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Inverse Soft Rough Sets and Its Application in Decision Making

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Abstract: ***In present study, we have established a new approach called inverse soft rough set by combining inverse soft set and rough set. We have defined inverse soft approximation space. Basic properties of inverse soft approximation space are presented and supported by some illustrative examples. We have defined new types of inverse soft sets such as inverse full soft set and inter- section complete inverse soft set. The notion of inverse soft rough equal relations is proposed and related properties are examined. An algorithm is established to handle decision making problems by using inverse soft rough set.***

Keywords: Rough set, Soft set, Inverse Soft set, Soft rough set, Inverse soft rough set, Decision making problem

1. Introduction

In almost all concepts, we are meeting in various research field deal with imprecise data. Many researchers have worked on and they use mathematical principles based on uncertainty and imprecision. In 1965 Zadah proposed Fuzzy set theory [8] based on membership function to describe uncertainty. Rough set theory was initiated by pawlak [6] in 1982 for dealing with vagueness and granularity in information system based on equivalence relation. In 1999 Molodstov [7] proposed soft set theory as a new mathematical tool for dealing with uncertainties and vagueness. Application of soft set theory in other disciplines and real life problems are progressing rapidly in recent years. Majhi et al. [9] (2002), presented the application of soft set theory in decision making problem. Based on fuzzy soft sets, Ray and Majhi[10] (2007) presented method of object recognition from an imprecise multi-observation data and applied in decision making problems. F. Feng et al. [2] (2010) introduced soft rough fuzzy sets. F.Feng et al.[1] (2011) de- fined soft rough set by combining rough set and soft sets. In 2016, cetkin, Aygunolu and Agum [3] initiated a new approach of inverse soft set to handling soft decision making problems. In 2019 [4] Khalid and Hasson introduce the notion of inverse fuzzy soft set and its application in decision making problems .In 2020 [5] Demirtas, Hussin, Dalkilic initiated inverse soft rough sets and develop an algorithm to solve decision making problems.

The aim of this paper is to further study the properties of inverse soft rough set and some related notions. In section 2,the basic definitions of soft set ,soft rough set are presented. In section 3,the concept of inverse soft set and some of its operations are defined. In section 4 , definition of inverse soft rough set and some of its properties are studied. Full inverse soft set and intersection complete inverse soft set are introduced. In section 5, we define inverse soft rough equal relation in terms of inverse soft rough approximation and present some related properties. In section 6,an algorithm is constructed by using inverse soft rough set theory. A decision making problem is discussed by using that algorithm .Finally, conclusions are presented in last section.

2. Preliminaries

Definition 2.1 (Pawlak 1982): Let U be a non empty set of objects called Universe, and R be an equivalence relation on U. If X U can be written as union of some equivalence classes of U/R then we say X is definable otherwise it is not definable with respect to the knowledge R. If X is not definable then we can approximate it by two definable sets.

$\underline{R}X = \{x \in U \mid [x]_R \subseteq X\} = \cup\{[x]_R \mid [x]_R \subseteq X\}$

$RX = \{x \in U \mid [x]_R \cap X \neq \Phi\} = \cup\{[x]_R \mid [x]_R \cap X \neq \Phi\}$

be known as the R-lower and R-upper approximations of X respectively, where $[x]_R$ denote the equivalence class of x ∈U by an equivalence relation (knowledge) R, that is $[x]_R \in$ U/R

If $\underline{R}X = \bar{R}X$ then X is definable otherwise X is Rough
The set $\bar{R}X \setminus \underline{R}X$ is called Boundary region of X through the knowledge R.

Let U be an universe of objects, E denotes the set of certain parameters, ℘ (U) denote power set of U.

Definition 2.2 (Malodtsov 1999): A pair S= (F, A) is called soft set over U where A$\subseteq$ *E, and F*: *A* → ℘ (U) is a mapping i.e. softest over U is parameterized family of subset of the universe U.

