

# Numerical Modeling of Vibratory Motion of Shock Absorber Performance in Automobiles

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**Abstract:** *Vibration is essentially an oscillatory motion. There is a force that initiates the vibratory motion. The research involves improving the shock absorber by using variable external disturbance force, spring and damper to come up with variable shock absorber model which adjusts depending on the shock. Using MATLAB software, a response of the system to a harmonic disturbance of variable automobile weight and to an impulse excitation is found. The objectives of the study are to determine the effects of both the distribution of spring - supported automobile weights and the shock absorber thickness on vibrations of the mounting points of the shock absorber. The equation governing vibratory motion of shock absorber is solved using Finite difference method with central difference scheme. The effects of both the distribution of spring - supported automotive weights and the shock absorber thickness on vibration are discussed and presented graphically. Various values of automobiles weights and thickness are used in the study. The findings obtained reveal that, an increase in automobiles weights leads to an increase in the impulse excitation of the shock absorber. It was also found that an increase in shock absorber thickness leads to a decrease in vibrations motion. The study provides insight knowledge which is important in elimination of vibrations in automobiles in order to improve both the comfort and the safety of the passengers.*

**Keywords:** Finite Difference Method, Central Difference Scheme, Vibration Damping, Shock absorber, Automobiles weights, Shock absorber thickness

## 1. Introduction

### 1.1 Background of Study

The main causes of vibrations affecting the users of automobiles are kinematic disturbances resulting from road surface irregularities. Elimination of these vibrations is essential in order to improve both the comfort and the safety of the passenger. When the vehicle is driving across a road with large irregularities (obstacles), its wheels might get separated from the surface of the road, which in turn decreases the efficiency of force transmission of the drive, braking and steering systems of the car. An improved driving dynamics and better road traction on curves and bumps can be achieved by using the so - called "hard suspension". However, the cost is the reduction of comfort of the passenger. The criteria for assessing the quality of shock absorbers should therefore include both the minimization of car body vibration and appropriate wheel - road adhesion (Łuczko and Ferdek, 2012).

### 1.2 Modeling of Mechanical Vibrations

Vibration is essentially a to - and - fro motion. Thus there is a force (excitation/disturbance) that initiates the motion. Under the influence of the external disturbance excitation, the system masses move i. e., they accelerate and decelerate setting up inertia forces. If external excitation were the only type of force on the system, the system would exhibit a rigid body motion. However since most systems are elastic, the movement of the masses invariably causes stretching or compression of springy elements setting up elastic restoring forces. For example when an automobile passes on a road, the road roughness is the external excitation. The mass of the vehicle moves up - down (pitch and bounce), left - right

(roll) setting up inertia forces. The suspension spring gets stretched and compressed as the vehicle mass moves up and down. When a spring is stretched or compressed from its free length position, it exerts a restoring force on the mass trying to bring it back to its free length position. In the process of course the mass would have gained momentum and continues to travel farther than the static equilibrium, free length position. Once again the spring tries to pull the mass back to its free length position and the cycle repeats. An automobile suspension always has a shock absorber i. e., a damper that dissipates the energy of vibration into friction against a moving fluid. Thus the four fundamental elements of a vibrating system are: mass or inertia, springiness or restoring element, dissipative element (often called damper) and external excitation.

In modeling a vibrating system, there are two types of models

- Physical model. Physical model of a system is a representation of the physics of the system that we would like to include in our study. For example, we may consider dissipation negligible in a system for a given study and may say that the physical model of the system consists of purely spring mass systems.
- Mathematical model. It refers to a mathematical relation that defines the input - output relation of the elements. For example, we could have a simple linear relation between force and deflection for a spring or a more complicated non - linear relation (for example softening or hardening types).

For a given problem of engineering, we typically develop first the physical model i. e., decide which physics is important to include in the model and then build the mathematical description of the various elements to develop

the mathematical model. Modeling of mass or inertia of a system seems fairly straight forward, at a first glance. We just need to worry about the total mass/inertia of the system and use it in system models. For example in an automobile, there is a certain mass of the vehicle – chassis, body, engine and we can include these masses in the model of the system. The vehicle mass changes with the number of passengers and the luggage but that can also be determined and taken into account.

