Flow Pattern of Compressible Fluids in Steady State

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Abstract: ***Many engineering tasks need the compressible flow applications usually within the style of a building/tower to face up to winds, high speed flow of air over cars/trains/airplanes etc. Thus, gas dynamics is that the study of fluid flows wherever the squeezability and also the temperature changes become vital [49]. Here, the complete flow field is dominated by physicist waves and shock waves once the flow speed becomes supersonic.***

Keywords: Steady flow, density, entropy, temperature, enthalpy, internal energy, pressure, basic thermodynamic, Critical speed and Maximum speed

1. Introduction

Envisage of a complex fluid flow is not many difficult, waves on the beaches; Spray behind the car; tornadoes and hurricanes or other atmospheric phenomenon these all are example for highly complex fluid flows that may be analysed with the varying degrees of success. There are some more common situations that is analysed easily. It is expected that the motion of fluids is that the same manner because the motion of solids that are expected victimization elementary laws of physics each with the physical properties of the fluid.

Generally, the liquids and gases come beneath identical class as "fluids". Incompressible flows principally deals with cases of constant density. Once there's negligible in variation of density within the flow domain, then flow will be thought-about as incompressible. Since liquids are thought-about as incompressible, density of liquid over a broad vary of operational conditions decreases slightly with temperature and moderately with pressure [47-48]. On the contrary, the compressible flows are outlined as "variable density flows". Hence, it's applicable just for gases wherever, looking on the conditions of operation they'll be thought-about as incompressible/compressible.

During flow of the gases beneath bound conditions, the modification in density square measure little that the belief of the constant density are often created with cheap accuracy and in few different cases the density changes of the gases square measure considerably vital. Special attention is needed as a result of twin nature of gases and broad space within the study of motion of compressible flows is dealt severally within the subject of "gas dynamics".

Many engineering tasks need the compressible flow applications usually within the style of a building/tower to face up to winds, high speed flow of air over cars/trains/airplanes etc. Thus, gas dynamics is that the study of fluid flows wherever the squeezability and also the temperature changes become vital [49]. Here, the complete flow field is dominated by physicist waves and shock waves once the flow speed becomes supersonic.

In steady flow the conditions (velocity, pressure and cross-section) may differ from point to point but DO NOT change with time. Almost all fluids are compressible including water their density will be changed as pressure changes. Under steady conditions, provided that changes in pressure are small, it is usually possible to simplify analysis of the flow by assuming it is incompressible and has constant density [50]. By analysing fluid flow it is useful to visualise the flow pattern, this achieved by drawing lines joining points of equal velocity-velocity contours. These lines are called as streamlines.

When fluid is flowing over a solid boundary, e.g. the wall of a pipe or the surface of an aerofoil, obviously fluid does not flow into or out of the surface. It flows very close to a boundary wall and the direction must be parallel to the boundary. Streamlines are parallel Close to at solid boundary. At every points the flow direction of streamline is the direction of the fluid velocity, this is how they are defined, at close to the wall the velocity is parallel to the wall hence streamline is also parallel to the wall [51-53]. It is also important to recognise that the position of streamlines can change with time this is the case in unsteady flow. In steady flow, the position of streamlines does not change.

- Since the direction of movement of fluid same as direction of the streamlines, fluid can't cross a streamline.
- Streamlines cannot cross each other. It would indicate two different velocities if they were to cross at the given same point which is physically impossible.
- From the above point we can conclude that any particles of fluid starting on one streamline will remain on that same streamline throughout the fluid

The most useful technique in fluid flow analysis is by considering only a part of the total fluid in isolation from the rest. This can be achieved by imagining a tubular surface formed by streamlines along which the fluid flows. This tubular surface is referred as a streamtube. (2.4)

$$A_1 u_1 = A_2 u_2 = Q \quad (2.5)$$

This represents form of the continuity equation which is used most often; this equation is a very powerful tool in fluid.