For each e ∈ A, F (e) is considered as the set of e-approximate elements in the softest S=(F,A)

Example 2.1: Suppose soft set (F, E) describe attractiveness of the houses in which Mr. X is going to buy U=set of houses under consideration=$\{h_1, h_2, h_3, h_4, h_5, h_6\}$, E= set of parameters =$\{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ where e_1=expensive , e_2=beautiful, e_3=wooden, e_4=cheap, e_5=in the green surrounding, e_6=modern and e_7=in good repair; A=$\{e_1, e_2, e_3,$

e_4, e_5} ⊆E, F(e_1)={h_1, h_3, h_4, h_5}, F(e_2)={h_1, h_2, h_3, h_4, h_5}, F(e_3)={h_2, h_3, h_6}, F(e_4)={h_1, h_4, h_5, h_6}and F(e_5)={h_1, h_3, h_4, h_5, h_6}. Hence (F, A)={*expensive* = {h_1, h_3, h_4, h_5}, *beautiful* = {h_1, h_2, h_3, h_4, h_5}, *wooden* = {h_2, h_3, h_6}, *cheap* = {h_1, h_4, h_5, h_6}, *in the green* {h_1, h_3, h_4, h_5, h_6}}

Table 1: for soft set (F, A)

	h_1	h_2	h_3	h_4	h_5	h_6
e_1	1	0	1	1	1	0
e_2	1	1	1	1	1	0
e_3	0	1	1	0	0	1
e_4	1	0	0	1	1	1
e_5	1	0	1	1	1	1

Definition 2.3 (Feng et al. 2011): Let S= (F, A) be a soft set over U, P= (U, S) is called a soft approximation space. For any X⊆ U, we define the following two operations

$\underline{apr}_P(X)=\{u \in U : \exists a \in A, u \in F(a) \subseteq X\}$

$\overline{apr}_P(X) = \{u \in U : \exists a \in A, u \in F(a) \cap X \neq \emptyset\}$

for all X ⊆ U, are called lower Soft approximation and Upper lower Soft approximation of X with respect to P respectively.

If $\underline{apr}_P(X) = \overline{apr}_P(X)$, X is called Soft definable in P

Otherwise X is called Soft rough Set in P.

3. Inverse Soft set:

Definition 3.1(Cetkin et al. 2016): Let U be an universe of objects, E denotes the set of certain parameters, ℘ (E) denote power set of E i. e. the set of all subsets of the parameter set, E. A pair S= (F, U) is called an Inverse soft set over E where F:U → ℘ (E) is a mapping.

In Example 2.1.U=set of houses under consideration= {h_1, h_2, h_3, h_4, h_5, h_6}and E= set of parameters ={e_1, e_2, e_3, e_4, e_5, e_6, e_7} where e_1=expensive , e_2=beautiful, e_3=wooden, e_4=cheap, e_5=in the green surrounding, e_6=modern, e_7=in good repair and F(h_1)={e_1, e_2, e_4, e_5}, F(h_2)={e_2, e_3, e_6}, F(h_3)={e_1, e_2, e_3, e_5, e_7}, F(h_4)={e_1, e_2, e_3, e_4, e_5}, F(h_5)={e_1, e_2, e_4, e_5}, F(h_6)={e_3, e_4, e_5, e_7}.Hence (F,U)={*F* (h_1) = {*expensive, beautiful, cheap, green surrounding*}, *F* (h_2) = {*expensive, wooden, modern*}, *F* (h_3) = {*expensive, beautiful, wooden, green surrounding, good repair*}, *F* (h_4) = {*expensive, beautiful, wooden, cheap, green surrounding*}, *F* (h_5) = {*expensive, beautiful, cheap, green surrounding*}, *F* (h_6) = {*wooden, cheap, green surrounding, good repair*}}

That is h_1 has four parameters: expensive, beautiful, cheap, green surrounding. Similarly we can describe features of other house in the universe U. The Inverse soft set present's basic characters of given objects in the universe U in terms of parameters in the parameter set E.

Definition 3.2: Let (F,U) and (G,V) be two inverse soft sets over E, then (F,U) is called inverse soft subset of (G,V) symbolically written as (F,U) ⊆ (G,V) if U ⊆ V and for each u ∈ U there exist v ∈V such that F(u) ⊆ G(v)

Definition 3.3: Two inverse soft set (F,U) and (G,V) over E are said to be inverse soft equal set symbolically written as (F,U)=(G,V) if (F,U) ⊆ (G,V)and (G,V) ⊆ (F,U)

Definition 3.4: An inverse soft set (F, U) over E is said to be null inverse soft set written as N_E if F(u)=φ u U

Definition 3.5: An inverse soft set (F,U) over E is said to be whole inverse soft set or absolute inverse soft set written as A_E if F(u)=E ∀*u* ∈ U