### 1.3 Geometry of the Problem

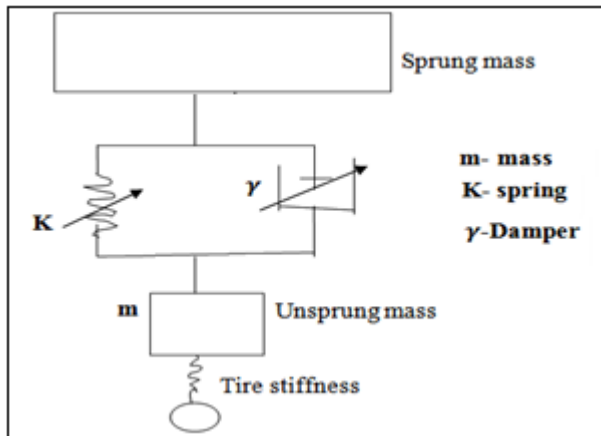


Figure 1: Variable Shock Absorber (Ogega *et al* 2015)

The Figure 1 shows the design of a variable shock absorber with variable spring,  $K$  and variable shock absorber, based on the influence of non - linear characteristics of spring and shock absorber. The unsprung mass consists mass below the shock absorbers which include the chassis and sprung mass include the vehicle body and its load.

### 1.4 Statement of the problem

The main causes of vibrations affecting the users of automobiles are kinematic disturbances resulting from road surface irregularities. Elimination of these vibrations is essential in order to improve both the comfort and the safety of the passenger. In this work, we determine the effects of the distribution of spring - supported automobile weights and the shock absorber thickness on vibration on the mounting points of the shock absorber. There is needed to look for mitigating solutions to the problem of disturbances arising from road surface irregularities with a view of improving performance of shock absorbers.

### 1.5 Objectives of the study

- To determine the effects of the distribution of spring - supported weights on vibrations of the mounting points of the shock absorber in automobiles.
- To determine the effects of shock absorber thickness on vibrations of the mounting points of the shock absorber in automobiles.

### 1.6 Assumptions

- The density of the fluid in the shock absorber is kept constant

- The flexural rigidity of the spring - supporting the shock absorber is kept constant

## 2. Literature Review

Dampers used in the suspension system can be passive, semi - active or active. Dynamical properties of the dampers are usually defined by models with hysteresis characteristics, such as Bingham (Prabakar *et al.*, 2009), Bouc - Wen (Dominguez *et al.*, 2008; Yao *et al.*, 2002) or Spencer model (Spencer *et al.*, 1996). Requirements set for the comfort and safety of driving can be fulfilled by using semi - active suspension systems, introduced by Crosby and Karnopp (1973). In comparison to passive ones, the semi - active systems allow the damping force to be adjusted depending on driving conditions. Additionally, they require less power than similar active systems. Several methods of control have been used, some of which can be found in the work of Ahmadian (2001). Liu *et al.* (2005) as well as Wu and Griffin (1997), when analyzing on - off control, assume that the damping force should be high if the product of relative and absolute velocity is more than zero. Fischer and Isermann (2004) analyzed the relation between parameters of the car suspension system and the driving comfort as well as the safety indexes. They defined the comfort index as the effective acceleration value while the safety index as the effective ratio of the dynamic and static response. In the study by Sapiński and Martynowicz (2005), the results were presented for the theoretical and experimental half - car model, in which the car suspension was controlled by two separate magneto - rheological dampers (MR damper).

Some interesting options for control of a semi - active car suspension were presented by Ahmadian (2001). The most common model analyzed was the quarter - car one. In the steady - state case, the response to the harmonic excitation was analyzed, while in the transient one (Ahmadian and Vahdati, 2006), the response to the unit step. To ensure a compromise between the requirements for both comfort and safety, hybrid control with a MR damper is used (Goncalves and Ahmadian, 2003) and a combination of sky - hook and ground - hook control.