1.1 The Bernoulli Equation-Work and Energy

Everyone knows that if we drop a ball than it accelerates towards down with an acceleration (neglecting the frictional resistance due to air). Speed of the ball after falling a distance h can be calculated by the formula $v^2 = u^2 = 2as$ (a = g and s = h) [55] [56]. This equation can be applied to a falling droplet of water as the same laws of motion, Applying the principle of conservation of energy is a general approach for obtaining the parameters of motion (of both solids and fluids). When friction is negligible the

Kinetic energy $\frac{1}{2}mv^2$ Gravitational potential energy m g h

(m is the mass, v is the velocity and h is the height above the datum).

To apply this to a falling droplet we have an initial velocity of zero, and it falls through a height of h. We know that kinetic energy + potential energy = constant so Initial kinetic energy + Initial potential energy = Final kinetic energy + Final potential energy

$$mgh = \frac{1}{2}mv^2 \quad (2.6)$$

So

$$v = \sqrt{2gh} \quad (2.7)$$

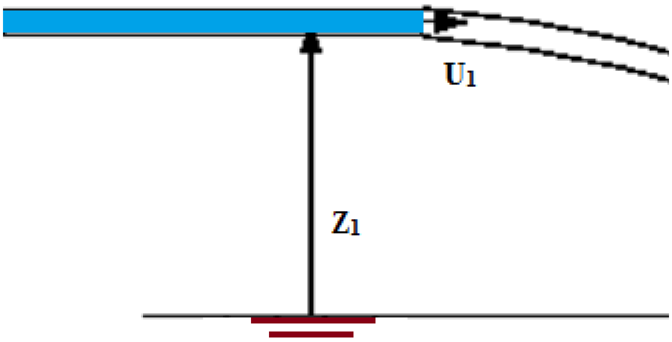

Figure 2.3: Showing the Trajectory of a jet of water

Consider the situation as in the figure above, from a pipe a continuous jet of water coming with velocity u1. A single particle of the liquid of mass m travels with jet and falls from height z1 to z2 and its velocity also changes from u1 to u2. In air the pressure is atmospheric everywhere hence there won't be no force due to pressure acting on the fluid when the jet is travelling in air. The only force which is acting on that is due to gravity. The sum of the potential energies and kinetic remains constant so

$$mgz_1 + \frac{1}{2}mu_1^2 = mgz_2 + \frac{1}{2}mu_2^2 \quad (2.8)$$

As m is constant this becomes

$$\frac{1}{2}u_1^2 + gz_1 = \frac{1}{2}u_2^2 + gz_2$$

We can expect reasonably accurate result as long as the weight of jet is large compared to frictional forces. It can be only applied while the jet is whole before it breaks up into droplets.

1.2 Method of solution

Stagnation / Total condition/state is referred when a moving fluid is decelerated isentropically to reach zero speed. For example, a gas has no thermodynamic state and the velocity when contained in a high pressure cylinder which is also known as stagnation/total condition. In a real flow field, if the actual conditions of density (ρ), entropy (s), temperature (T), enthalpy (h), internal energy (e), pressure (p) etc. are referred to as static conditions while the associated stagnation parameters are denoted as $p_0, T_0, \rho_0, h_0, e_0$ and s_0, respectively [57-60]. The simplified form of energy equation for steady, with no heat addition in one-dimensional flow across two regions 1 and 2 of a control volume is given by

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \quad (2.9)$$

1.2.1 Characteristics Conditions

Suppose take into consideration of an arbitrary flow field, where a fluid element is travelling at some velocity (V) and Mach number (M) at a given point 'A'. The static pressure, temperature and density are p T, and ρ, respectively. Now, imagine that fluid element is slowed down (if 1 M >) or speeded up (if 1 M <) unless and until the Mach number at 'A' reaches the sonic state. The temperatures will also changes in this process. When a real state in the flow is brought to sonic state with the imaginary situation of the flow field is known as the characteristics conditions [61]. The associated parameters are denoted as

$p^\bullet, T^\bullet, \rho^\bullet, a^\bullet$ etc.

Now, revisit the equation and use the relations for a calorically perfect gas, by replacing,

$$c_p = \frac{\gamma R}{\gamma - 1} \text{ and } a = \sqrt{\gamma R T}$$

Another form of energy equation is obtained as below;

$$\frac{a_1^2}{\gamma - 1} + \frac{u_1^2}{2} = \frac{a_2^2}{\gamma - 1} + \frac{u_2^2}{2} \quad (2.10)$$

At the imagined condition (point 2) of Mach 1, the flow velocity is sonic and u a 2 = ∗.