Definition 3.6: The complement of an inverse soft set (F, U) denoted by $(F, U)^r$ is defined as the soft set (F^r ,U) where F^r:U→℘ (E) and F^r(u)=E-F(u) ∀*u*∈U

Definition 3.7: If (F,U) and (G,V) are two inverse soft sets over E then (F,U) AND (G,V) denoted by (F,U) ⋀(G,V)= (H,U X V) where H (u, v)= F (u) ∩ G(v) ∀ (u, v) ∈ U X V

Definition 3.8: If (F,U) and (G,V) are two inverse soft sets over E then (F,U) OR (G,V) denoted by (F, U) ∨ (G,V) = (H, U X V) where H (u, v)= F(u) ∪ G (v) ∀ (u, v) ∈U X V

Definition 3.9: Let (F, U) and (G, V) be two inverse soft sets over E,

(i) The extended intersection of (F,U) and (G,V) denoted by (F,U) $\cap_E$ (G,V) is defined as the inverse soft set (H,W) where W=U∩V and ∀*w* ∈ W

$$H(w) = \begin{cases} F(w) & \text{if } w \in U \setminus V \\ G(w) & \text{if } w \in V \setminus U \\ F(w) \cap G(w) & \text{if } w \in U \cap V \end{cases}$$

(ii) The restricted intersection of (F,U) and (G,V) denoted by (F,U) $\cap_R$ (G,V) is defined as the inverse soft set (H,W) where W=U∩V and H(w)=F(w)∩ G(w) ∀*w* ∈ W

(iii) The extended union of (F,U) and (G,V) denoted by (F,U) $\cup_E$ (G,V) is defined as the inverse soft set (H,W) where W=U∪V and ∀*w* ∈ W

$$H(w) = \begin{cases} F(w) & \text{if } w \in U \setminus V \\ G(w) & \text{if } w \in V \setminus U \\ F(w) \cup G(w) & \text{if } w \in U \cap V \end{cases}$$

(iv) The restricted union of (F,U) and (G,V) denoted by

(F,U) $\cup_R$ (G,V) is defined as the inverse soft set (H, W) where W=U∩V and H(w)=F(w) ∪ G(w) $\forall \omega \in W$

Remarks: 1) In case of soft set the objects in the universe (which owned some characters)are described by determining the object set which correspond to any parameter of the parameter set. i. e. F (e_1)= {h_1, h_3, h_4, h_5} show that houses 1, 3, 4, 5 have the characters 'expensive'

2) The inverse soft set provides an another opinion for the universe U is described by determining the parameter $e_i \in$ E which correspond to any object $h_j \in$ U For a house $h_i \in$ U , one wants to know that the features are owned b the house $h_i \in$ U ,that answer provides by inverse soft set i. e. F(h_1)= {e_1, e_2, e_4, e_5} shows that house h_1 has four features :expensive, beautiful, cheap and in the green surrounding

3) An inverse soft set provides a new approach to describe the essence character of every object in the universe U.

4. Inverse Soft rough set:

Definition 4.1: Let S=(F,U) be a inverse soft set over E. Then the pair I=(E,S) is called Inverse soft approximation space. Based on inverse soft approximation space I ,we define two operators , for every subset A⊆ E

$\underline{isap}_I$ $(A)=\{e \in E : \exists u \in U, e \in F(u) \subseteq A\}$

$isap_I$ $(A)=\{e \in E : \exists u \in U, e \in F(u) \cap A /= \varphi\}$

Two sets $\underline{isap}_I$ (A), $\overline{isap}_I$ (A) are called lower inverse soft approximation and upper inverse soft approximation of A in I respectively.

$IPos_I(A)= \underline{isap}_I(A)$

$INeg_I(A)=isap_I\ (A)$

$IBad_I(A) =\overline{isap}_I(A)- \underline{isap}_I(A)$

The above sets are called soft-I positive region, the soft-I negative region and Soft-I boundary region of A respectively.

If $\overline{isap}_I$ (A)=$\underline{isap}_I$ (A), A is said to be inverse soft definable in I. Otherwise A is said to be inverse soft rough set in I

$\underline{isap}_I(A) \subseteq A$ and $\underline{isap}_I(A) \subseteq \overline{isap}_I(A)$ $\forall A \subseteq E$

But A ⊆ $\overline{isap}_I(A)$ does not hold in general.