The objective of the paper is to developed a model that will provide information to engineers and designers for making decisions associated with damping and shock absorbers so as to increase comfortability and durability of vehicles) and provided information to researchers for making decision associated with damping.

Ogega *et al* (2015) developed a variable shock absorber model by varying the spring constant achieved by using helical spring with variable pitch in coils and varying coil diameter and a variable damper. The governing equations were solved and analysis of various parameters such as damping coefficient, spring constant and damper fluid temperature were considered and their effect on damping. The revealed that if the damper fluid density reduces, the vehicle's damper fluid temperature increases. The fluid density increases with time as the vehicle moves and as the damper mass reduces, damping increases. Damper vibration is low and as time increases damping also increases.

Urszula and Jan (2016) analysed a half - car model with linear and nonlinear semi - active dampers performance. Using MATLAB - Simulink software, a response of the system to a harmonic excitation of variable frequency and to an impulse excitation was found. The effect of both the distribution of spring - supported mass and the asymmetry of the support on the frequency characteristics of velocities and displacements at the mounting points of the dampers were analyzed. It was found that the algorithms for control of a semi - active on - off damper, in which the switching is related to the actual power, enable improvement the driving comfort, especially within the low frequency excitation range. Also introduction of an additional spring and damping elements to the front and back car suspension in which the center of stiffness overlaps the center of mass, does not cause a decrease in the indexes describing the driving comfort which is, however, beneficial when considering the safety.

The objective of the study in this article is to make a closer investigation of the application of numerical solution to a partial differential equation of mechanical system dynamics with focus on effects of automobiles weights onshock absorber by using variable spring dynamics as a case study. Upon reviewing the previous scientific works related to this area and exploring the methods of numerical solution to the partial differential equation as well as the concept of vehicle suspension system, a mathematical model of a selected suspension system is described using partial differential equation. The application of the central difference numerical method to partial differential equation of vibration analysis is studied and the solutions are simulated using MATLAB computer codes.

### 3. Method of Solution

Thohura and Rahman (2013) explained in their comparative study, different techniques of numerical approaches that are available for solving differential equations including powerful numerical analysis software packages. For vibration analysis, in particular, numerical approaches are to be used if the free or forced vibration of a system cannot be integrated in closed form for the differential equation governing the system. The numerical methods employed for this purpose include the finite difference method and central difference scheme, Rao *et al* (2010). These methods are based on the approximation of the derivatives appearing in the equation governing shock absorber vibrations and the boundary conditions are used for solving the partial differential equations in the numerical method.

$$7000 \times t_s \left( \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta t)^2} \right) + 80 \left( \frac{U_{i+2,j} - 4U_{i+1,j} + 6U_{i,j} - 4U_{i-1,j} + U_{i-2,j}}{(\Delta x)^4} \right) + \left( \frac{U_{i,j+2} - 4U_{i,j+1} + 6U_{i,j} - 4U_{i,j-1} + U_{i,j-2}}{(\Delta y)^4} \right) + 2 \left( \frac{2U_{i,j+1} - 4U_{i,j} + 2U_{i,j-1} - U_{i-1,j-1} + 2U_{i-1,j} - U_{i-1,j-1} - U_{i+1,j+1} + 2U_{i+1,j} - U_{i+1,j-1}}{(\Delta x)^2(\Delta y)^2} \right) = F \quad (3)$$

Taking  $(\Delta x) = (\Delta y)$  and multiply both sides by  $(\Delta t)^2$  while we let

$$\phi = \frac{(\Delta t)^2}{(\Delta x)^4}$$

We get

### 3.1 Vibration Governing Equation

The transverse vibration of damper shock absorber  $u(x, t)$  can be described by the following equation which is non dimensional in the spatial variable (Dino Sciulli, 1997).

