.

x ∈ S and x ∉ P_x

S ⊄ **Px** or S ∩ P_x = ∅.

Therefore, the composite X -power set $P_{x \text{ does}}$ not contain multiples of 5.

Hence, it is proved that if X is a composite X -power number, P_x is a composite X -power set and S is the set of multiple of 5, then S ∩ P_x = ∅ where S ⊄ Px .

EXAMPLE:

Let X = 48 be a composite X -power number and

P_x = {4, 8, 48, 84}. In this set we have not multiples of 5 and

S= {5,10,15, 20,……}, where S is a set of multiples of 5
Here S ∩ P_x = {5,10,15, 20,……}∩ {4, 8, 48, 84 }= ∅.

Lemma2:
Every Composite X -power number is a Composite number, but every composite number is not a composite X -power number.

Proof:
Let X be a positive composite number, X is called a composite X -power number when all the elements of X-power set formed by the digits of X are composite numbers.

So, clearly by the definition of Composite X -power number, every composite X -power number is a composite number. Now we need to prove that every composite number is not a composite X -power number.

For that, let us take a composite number $X=x_1x_2x_3….x_n$ where $x_1,x_2,…x_n$ are the digits of X and P_x be the X -power set of X. Here, it will be arised two cases.

Case(i):
Let us assume that any one of the digits of X say x_k belongs to the set M= { $x_{1,}$ $x_{2\ ,}$ $x_{3,}$ ….x_k }where k is any positive integer but not composite numbers. i.e x_k ∈ M

By the definition of X -power set of X, x_k belongs to P_x.
i.e. x_k ∈ P_x which implies P_x contains an element x_k which is not a composite number.
So, P_x is not a composite X -power set.
∴ X is not a composite X -power number.

Example:
Consider M = {0, 1, 2, 3, 5, 7}

Let us take the composite number X=153 and P_X = P_{153} = {1, 3, 5, 13, 15, 31, 35, 51, 53, 135, 153, 315, 351, 513, 531}. Here 153 is a digit number of M and P_{153} contains prime numbers.
∴ 153 is not a composite X -power number.

Case(ii):
Let us assume that any one of the digits of X say x_k belongs to the set S= {4, 6, 8, 9}.

By the case (ii) in forth coming theorem 2, not all the numbers formed by the elements of set S= {4, 6, 8, 9} are composite X -power numbers.

So, the from case (i) and case (ii) we can clearly say that every composite number cannot be a composite X -power number.

4. Main Theorems

Theorem1: 73 is the only largest prime X -power number.
(or)

There is no other prime X -power number which is greater than 73.

Proof: Before proving that 73 is the largest prime X -power number,
let us first show that 73 is a prime X -power number.

We know that 73 is the prime number as 1 and 73 are only the factors of 73.

Let P_{73} be the 73-prime power set i.e. P_{73} = {3, 7, 37, 73}.

Here all the elements P_{73} are prime numbers and we know that X is a prime X -power number, when all the elements of the X-power set are prime.

Therefore, By the definition P_{73} is a prime X -power set and 73 is a prime X -power number.

Now, let us prove that 73 is the largest prime X -power number.

To prove that, firstly, let us check which digits should be the prime X -power number.

Let p be the prime number and P_p be the p-prime power set.

Case (1):
Let S = {0, 1, ,4 ,6 ,8 ,9}

We know that elements of S are not prime numbers.
Let x ∈ S and x is one of the digits of p.
By the definition of X-power set, x ∈ P_p
Since x is not a prime, Pp is not a prime X -power set. So, p is not a prime X -power number.
Therefore, the elements of S cannot be the digits of p.

Example:
Let prime number X = 109. P_{109} = {0, 1, 9, 10, 19, 90, 91, 109, 190, 901, 910}.Here 0, 1, 9, 10, 90, 91, 190, 901, 910 are not prime numbers.

Therefore 109 is not prime X -power number.

Case (2):
Let p be the prime numbers which has 2 as one of its digits.
Let x be a positive integer which has 2 in its ones place (ex: 12, 32, 432….etc.)
We know that x is divisible by 2 according to divisibility rule of 2.
By the definition, x ∈ P_p, x is not a prime number.
i.e. P_p is not prime X -power set.
So, p is not a prime X -power number.
Hence, prime X -power number does not contain 2 as one of its digits.