Example 4.1: Tabular representation of inverse soft set S=(F,U) in example 2.1 is given below

Table 1: invese soft set S=(F,U)

U	e_1	e_2	e_3	e_4	e_5	e_6	e_7
h_1	1	1	0	1	1	0	0
h_2	0	1	1	0	0	1	0
h_3	1	1	1	0	1	0	1
h_4	1	1	1	1	1	0	0
h_5	1	1	0	1	1	0	0
H_6	0	0	1	1	1	0	1

Let A={e_1, e_2, e_3, e_4, e_5} ⊆ E ,I=(E,S)

$\underline{isap}_I$ (A)={e_1, e_2, e_3, e_4, e_5}, $\overline{isap}_I$ (A)={e_1, e_2, e_3, e_4, e_5, e_6, e_7}

$isap_I$ (A)=$\underline{isap}_I$ (A) so A is an inverse soft rough set, $\underline{isap}_I(A) \subseteq A$, $\overline{isap}_I(A) \subseteq \underline{isap}_I(A)$

Proposition4.1: Let S=(F,U) be an inverse soft set over E and I=(E,S) be an inverse soft approximation space .Then we have

$$\underline{isap}_I(A)=\bigcup_{u\in U}\{\ \mathrm{F(u)}\mid \mathrm{F(u)}\subseteq A\}$$

$$\overline{isap}_I(A)=\bigcup_{u\in U}\{\ \mathrm{F(u)}\mid \mathrm{F(u)}\cap A\neq\phi\}$$

for all A ⊆ E

Definition 4.2: Let S=(F,U)be an inverse soft set over E if

$$\bigcup_{u\in U} F\ (u) = E$$

then S is said to be Inverse full soft set.

Let S=(F, U) be an inverse full soft set and I=(E,S) be an inverse soft approximation space and A ⊆ E

We define four basic classes of inverse soft rough sets

* A is said to be roughly inverse soft definable if $\underline{isap}_I(A) \neq \varphi$ and $\overline{isap}_I(A) \neq$E
* A is said to be internally inverse soft definable if $\underline{isap}_I(A) = \varphi$ and $\overline{isap}_I(A) \neq$E
* A is said to be externally inverse soft definable if $\underline{isap}_I(A) \neq \varphi$ and $\overline{isap}_I(A)$=E
* A is said to be totally inverse soft definable if $\underline{isap}_I(A) = \varphi$ and $\overline{isap}_I(A)$=E

Proposition 4.2: Let S=(F,U) be an Inverse soft set over E ,I=(E,S) an inverse soft approxi- mation space and A,B ⊆ E ; we have

1)$\underline{isap}_I(\phi)=\overline{isap}_I(\phi)=\phi$

2)$\underline{isap}_I(E)=\overline{isap}_I(E)=\bigcup_{u\in U}\mathrm{F(u)}$

3)A ⊆B $\Rightarrow \underline{isap}_I(A)\subseteq \underline{isap}_I(B)$

4)A ⊆B $\Rightarrow \overline{isap}_I(A)\subseteq \overline{isap}_I(B)$

5)$\overline{isap}_I(A\cup B) = \overline{isap}_I(A) \cup \overline{isap}_I(B)$

6)$\overline{isap}_I(A\cap B) \subseteq \overline{isap}_I(A)\cap \overline{isap}_I(B)$

7)$\underline{isap}_I(A\cup B) \supseteq \underline{isap}_I(A)\cup \underline{isap}_I(B)$

8)$\underline{isap}_I(A\cap B)\subseteq \underline{isap}_I(A) \cap \underline{isap}_I(B)$

Theorem 4.1: Let S=(F,U) be an Inverse soft set over E ,I=(E,S) an inverse soft approximation space and A ⊆ E ; we have

1) $\underline{isap}_I(\overline{isap}_I(A))=\overline{isap}_I(A)$
2) $\overline{isap}_I(\underline{isap}_I(A)) \supseteq \underline{isap}_I(A)$
3) $\underline{isap}_I(\underline{isap}_I(A))=\underline{isap}_I(A)$
4) $\overline{isap}_I(\overline{isap}_I(A)) \supseteq \overline{isap}_I(A)$