$$M \frac{d^2 u}{dt^2} + \gamma \frac{du}{dt} + ku = F \quad (1)$$

The system has inhomogeneous boundary conditions represented by  $U(x, t) = 0$ . The parameters  $M, \gamma, k$  and  $F$  in Equation (1) represent mass of damper, damping coefficient, damper spring constant and the external disturbance force (in our case, the mass of the vehicle) (Dino Sciulli, 1997). Where  $u(x, t)$  is the displacement of two vibrating springs measured from their equilibrium positions. Equation (1) cannot be used for shock absorber - like problems, since no exact solution of the governing equation of motion is available. Therefore, an alternative model is derived, which does not account for the mass and stiffness of shock absorber. The equation of motion for the transverse shock absorber vibration is given by (Oude and Marco, 2003):

$$\rho_s t_s \frac{\partial^2 u}{\partial t^2} + M \left( \frac{\partial^4 u}{\partial x^4} + 2 \frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4} \right) = F(x, t) \quad (2)$$

Where  $u(x, y, t)$  is the shock absorber displacement in the transverse direction,  $\rho_s$  is the density of the shock absorber material,  $t_s$  is the shock absorber thickness,  $M$  is momentum distribution and  $F$  is an external distributed load on the shock absorber (automobiles weights). This equation follows from classical shock absorber theory, i. e. the Kirchhoff hypothesis is used, and shear effects and rotary inertia effects are neglected.

### 3.2 Discretization of governing equation

We consider the equation (2) and use the central difference scheme to solve this equation. We investigate the effect of on shock absorber transverse vibration using equation (2).

For the Central Difference scheme (CDS), the values  $u_{tt}$ ,  $u_{xxxx}$ ,  $u_{xyxy}$  and  $u_{yyyy}$  are replaced by the central difference approximation. If we take  $\rho_s = 7000 \text{kg/m}^3$  and  $D = 80 \text{Kn/m}^2$  to be constants and the values substituted into Equation (2), to get central difference numerical scheme,

$$\begin{aligned}
 & (14000t_s + 320\phi)U^n_{i,j} - 160\phi U^n_{i+1,j} + 80\phi U^n_{i+2,j} \\
 & = 160\phi U^n_{i+1,j+1} + 160\phi U^n_{i-1,j} - 80\phi U^n_{i-2,j} - 80\phi U^n_{i,j+2} - 7000t_s\phi U^{n+1}_{i,j} - 7000t_s\phi U^{n-1}_{i,j} + F
 \end{aligned} \tag{4}$$

Equation (4) is used to investigate the effects of automobile weights and thickness on absorber transverse vibrations. We take  $i$  to represent shock absorber transverse vibration along  $y$  - axis,  $j$  to represent thickness of the shock absorber. We consider the initial and boundary conditions as

$$\left. \begin{aligned}
 \mathbf{u}(\mathbf{x}, \mathbf{y}, 0) &= \mathbf{0}, \text{ for all } \mathbf{x}, \mathbf{y}, \mathbf{t} > \mathbf{0} \\
 \mathbf{u}(0, \mathbf{y}, \mathbf{t}) &= \mathbf{0}, \text{ for all } \mathbf{y}, \mathbf{t} > \mathbf{0} \\
 \mathbf{u}(\mathbf{x}, 0, \mathbf{t}) &= \mathbf{0} \text{ for all } \mathbf{x}, \mathbf{t} > \mathbf{0}
 \end{aligned} \right\} \tag{5}$$

Using the initial and boundary conditions in (5) and taking  $(\Delta x) = (\Delta y) = 0.02, (\Delta t) = 0.0001$ , so that  $\phi = 0.0625$  substituted into (4), with  $i = 1, 2, \dots, 6, j = 0, n = 0$  and writing (4) in matrix form, we obtain the following matrix equation

$$\begin{pmatrix} -10 & 5 & & & & \\ (1400t_s + 20) & -10 & 5 & & & \\ & (1400t_s + 20) & -10 & 5 & & \\ & & (1400t_s + 20) & -10 & 5 & \\ & & & (1400t_s + 20) & -10 & 5 \\ & & & & (1400t_s + 20) & -10 \end{pmatrix} \begin{pmatrix} u_{1,1}^1 \\ u_{2,1}^1 \\ u_{3,1}^1 \\ u_{4,1}^1 \\ u_{5,1}^1 \end{pmatrix} = \begin{pmatrix} 30.8616 + F \\ 30.8616 + F \\ 30.8616 + F \\ 30.8616 + F \\ 30.8616 + F \end{pmatrix} \tag{6}$$