Then the Eq. (2.10) becomes,

$$\frac{a^2}{\gamma - 1} + \frac{u^2}{2} = \frac{a^{\bullet 2}}{\gamma - 1} + \frac{a^{\bullet 2}}{2} \quad (2.11)$$

or, $$\frac{a^2}{\gamma - 1} + \frac{u^2}{2} = \frac{\gamma + 1}{2(\gamma - 1)} a^{\bullet 2} \quad (2.12)$$

1.2.2 Critical speed and Maximum speed

The critical speed of sound (a∗) is same as that speed of the gas (u∗) at the sonic state i.e. u a M ∗∗ = = at 1. A gas can attain to its maximum speed (umax) when it is expanded hypothetically to zero pressure. Static temperature corresponding to this state is also zero [62] [63]. The maximum speed of the gas represents the speed corresponding to the complete transformation of kinetic energy associated with the random motion of gas molecules into the directed kinetic energy.

Rearranging Eq. (2.12), one can obtain the following equation;

$$T_0 = T + \left(\frac{\gamma - 1}{2\gamma R}\right)u^2;\ \text{At } T = 0;\ u = u_{\max} = \sqrt{\frac{2\gamma R T_0}{\gamma - 1}}$$

(2.13)

Or, $$\left(\frac{u_{\max}}{a_0}\right)^2 = \frac{2}{\gamma - 1} \quad (2.14)$$

Now, the Eq (2.13) & (2.14) can be simplified to obtain the following relation;

$$\frac{u_{\max}}{a^{\bullet}} = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \quad (2.15)$$

1.3 Steady Flow Adiabatic Ellipse

In an ellipse all the points have same total energies; each point differs from one another owing to relative proportions of kinetic and thermal energies corresponding to different Mach numbers. Now by replacing

$$c_p = \frac{\gamma R}{\gamma - 1} \text{ and } a = \sqrt{\gamma R T};$$

$$\frac{u^2}{2} + \frac{\gamma R}{\gamma - 1}T = c_1 \Rightarrow u^2 + \left(\frac{2}{\gamma - 1}\right)a^2 = c \quad (2.16)$$

When, T u u= = 0, max u= max 2. Then, Eq. (2.16) is written as follows;

$$u^2 + \left(\frac{2}{\gamma - 1}\right)a^2 = u_{\max}^2 \Rightarrow \frac{u^2}{u_{\max}^2} + \left(\frac{2}{\gamma - 1}\right)\frac{a^2}{u_{\max}^2} = 1$$

(2.17)

Replacing value of umax 2 from Eq. (2.17), we can write the following expression;

$$\frac{u^2}{u_{\max}^2} + \frac{a^2}{a_0^2} = 1 \quad (2.18)$$

This represents the equation of an ellipse with major axis as umax and minor axis as a0. Now, rearrange Eq. (4.3.20) in the following form.

$$a^2 = a_0^2 - \left(\frac{u^2}{u_{\max}^2}\right)a_0^2 \quad (2.19)$$

Now, differentiate Eq. (2.19) with respect to u and simplify.

$$\frac{da}{du} = -\left(\frac{\gamma - 1}{2}\right)\left(\frac{u}{a}\right) = -\left(\frac{\gamma - 1}{2}\right)M \Rightarrow M = -\left(\frac{2}{\gamma - 1}\right)\frac{da}{du}$$

(2.20)

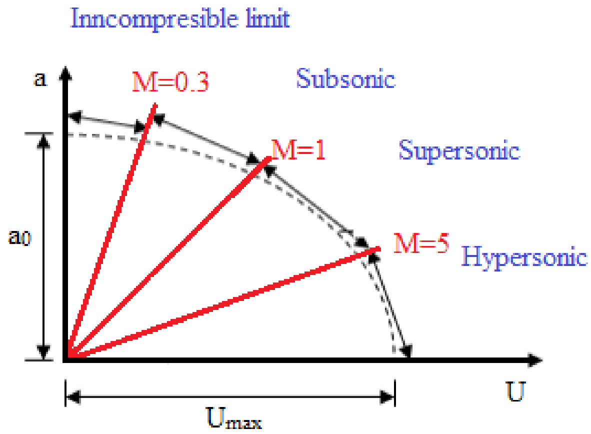

Figure 2.4: Showing steady flow adiabatic ellipse

Hence, the point to point change of slope on the ellipse indicates change in Mach number and hence the speed of velocity and sound. Thus, it gives the direct comparison of the relative magnitudes of kinetic and thermal energies [64]. Different compressible flow can be obtained with the knowledge of slope in Fig. 2.4. The following important inferences may be drawn;