Example:
Let prime number X = 23, P_{10} = {2, 3, 23, 32}.
Here 32 is not prime number because it is divisible by 2 according to divisibility rule of 2.
∴ 23 is not a prime X -power number.

Case (3):
Let us assume that x is a positive integer which has 5 in its unit place.

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Let p be the prime number which has 5 as one of its digits.

We know that x is divisible by 5 according to divisibility rule of 5.

By the definition, we know x ∈ Pp .

Since x is not a prime number, P_p is not prime X -power set So that, p is not a prime X -power number.
∴ prime X -power number does not contain 5 as one of its digits.

Example:
Let prime number x=53, P_{10} = {5, 3, 35, 53}

Here 35 is not prime number because it is divisible by 5 according to divisibility rule.
∴ 53 is not prime power number.

Case (4):
Let x be positive integer.

Let p is prime number which has x as its digit repeated two or more times in it (ex. 773) .

In this case P_p contains multiples of 11 or 111 or 1111 or etc.

i.e. P_p is not prime X -power set and p is not a prime X -power number.

Therefore, prime X -power number doesn't contain repeated digits.

Example:
Let us consider the prime number x = 337, P_{337} = {3, 7, 33, 37, 73, 337, 373, 733}

In this case 33 is divisible by 11 according to the divisibility rule of 11.
∴ 337 is not prime X -power number.

From the above four cases, we have, the prime X -power number p contains 3 and 7 as its only digits. So, p = 3 or 7 or 37 or 73. In this case 73 largest numbers.

∴ 73 is the largest prime X -power number.

Theorem2:
1) Let X = x_1x_2 or y_1y_2 be a composite X -power number, then the digits of X must be formed by the elements of the set S = {4, 6, 8, 9}.
2) Not all the numbers formed by the elements of set S = {4, 6, 8, 9} are composite X -power numbers i.e the converse of statement (i) is not true.

Proof:
1) Given X = x_1x_2 or y_1y_2 or z_1z_2 be a composite X -power number. Let P_x be the X -power set formed by the digits of X.

Let X= $x_1x_2x_3\ldots x_n$ be a positive composite number, we know that X is called a composite X -power number when all the elements of X-power set formed by the digits of X are composite numbers.

By the definition of composite X -power number, all the elements of P_x are composite numbers.

Now we need to prove that the digits of X must be formed by the elements of the set S= {4, 6, 8, 9}.

Let us prove this theorem by contradiction. Let us assume that any one of the digits of X say x_k does not belong to S i.e x_k ∈ M = {0, 1, 2, 3, 5, 7}.

By definition of the X -power set of X, x_k belongs to $P_{x.}$

i.e.,x_k ∈ Px. This contradicts the definition of Composite X -power number that all the elements of its X -power set are composite numbers as none of the elements of M is a composite number.
∴ the digits of X must be formed by the elements of the set S= {4, 6, 8, 9}.

2) Now we will prove the second statement, that not all the numbers formed by the elements of set S= {4, 6, 8, 9} are composite X -power numbers i.e the converse of statement (i) is not true.

Let us take a number X = $x_1x_2\ldots x_n$ which is formed by the elements of S= {4, 6, 8, 9} and $P_{x\,,}$ be the X -power set of X .

Let us assume that all the elements of X be formed by only 4,6and 8.

All the elements of P_x are clearly divisible by 2 Which means all the elements of P_x have a factor 2 other than 1 and itself.

Therefore, in this case the set P_x contains the elements which are composites which makes X a composite X -power number.

But if any one of the digits of X is 9, then there is a possibility of that number being a prime number.

Example, let us take the X=98 where x_1=9 and x_2=8, which is formed by the elements of set S.

We know that 98 is a composite number. Now let us look at the X -power set of 98.
P_{98} be the X -power set of 98 and P_{98}= {8, 9, 89, 98}.
In this $P_{98,}$ the numbers 8, 9, 98 are all composite numbers but 89 is a prime number.
We know that for X being a positive composite number, X is called a composite X -power number when all the elements of X-power set formed by the digits of X are composite numbers.