The effects of automobiles weights,  $F$  and shock absorber thickness,  $t_s$  on the shock absorber transverse vibration are obtained using equation (6). This is done by varying one parameter at a time while holding the other constant.

discussed. Solving the matrix equation (6) using MATLAB, the results for varying automobiles weights  $F$  and shock absorber thickness,  $t_s$  are obtained and presented graphically in the figure2 and figure 3.

### 4. Results and Discussion

#### 4.1 The effects of effect of automobiles weights $F$ on the shock absorber vibrations

The effects of automobiles weights  $F$  and shock absorber thickness,  $t_s$  on the shock absorber transverse vibration are

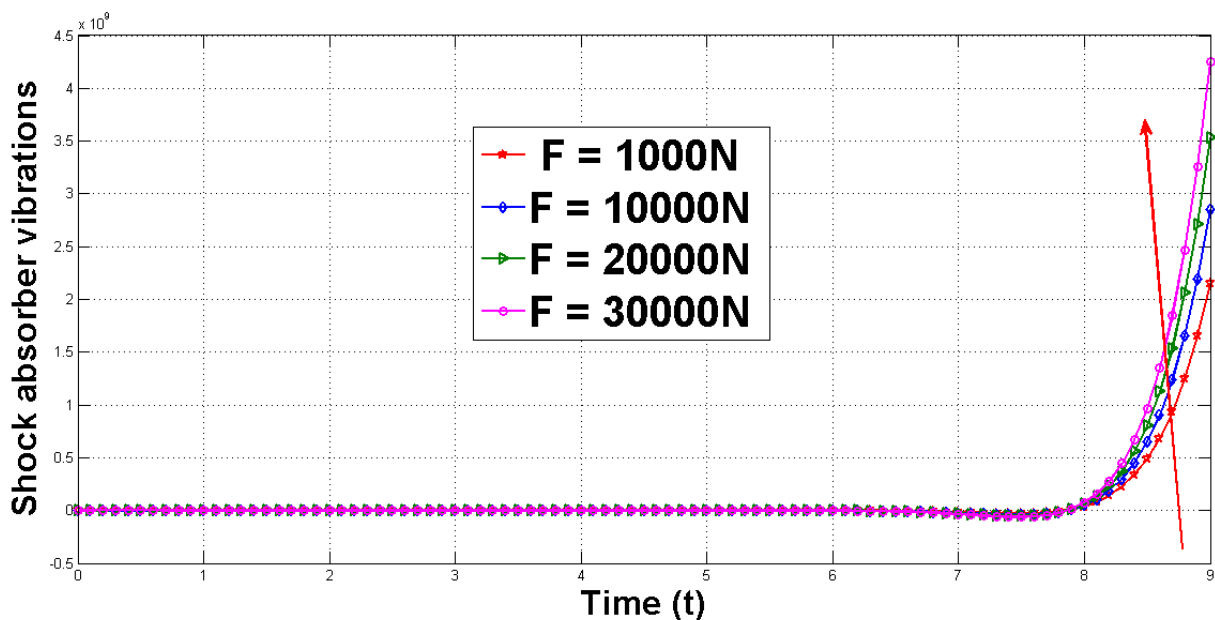


Figure 2: Shock absorber vibration against time at varying  $F$

Fig 2 shows that there is a decrease in shock absorber vibration initially then shock absorber and later vibrations increase with time as automobile speed increases. Also it is seen that as the automobiles moves with time with increase in speed, the vibrations keep on increasing. When the automobile weight,  $F = 1000$  N. the shock absorber vibration is small, but when the automobile weight increases to  $F =$

10000 N, 20000N, and 30000N, the shock absorber vibration also increases. This is due to the fact that, when the automobile weight increases, the stiffness of the shock absorber increases hence the shock absorber vibration increases and when weight reduces the stiffness also reduces resulting to a reduction in shock absorber vibration. As the speed increases, vibration approaches the same value.

