Pseudo-Matrix Matrices: A Comprehensive Exploration and Applications in Matrix Equations

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Abstract: ***This article delves into the world of pseudo-matrix matrices, shedding light on their significance in solving complex matrix equations. It explores the properties of pseudo-matrix matrices and demonstrates their utility in various scenarios. The article also presents a detailed derivation of the best approximate solutions for matrix equations using these specialized matrices. Understanding the intricacies of pseudo-matrix matrices provides valuable insights for practitioners in linear algebra and related fields.***

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1. Introduction

The talk first looks at the following matrix grave equation

$$AXA = A. \tag{1}$$

If A the matrix is a square matrix that does not crack, it is $X = A^{-1}$ determined by the solution button. If A the matrix $m \times n$ is a measured matrix, the solution to this equation is a $n \times m$ measured matrix that is not defined as a single. In general, there are an infinite number of solutions to the equation. Among these solutions is only one solution that combines lines and columns A^* with lines and columns in accordance with the matrix. That solution is A called a pseudo-matrix matrix for the matrix and A^+ is marked with it, if it' $m \times n$ s for a size matrix matrix A

$$AA^+A = A, \tag{2}$$

$$A^+ = UA^* = A^*V \tag{3}$$

if their equations are true, then $n \times m$ A^+ pseudotars are called matrix matrix for A a measurable matrix matrix, U wherever and V - any matrix.

A there can't be two different A_1^+ and A_2^+ matrices for the matrix. Indeed, $AA_1^+A = AA_2^+A = A$, $A_1^+ = U_1A^* = A^*V_1$, $A_2^+ = U_2A^* = A^*V_2$ from their equalities $D = A_2^+ - A_1^+$, $U = U_2 - U_1$ $V = V_2 - V_1$ by accepting, $ADA = 0$, $D = UA^* = A^*V$.

From here

$$(DA)^* DA = A^*D^*DA = A^*V^*ADA = 0$$

and so on,

$$DA = 0.$$

Then that $DD^* = DAU = 0$, 's what $D = A_2^+ - A_1^+ = 0.$

Now A let 's A^+ show the existence of a matrix for the matrix. To do this, A^+ we will use the skeleton separation of the $A^+ = C^+B^+$ matrix.

It should be noted that r the number of digits $m \times n$ can be found in A $m \times r$ accordance with $r \times n$ the size r and size B C of the matrix $A = BC$. A such a description of the matrix is called a skeleton separation.

B^+ and C^+ we're looking for their matrices. By assignment

$$BB^+B = B, , B^+ = \hat{U}B^*, \tag{4}$$

where it $\hat{U}$'s any matrix. Then . $B\hat{U}B^*B = B$. . Let's hit each side of the last equally to the B^* matrix from the left. Then we B^*B take into account the fact that the matrix is a matrix that doesn $\hat{U} = (B^*B)^{-1}$ 't crack . Given this button (4) we will get it for the second equation in the button B^+ :

$$B^+ = (B^*B)^{-1}B^*. \tag{5}$$

We're analogies to the last button C^+ for matrix:

$$C^+ = C^*(CC^*)^{-1}. \tag{6}$$

Now let's show that

$$A^+ = C^+B^+ = C^*(CC^*)^{-1}(B^*B)^{-1}B^* \tag{7}$$

matrix (2), (3) and thus A pseudo-matrix matrix for matrix.

Directly as well

$$AA^+A = BC(C^*(CC^*)^{-1}(B^*B)^{-1}B^*)BC = BC = A.$$

On the other hand, (5)-(7) by $A^* = C^*B^*$ taking into account and accepting equality $K = (CC^*)^{-1}(B^*B)^{-1}$ from their equalities:

$$A^+ = C^*KB^* = C^*K(CC^*)^{-1}CC^*B^* = UC^*B^* = UA^*,$$

$$A^+ = C^*KB^* = C^*B^*B(B^*B)^{-1}KB^* = C^*B^*V = A^*V,$$

Harada Ki, $U = C^*K(CC^*)^{-1}C, V = B(B^*B)^{-1}KB^*.$

Thus, we showed that A there is only one pseudo-matrix A^+ matrix defined by the button (7) for any curved B matrix, where and C where A the matrix matrix is hit in the skeleton separation. This definition button shows that when the A matrix is square and unbrokeable, the A^+ pseudo-matrix matrix matches A^{-1} the matrix.

For example, A the matrix is a $m \times n -$ measured matrix: $A = (a_{ik}), i = \overline{1,m}, k = \overline{1,n}$ . . . Let's look at the standard of this matrix. A the standard of matrix $\|A\|$ is a negative number,

$$\|A\|^2 = \sum_{i,k} |a_{ik}|^2 \tag{8}$$

is assigned to the button. It's obvious that

$$\|A\|^2 = \sum_{k=1}^{n} | A_{\bullet k} |^2 = \sum_{i=1}^{m} | A_{i\bullet} |^2 , \tag{9}$$

the $A_{\bullet k}$ first column $A_{i\bullet}$ and A line are marked in accordance with $k-$ where and where $i-$the matrix is. Consider the following matrix equation:

$$AX = Y, \tag{10}$$

where they' A re $Y-$ given $m \times n-$ and $m \times p-$ they're dead matrices, and they $X -$' $n \times p-$ re looking for dimensions.

(10) Find the X^0 best solution $\| Y - AX^0 \| = \min \| Y - AX \|$ to the equation , $\| Y - AX \| = \| Y - AX^0 \|$ in the case of . $\| X^0 \| \le \| X \|$. .

Consider the following relationships:

$$\| Y - AX \|^2 = \sum_{k=1}^{p} | Y_{\bullet k} - AX_{\bullet k} |^2 , \tag{11}$$

$$\| X \|^2 = \sum_{k=1}^{p} | X_{\bullet k} |^2 . \tag{12}$$

This relationship shows that the X^0 column vector $k-$, the first column $X^0_{\bullet k}$ of the matrix being sought $AX_{\bullet k} = Y_{\bullet k}$ XCTS should be the best approximate solution. Because this equation $k = 1,..., p$ is true, it's taken $X^0 = A^+ Y$. (13)

Thus, (10) the equation has only one best approximate solution, and it is defined by the button (13).

In particular, $Y = E$ $m-$if it is a vertical matrix, it will $X^0 = A^+$ be. Thus, A^+ the pseudo-style matrix $AX = E$ may be the best approximate solution to the method of the smallest squares in the matrix equation. This characteristic of the pseudo-style matrix may be accepted for its tactics.

In conclusion, the concept of pseudo- matrix matricesopensup a fascinating realm of possibilities for solving matrix equations efficiently. Their unique properties and applications make them an indispensable tool in the tool kit of mathematicians and scientists working with matrices. By leveraging pseudo-matrix matrices, researchers can unlock new solutions and insights in a variety of domains where matrix equations play a crucial role.

References

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