Proof: 1) Let X= $\overline{isap}_I$ (A) and e∈ X

$\Rightarrow e \in F(u)$ and $F(u) \cap A \neq \phi$ for some $u \in U$
But $X=\overline{isap}_I(A)= \bigcup_{u\in U} \{ F(u) \mid F(u) \cap A \neq \phi\}$
there exist u∈ U such that e ∈F(u) ⊆X
so $X \subseteq \underline{isap}_I(X)$)..........(1)
We know $\underline{isap}_I(X)) \subseteq X$........(2)for any $X \subseteq E$
combining (1) and (2) $\underline{isap}_I(X)=X$
$\Rightarrow \underline{isap}_I(\overline{isap}_I(A))=\overline{isap}_I(A)$
(2)Let $X=\underline{isap}_I(X)$ and e∈ X
⇒ e∈F(u)⊆A for some u ∈U
but $X=\underline{isap}_I(A)=\bigcup_{u\in U} \{ F(u) \mid F(u)\subseteq A\}$
there exist $e \in F(u)$ and $F(u) \cap X = F(u)\neq\phi$
$\Rightarrow e\in \in \overline{isap}_I(X)$
so $X \subseteq \overline{isap}_I(X)$
$\underline{isap}_I(A) \subseteq \overline{isap}_I(\underline{isap}_I(A))$
3)Let $X=\underline{isap}_I(X)$ and e∈ X
⇒ e∈F(u)⊆A for some u ∈U
but $X=\underline{isap}_I(A)=\bigcup_{u\in U} \{ F(u) \mid F(u)\subseteq A\}$
there exist u ∈U such that e ∈F(u) ⊆X
$\Rightarrow e \in \underline{isap}_I(X)$
$\Rightarrow X \subseteq \underline{isap}_I(X)$.......(1)
we know $\underline{isap}_I(X) \subseteq X$.......(2) for any $X \subseteq E$
Combing (1) and (2) $X = \underline{isap}_I(X)$
$\underline{isap}_I(\underline{isap}_I(A))=\underline{isap}_I(A)$
4)Let $X=\overline{isap}_I(A)$ and e∈ X
$\Rightarrow e \in F(u)$ and $F(u) \cap A \neq \phi$ for some $u \in U$
but $X=\overline{isap}_I(A)= \bigcup_{u\in U} \{ F(u) \mid F(u) \cap A \neq \phi\}$
there exist $e \in F(u)$ and $F(u) \cap X = F(u)\neq\phi$
$\Rightarrow e \in \overline{isap}_I(X)$
so $X \subseteq \overline{isap}_I(X)$
$\overline{isap}_I(A) \subseteq \overline{isap}_I(\overline{isap}_I(A))$

Example 4.2: Let U={$v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8$}
E = {$e_1, e_2, e_3, e_4, e_5, e_6$}, A= {$e_1, e_2, e_3, e_4$}, A ⊆ E

Let S=(F,U) be a inverse soft set over E given in table 2 and inverse soft approximation space I= (E, S)

Table 2: Inverse soft set S= (F, U)

U	e_1	e_2	e_3	e_4	e_5	e_6
v_1	0	1	1	0	1	0
v_2	0	0	1	o	1	0
v_3	0	0	0	0	1	1
v_4	0	0	0	1	0	0
v_5	0	1	0	0	1	0
v_6	0	0	0	0	1	0
v_7	1	0	0	1	0	0
v_8	1	0	0	0	1	0

$\underline{isap}_I$ (A) = {e_1, e_4}
$isap_I$ (A)={e_1, e_2, e_3, e_4, e_5}

$\underline{isap}_I$ ($\overline{isap}_I$ (A))={e_1, e_2, e_3, e_4, e_5}

$isap_I$ ($\underline{isap}_I$ (A))={$e_1, e_2, e_3, e_4, e_5, e_6$}

$\underline{isap}_I$ ($\underline{isap}_I$ (A))= {e_1, e_4}

$isap_I$ ($\underline{isap}_I$ (A))=$\underline{\{e_1}$, e_2, e_3, e_4, e_5, e_6}

$\underline{isap}_I$ ($\overline{isap}_I$ (A))= $\overline{isap}_I$ (A)

$isap_I$ ($\underline{isap}_I$ (A) ⊇ $\underline{isap}_I$ (A)

$\underline{isap}_I$ ($\underline{isap}_I$ (A))= $\underline{isap}_I$ (A)

$isap_I$ ($isap_I$ (A)) ⊇ $isap_I$ (A)

Proposition 4.3: Let S=(F,U) be an Inverse full soft set over E, I=(E,S) an inverse soft approximation space and A ⊆ E ; we have
(1)-$\underline{isap}_I$ (A) ⊆ $\overline{isap}_I$ (−A)
(2)-$isap_I$ (A) ⊆ $\underline{isap}_I$ (−A)
Proof: 1) let e ∈ -$\underline{isap}_I$ (A)