By the definition of composite X -power number, P_{98} cannot be a Composite X -power set and X=98 is also not a Composite X -power number.

6899, 4899, 6689, 6949, 9949, 8669…. etc are some prime numbers which are formed by the elements of set S.

So, therefore this proves that not all the numbers formed by the elements of set S= {4, 6, 8, 9} are composite X -power numbers.

5. To find the X-Power set and Order of X-Power set to wards Algorithm

Algorithm:

Start the program.

- Read the input number from new **Scanner** object to the user.
- Create a read input from the console.
- Print the message asking the user to enter a number.
- Read the input number using **nextInt()** method of the **Scanner** object and store it in the variable **number**.
- Close the **Scanner** object.
- Call the **generateX_PowerSetNumbers ( )** method with the input number and store the result in the **power Set** variable.
- Print the X -power set by iterating over the elements in the **X_power Set** and displaying them.
- Print an empty line for formatting.
- Print the order of the X-power set, which is the size of the X**_power Set**.
- End the program.

The **generate X_Power Set Numbers ()** method:

- Convert the input number to a string using **String. Value Of(number)** and store it in the **number String** variable.
- Create a new **TreeSet** called **formed Numbers** to store the generated numbers.
- Call the **generate Permutations()** method with the number string, an empty string, and the **formed Numbers** set.
- Return the **formed Numbers** set.

The **generate Permutations( )** method:

- Check if the length of **currentNum** is greater than 0.
- If true, parse **currentNum** as an integer and add it to the **formedNumbers** set using **Integer.parseInt( )**.
- Iterate over the digits array using a for loop.
- Inside the loop, append the current digit to **currentNum** and store it in the **newNum** variable.
- Call the **generatePermutations( )** method recursively with the updated **newNum** and the reduced digit array obtained by calling **removeElement( )**.
- Repeat the above steps for each digit in the array.

The **removeElement( )** method:

- Create a new character array **newArr** with a length of **arr.length - 1**.
- Initialize **newIndex** to 0.
- Iterate over the elements of **arr** using a for loop.
- Check if the current index is not equal to the specified index.
- If true, add the current element to **newArr** and increment **newIndex**.
- Return the **newArr** array.

Note: The **generateX_PowerSetNumbers( )** method and the **generatePermutations** method are updated as well.

To find the X-Power set and Order of X-Power set towards java program

```
import java.util.Scanner;
import java.util.Set;
import java.util.TreeSet;

public class X_PowerSetFormation {
   public static void main(String[] args) {
   Scanner sc =new Scanner(System.in);
   System.out.println("Enter number :");
      int number = sc.nextInt();
      Set<Integer> powerSet =
generateX_PowerSetNumbers(number);
      System.out.println(number+"-Power set is
"+powerSet);
      System.out.println();
      System.out.println("order of "+number+"-Power set =
"+powerSet.size());
      sc.close();
   }
   public static Set<Integer>
generateX_PowerSetNumbers(int number) {
      String numberString = String.valueOf(number);
      Set<Integer> formedNumbers = new TreeSet< Integer
>();
      generatePermutations(numberString.toCharArray(), "",
formedNumbers);
      return formedNumbers;
   }
   public static void generatePermutations(char[ ] digits,
String currentNum, Set<Integer> formedNumbers) {
      if (currentNum.length() > 0) {
         formedNumbers.add(Integer.parseInt(currentNum));
      }
      for (int i = 0; i < digits.length; i++) {
         String newNum = currentNum + digits[i];
         generatePermutations(removeElement(digits, i),
newNum, formedNumbers);
      }
   }
   public static char[] removeElement(char[] arr, int index)
{
      char[] newArr = new char[arr.length - 1];
      int newIndex = 0;
      for (int i = 0; i < arr.length; i++) {
         if (i != index) {
            newArr[newIndex++] = arr[i];
         }
      }
      return newArr;
   }
}
```


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6. Conclusion

In this paper we discussed the concept of largest power X-prime number, composite X-power set and composite power number. Along with we are presenting the java program for getting X-power set of a positive integer and in future we are going to find an anther concept of the X-power numbers with algorithm.

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