-The changes in Mach number are caused mainly due to the changes in speed of sound in high Mach numbers flows.
-The changes in Mach number are caused mainly due to the changes in the velocity in low Mach numbers flows.
-When the value of flow Mach number is below 0.3, the changes occurred in speed of sound is very negligible small and the flow is treated as incompressible.

2. Analysis of Flow Patterns (Analysis of Flow)

2.1 Fluid Dynamics Background

Liquids, such as water and gases, such as air are both fluids. Before proceeding to the partial deferential equations that describe the motion of such fluids, we shall know the basis for the distinction between these two types of fluids and then know the relevant thermodynamic relationships.

2.1.1 Phases of Matter

Phases of matter are broadly categorized into solids and fluids. Simply stated, retain a rigid shape and solids resist deformation; in particular, stress is a function of the strain. Fluids does not resist deformation and it take on the shape same of its container owing to their inability to support shear stress in static equilibrium. More precisely, the stress is a function of the strain rate [65]. The distinction between fluids and solids is not so simple and is based upon both the time scale of interest and the viscosity of the matter [66] [67] [68]. Consider inorganic polymer-based toy known as Silly Putty™ in the U. S. (and Pongo™ in Italy), for instance, it is conserved either as a solid or as a fluid, depending on the time period over which it is observed. Fluids include liquids, gases and plasmas. A liquid is a fluid which has a property that it conforms to the shape of its container while retaining its constant volume independent of pressure. A gas is known has a compressible fluid which not only conforms to the shape of its container but also it expands to occupy the whole container. Plasma is a ionized gas.

The mathematical description of the states of matter appears implausible. Goodstein (1975) states that “Precisely what do we mean by the term liquid? Asking what is a liquid is like asking what is life; we usually know it when we see it, but the existence of some doubtful cases makes it hard to define precisely.” In his book, Anderson (1963) asks “How does one describe a solid from a really fundamental point of view in which the atomic nuclei as well as electrons are treated truly quantum mechanically? How and why does a solid hold itself together? I have never yet seen a satisfactory fully quantum mechanical description of a solid.” Phase diagrams seem to require well-defined criteria, but always there are states at the boundaries of the phases that cannot be defined precisely [69] [70].

To be more precise, one should speak of “fluid-like” and “solid-like” behaviour; time and the length scales of interest will predict the distinction. Earth’s mantle flow and Glaciers are thus fluid-like if time scale is very large, while if the time scale is very small they can be considered as solids. However, it is not a matter of the time scale, even if the length scale is on the order of miles and time scale is on the order of months, the Earth’s mantle shall be modelled as a solid, suppose if length scale is the order of nanometres or lesser it is appropriate to a fluid. This suggests the need for appropriate non dimensional numbers. In rational mechanics, the non dimensional Deborah number is the ratio of a process time to an inherent relaxation time. The same lump of borosiloxane can be observed to flow as a fluid if the time scale is sufficiently large, to deform like an elastic solid, or to shatter like brittle glass.

Perhaps, the most accurate and lucid discussion of the distinction between solid-like and fluid-like behavior was given by Maxwell (1872): “What is required to alter the form of a soft solid is a sufficient force, and, this when applied produces its effect at once. In the case of viscous fluid it is time that is required, and if enough time is given, the very smallest force will produce a sensible effect, such as would require a very large force if suddenly applied. Thus a block of pitch may be so hard that cannot make a dent in it by striking it with = knuckles; and yet it will in course of time, flatten itself by its own weight, and glide downhill like a stream of water.” Maxwell is clearly referring to the importance of time scales and to a material that seems to have two time scales [71]. However, materials can have more than two time scales.