Example 4.3: In example 3 U= {$u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8$}
E ={$e_1, e_2, e_3, e_4, e_5, e_6$} , A={$e_1, e_2, e_3, e_4$}, A ⊆ E
$\underline{isap}_I$ (A)={e_1, e_4}, $\overline{isap}_I$ (A)={e_1, e_2, e_3, e_4, e_5}, -A={e_5, e_6}, -$\underline{isap}_I$ (A)={e_2, e_3, e_5, e_6}, -$\overline{isap}_I$ (A)={e_6}, $\underline{isap}_I$ (-A)= {e_5, e_6}, $\overline{isap}_I$ (-A)= {e_1, e_2, e_3, e_5, e_6}, -$\underline{isap}_I$ (A) ⊆ $\overline{isap}_I$ (-A) and -$\overline{isap}_I$ (A) ⊆ $\underline{isap}_I$ (-A)

Proposition 4.4: Let S= (F, U) be an Inverse full soft set over E and I= (E, S) an inverse soft approximation space then for any A ⊆ E; A is soft definable iff $\overline{isap}_I$ (A) =A

Proof: Suppose A is soft definable then $\underline{isap}_I$ (A)= $\overline{isap}_I$ (A) and $\underline{isap}_I$ (A) ⊆ A from definition
⇒ $\overline{isap}_I$ (A) ⊆A................ (1)

Since S= (F, U) be an Inverse full soft set over E , A ⊆ E

for any e∈A there exist some u ∈ U such that e∈F(u)
⇒ $e \in A \cap F(u)$

Since e ∈ A∩F(u), A∩F(u)/= φ

⇒e ∈$\overline{isap}_I$ (A)

A $\subseteq \overline{isap}_1(A)$..................(2)

combining 1 and 2 , $\overline{isap}_1\ (A)$ =A

Conversely Suppose $\overline{isap}_1\ (A)$ =A
i.e. $\overline{isap}_1\ (A) \subseteq$A ∀ A ⊆E. ....(3)
let e ∈ $\overline{isap}_1(A)$
⇒ $e \in F\ (u)$ such that F(u)∩A /= φ for some u ∈U
⇒ $e \in F\ (u) \subseteq \overline{isap}_1\ \overline{(A)}$
⇒ $e \in F\ (u) \subseteq \overline{isap}_1\ (A) \subseteq$ A by (3)
⇒ $e \in F\ (u) \subseteq$ A
⇒ e ∈$\underline{isap}_I\ (A)$

$\underline{isap}_I\ (A) \subseteq \overline{isap}_1(A)$

from definition $\overline{isap}_1(A) \subseteq \underline{isap}_I\ (A)$

combining $\underline{isap}_I\ (A)$= $\overline{isap}_1(A)$

so A is soft definable

Example 4.4: Consider S= (F, E) which describes 'system of influenza'. Suppose that the universe U={u_1, u_2, u_3, u_4, u_5, u_6} consists of 6 patients
E ={e_1, e_2, e_3, e_4} is a set of parameters, e_1 stands for fever ;e_2 stands for dry cough;e_3 stands
for headache ;e_4 stands for muscle pain

F(e_1)={u_3, u_4, u_5, u_6}, F(e_2)={u_1, u_2, u_3, u_6}, F(e_3)={u_3}, F(e_4)={u_1, u_2, u_3}
Let X= {u_1, u_2, u_3},
$\underline{apr}_P\ (X)$={u_1, u_2, u_3}
$\underline{aprP}\ (X)$= {u_1, u_2, u_3, u_4, u_5, u_6}

Remarks: By using available knowledge i.e. using soft rough set, we can decide infected patients. Hence patients u_1, u_2, u_3 are certainly infected patients but u_4, u_5, u_6 may probably infected patients

Example 4.5: Consider (F, U) be an 'inverse soft set' which describes 'symptoms of patients'. Suppose that the universe U= {u_1, u_2, u_3, u_4, u_5, u_6} consists of 6 patients
E = {e_1, e_2, e_3, e_4} is a set of parameters, e_1 stands for fever; e_2 stands for dry cough; e_3 stands for headache ;e_4 stands for muscle pain
F(u_1)={e_2, e_4}, F(u_2)={e_2, e_4}, F(u_3)={e_1, e_2, e_3, e_4}, F(u_4)={e_1}, F(u_5)={e_1}, F(u_6)={e_1, e_2}

Let A= {e_1, e_2, e_4}; $\underline{isap}_I\ (A)$= e_1, e_2, e_4 , $isap_I\ (A)$= e_1, e_2, e_3, e_4

Remarks: By using inverse soft rough set ,we can decide most appropriate symptoms (parameters) set for detections of infected persons. Hence e_1, e_2, e_4 are sure symptoms for detection of influenza. But e_3 may probable symptom for detection of influenza.