#### 4.2 The effects of effect of shock absorber thickness, $t_s$ on the shock absorber vibrations

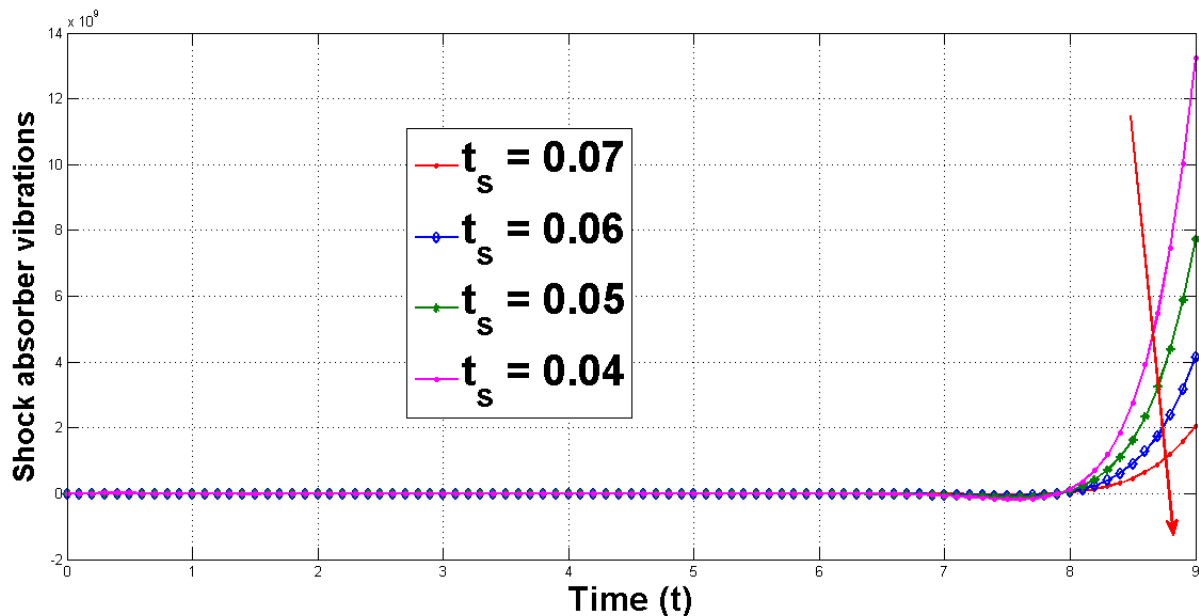


Figure 3: Shock absorber vibration against time at varying  $t_s$

In Fig 3, it is revealed that as the shock absorber thickness increases, the automobile shock absorber vibrations reduce. Also it is seen that as the automobiles moves with time with increase in speed, the vibrations keep on increasing. This is due to the fact that an increase in thickness or cross-sectional area leads to reduced shock resistance absorption and hence leads to small vibrations whereas decrease in thickness (cross-sectional area) leads to increased shock resistance absorption and hence leads to increased vibrations.

## 5. Conclusion

The objectives of this study have been to find the effects of automobiles weights and shock absorber thickness on shock absorber vibrations. This is achieved by using a variable spring constant done by using a helical spring with variable wire diameter and coil diameter. Variable shock absorber dissipates the energy to avoid oscillations. It was found that increase in automobile weights leads to increase in shock absorber vibrations while increase in thickness leads to decrease in vibrations. A vehicle varies its automobile weights when it is loaded or unloaded or when passengers get into it or alight. Also the absorption of a shock by shock absorber depends on the extent of the shock vibration and the speed of the vehicle.

## 6. Recommendations

For the shock absorber to absorb varying shocks effectively, the following recommendations are proposed;

- Maintain the required automobile weights by avoiding overloading to reduces the stiffness of the shock absorber
- Making a shock absorber spring thicker using a thicker wire with varying coil diameter

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