2.1.2 Thermodynamic Relationships

The basic thermodynamic variables of most interest in fluid dynamics are density ρ (or specific volume V = 1/ρ), the temperature T, the pressure p, the specific entropy s, the specific internal energy e, and the specific enthalpy h. [72] (A specific quantity is one per unit mass.), Only two are independent of these state variables, and the remaining can be expressed as the functions of two independent variables. In the classical Gibbs axiomatic formulation, equation of state is

$$e = e(V, s) \quad (2.21)$$

and the variables temperature and pressure are defined by

$$p = -\frac{\partial e}{\partial V} \quad \text{and} \quad T = \frac{\partial e}{\partial s}, \quad (2.22)$$

Where p and T are positive, by forming the total deferential of the relation e =e (V, s), we obtain the fundamental thermodynamic relation

$$T\ ds = de + p\ dV. \quad (2.23)$$

This can be considered as a corollary of the second law of thermodynamics, and it defines the specific entropy, denoted by s.
The (specific) total energy is defined as

$$E = e + \frac{1}{2}(u \cdot u) \quad (2.24)$$

3. Results and discussion

The results of Newtonian theory for the in viscid flow over a flat plate are plotted in Fig. 2.5 and the following important observations can be made;

-The value of lift-to-drag ratio increases monotonically when the inclination angle decreases. It is mainly due to the fact that the Newtonian theory does not account for skin friction drag in the calculation. When skin friction is added, the drag becomes a finite value at 00 inclination angle and the ratio approaches zero.

-The lift curve reaches its peak value approximately at an angle of 550. It is quite realistic, because most of the practical hypersonic vehicles get their maximum lift in this vicinity of angle of attack.

-The lift curve at lower angle (0-150) shows the non-linear behavior. It is clearly the important characteristics feature of the hypersonic flows.

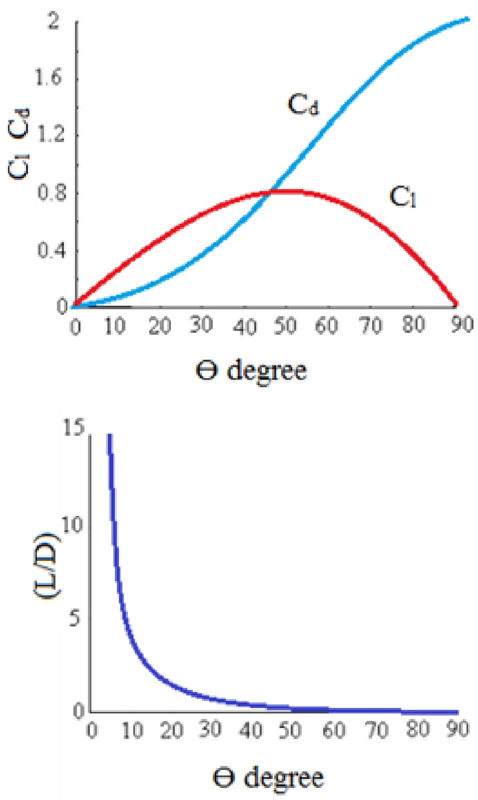

Figure 2.5: Aerodynamic parameters for a flat plate inclined at an angle

In the limit of M∞ → ∞, cpmax can be obtained as below;

$$c_{p\max} \rightarrow \left[\frac{(\gamma+1)^2}{4\gamma}\right]^{\frac{\gamma}{\gamma-1}}\left(\frac{4}{\gamma+1}\right) \quad (2.25)$$

$$\rightarrow 1.839\ (\gamma=1.4) \quad (2.26)$$

$$\rightarrow 2\ (\gamma=1)_s \quad (2.27)$$

The expression in Eq. (2.27) is called as the modified Newtonian law. The following important observation may be made.

- The modified Newtonian law does not follow the Mach number independence principle.
- When both M∞ → ∞ → and 1 γ, the straight Newtonian law is recovered from modified theory.
- The modified Newtonian theory is a very important tool to estimate the pressure coefficients in the stagnation regions in the hypersonic flow fields of the blunt bodies.

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