3. Decision making problem based on inverse soft rough sets

Algorithm:
1) Input the inverse soft set (F,U) on E
2) Compute lower inverse soft approximation and upper inverse soft approximation for each parameters set selected by decision makers.
3) Select the parameter set where boundary region is minimum
4) The selected parameter set in step 3 is the most appropriate parameter set which is the best for selection of candidates.

Example 5.1: A company advertises to fill some posts. There are 10 candidates to take part in this recruitment process. There are two decision makers. One of them is from the department of Human recourses and other one is from board of directors. They asked to indicate the most appropriate best parameter set for selection of candidates. Let U={x_1, x_2, ..., x_{10}} be the set of 10 candidates, E represents the characteristics of the candidates i.e. E= {e_1, e_2, , e_7 }where e_1 stands for experience, e_2 stands for computer knowledge, e_3 stands for trained, e_4 stands for young, e_5 stands for highly educated, e_6 stands for marital status, e_7 stands for good health The decision makers consider set of parameters A= {e_1, e_2, e_4, e_7} and B= {e_1, e_2, e_5} respectively to evaluate the candidates. Let S= (F, U) be a inverse soft set over E in following table 3 and the inverse soft approximation space I= (E, S)

Table 3: Inverse soft set S= (F, U)

U	e_1	e_2	e_3	e_4	e_5	e_6	e_7
x_1	0	1	0	0	0	0	1
x_2	0	1	0	1	0	0	0
x_3	0	1	0	1	0	0	0
x_4	1	0	0	0	0	0	0
x_5	0	0	0	0	0	0	1
x_6	0	1	0	0	0	0	0
x_7	1	0	0	1	0	0	0
x_8	1	1	0	1	0	0	1
x_9	1	1	0	1	0	0	0
x_{10}	1	0	0	0	1	0	0

For A= {e_1, e_2, e_4, e_7}
$\underline{isap}_I\ (A)$={e_1, e_2, e_4, e_7}
$isap_I\ (A)$={e_1, e_2, e_4, e_5, e_7}
thus $\underline{isap}_I\ (A)$/=$isap_I\ (A)$
A is an inverse soft rough set and $IBad_I(A)$={e_5}
For B ={e_1, e_2, e_5}
$\underline{isap}_I\ (B)$={e_1, e_2, e_5}
$isap_I\ (B)$={e_1, $\underline{e_2, e_4}$, e_5, e_7}
thus $\underline{isap}_I\ (B)$/=$isap_I\ (B)$
B is an inverse soft rough set and $IBad_I(B)$={e_4, e_7}

Here $IBad_I\ (A)$ is the minimal comparative to $IBad_I\ (B)$.So the set A is the most appropriate parameter set and best for selection of candidates. If we take the characters e_1, e_2, e_4, e_7 as criteria for selection of candidates, it makes the most accurate judgment of the two decision makers and with a high degree of responsibility and judgment. According to the above parameter sets, the most appropriate parameter set is A, which is also the best for selection of candidates.

Remarks: By using soft rough set we can select the person/ goods/ infected patients but by using inverse soft rough set, we can select the best parameters (characters)/ symptoms which are necessary for selection process/detection of diseases. In the above example there are two decision makers. They choose two set of parameters to evaluate the candidates. The company wanted to know which parameter set is best and most appropriate for selection candidates. Using above technique, company can select the best appropriate parameter set for selection of candidates to fill posts.

5. Inverse Soft Rough Equal Relation

Definition 6.1: Let S= (F,U) be an inverse soft set over E and I=(E,S)an inverse soft approximation space, for any A, B ⊆ E; we define

A $\simeq_1$ B iff $\underline{\underline{isap}}_I\,(A)=\underline{\underline{isap}}_I\,(B)$

A $\overline{\sim}_1$ B iff $isap_I\,(A)=isap_I\,(B)$

A$\approx_I$ B iff A$\simeq_1$B and A $\overline{\sim}_1$ B

These binary relation are called lower inverse soft rough equal relation, upper inverse soft rough equal relation and inverse soft rough equal relation respectively.

Proposition 6.1: The lower inverse soft rough equal relation, upper inverse soft rough equal relation and inverse soft rough equal relation are equivalence relation over P(E)

Proposition 6.2: Let S=(F,U) be an inverse soft set over E and I=(E,S)an inverse soft approximation space, for any A, B, A_1, B_1 ⊆ E; we define

1) A $\overline{\sim}_1$B ⇐⇒ A $\overline{\sim}_1$ $(A \cup B)$ $\overline{\sim}_1$ B
2) A $\overline{\sim}_1$ A_1, B $\overline{\sim}_1$ B_1 ⇒ $(A \cup B)$ $\overline{\sim}_1$ $(A_1 \cup B_1)$
3) A $\overline{\sim}_1$ B ⇒ A ∪$(-B)$ $\overline{\sim}_1$ U
4) A ⊆ B, B $\overline{\sim}_1$ φ ⇒ A $\overline{\sim}_1$ φ
5) A ⊆ B, A $\overline{\sim}_1$U ⇒ B $\overline{\sim}_1$ U
6) A ⊆ B, B $\simeq_1$ φ ⇒ A $\simeq_1$ φ
7) A ⊆ B, A$\simeq_1$U⇒ B$\simeq_1$U

Proposition 6.3: Let S= (F, U) be an intersection complete inverse soft set over E and I= (E, S) an inverse soft approximation space. Then for any A, B, A_1, B_1 ⊆ E; we have

1) A $\simeq_1$ B ⇐⇒ A $\simeq_1$ $(A \cap B)$ $\simeq_1$ B
2) A $\simeq_1$ A_1, B $\simeq_1$ B_1 ⇒ $(A \cap B)$ $\simeq_1$ $(A_1 \cap B_1)$
3) A $\simeq_1$ B ⇒ A ∩$(-B)$ $\simeq_1$ φ

6. Conclusion

In this paper, we have presented an alternative method for computation of decision making problems by application of inverse soft rough set theory. We have established important properties of inverse soft rough approximation space with interesting examples. The notion of inverse soft equal relation is proposed and related properties are discussed. We proposed an algorithm to get best parameters for selection process in decision making problems by using inverse soft rough sets theory. The purpose of the paper is to select the key parameters in selection of an object or candidates by inverse soft rough sets theory

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References

[1] F.Feng X.Liu,V.Fotea,Y.Jun, soft set and Soft rough sets ,Information science 181(2011) 1125-1137
[2] F.Feng ,C.Li, B.D avvad, M.I.Ali,Soft set combining with Fuzzy sets and rough sets,Soft computing 2010,14,899- 911
[3] V.Cetkin, A.Aygunoglu and H.Aygun,. A new approach in handling soft decision making problems, Journal Nonlinear science application 9(2016),231-239
[4] A.M. Khalil and N.Hassan,Inverse Fuzzy soft set and application in decision making problem vol-11,no 1;2019
[5] N. Demirtas ,S.Hussain and O. Dalkilic, New approaches of Inverse soft rough sets and application in a decision making problems ,J. Appl. Math. Information ,vol 38 (2020) no 3 ,pp 335-349
[6] Z.Pawlak,Rough sets,International Jouncy of Computer and information science 91982),341- 356
[7] D.Molodtsov ,Soft set theory first result,Computer math application 37 (1999),19-31
[8] L.A. zadeh ,Fuzzy sets,Information Control,8 (1965),338-353
[9] P.K.Majhi ,A.R.Ray,R.Biswas,An aplication of soft sets in decision making problems, computer and Math- ematics with applications (2002) vol-44,no 89,pp-1077-1083
[10] P.K.maji,A.R.Roy ;A Fuzzy soft set theoretic approach to decision making problems,Journal of computational and applied math vol-23 no 2 pp-412-418
[11] M.I.Ali,On some new operation in soft set theory;Computer and Math with applica- tion (2009) vol-57,no-9,pp 1547-1553
[12] G.Liu and W.Zhu ;The algebraic Structure of generalized rough set theory; Information sci- ence 178 (2008),4105-4113
[13] D.Molodtsov ,The theory of soft sets (in Russia), URSS Publication,Moscow (2004)