International Journal of Science and Research (IJSR)
ISSN: 2319-7064
SJIF (2022): 7.942

Numerical Study of Higher Order Differential Equations Using Differential Transform Method

P. L. Suresh[1], Ch. Satyanarayana[2]

[1]Lecturer, Department of Mathematics, B V Raju Degree College, Bhimavaram, West Godavari Dt. Andhra Pradesh, India
Email: *p.arunasuresh[at]gmail.com*

[2]Lecturer, Department of Mathematics, B V Raju Degree College, Bhimavaram, West Godavari Dt. Andhra Pradesh, India

Abstract: ***In this article the Differential Transform method is working for obtaining solutions for higher order differential equations. This proposed technique gives the series of solutions which can be easily converted to exact ones. The differential transform method was productively applied to higher order differential equations The results of the study has established that the method is easy, effective and flexible. The result of the differential transform method is in good agreement with those obtained by using the already existing ones.***

Keywords: Differential Transform, Higher order differential equations, Taylor's Series

1. Introduction

Nonlinear phenomena have significant effects in applied mathematics, physics and related to engineering; many such physical phenomena are modeled in terms of nonlinear differential equations [3,4,10]. A variety of numerical and analytical methods have been developed to obtain precise approximate and analytic solutions for the problems in the literature [3,7,8,10,11,12]. The classical Taylor's series method is one of the earliest analytic techniques to many problems, especially ordinary differential equations. However, since it requires a lot of symbolic calculation for the derivatives of functions, it takes a lot of computational time for higher derivatives. Here, we introduce the update version of the Taylor series method which is called the differential transform method (DTM)[4,5]. The (DTM) is the method to determine the coefficients of the Taylor series of the function by solving the induced recursive equation from the given differential equation. The basic idea of the (DTM) was introduced by Zhou [5]. In what follows we introduce a few notations for the (DTM).

2. The Differential Transform Method

The transformation of the k th derivative of a function y(x) in one variable is defined as follows

$$\mathrm{Y}\,(\mathrm{k}) = \frac{1}{k!}\left[\frac{d^k(y(x))}{dx^k}\right]_{x=0} \qquad (1)$$

and the inverse transform of Y(k) is defined as

$$\mathrm{y}(\mathrm{x}) = \sum_{\mathrm{k}=0}^{\infty} \mathrm{Y}(\mathrm{k})\mathrm{x}^{\mathrm{k}} \qquad (2)$$

The following are the important theorems of the one dimensional differential transform method

Theorem 1: If then $\mathrm{Y}(\mathrm{k}) = \mathrm{M}(\mathrm{k}) \pm \mathrm{N}(\mathrm{k})$

Theorem 2: If $\mathrm{y}(\mathrm{x}) = \alpha\mathrm{m}(\mathrm{x})$, they(x) $= \mathrm{m}(\mathrm{x}) \pm \mathrm{n}(\mathrm{x})$, $\mathrm{Y}(\mathrm{k}) = \alpha\mathrm{M}(\mathrm{k})$

Theorem 3: If $y(\mathrm{x}) = \frac{\mathrm{dm}\,(\mathrm{x})}{\mathrm{dx}}$, then $\mathrm{Y}(\mathrm{k}) = (\mathrm{K}+1)\mathrm{Y}(\mathrm{k}+1)$

Theorem 4: If $\mathrm{y}(\mathrm{x}) = \mathrm{m}(\mathrm{x})\mathrm{n}(\mathrm{x})$, then $\mathrm{Y}(\mathrm{k}) = \sum_{\mathrm{r}=0}^{\mathrm{k}} \mathrm{M}(\mathrm{r})\mathrm{N}(\mathrm{k}-\mathrm{r})$

Theorem 5: If $\mathrm{y}(\mathrm{x}) = \mathrm{x}^{\mathrm{l}}$, then $\mathrm{Y}(\mathrm{k}) = \delta(\mathrm{k}-\mathrm{l}) = \begin{cases} 1, \text{if } \mathrm{k}=\mathrm{l} \\ 0, \text{if } \mathrm{k} \neq \mathrm{l} \end{cases}$

Theorem 6: If $\mathrm{y}(\mathrm{x}) = \frac{\mathrm{d}^2\mathrm{g}(\mathrm{x})}{\mathrm{dx}^2}$ then $\mathrm{Y}(\mathrm{k}) = (\mathrm{k}+1)(\mathrm{k}+2\mathrm{G}(\mathrm{k}+2)$

Theorem 7: If $\mathrm{y}(\mathrm{x}) = \frac{\mathrm{d}^{\mathrm{m}}\mathrm{g}(\mathrm{x})}{\mathrm{dx}^{\mathrm{m}}}$ then $\mathrm{Y}(\mathrm{K}) = (\mathrm{k}+1)(\mathrm{k}+2+\ldots+\mathrm{k}+\mathrm{m}\mathrm{G}(\mathrm{k}+\mathrm{m})$

Theorem 8: If $\mathrm{y}(\mathrm{x}) = 1$ then $\mathrm{Y}(\mathrm{k}) = \delta(\mathrm{k})$

Theorem 9: If $\mathrm{y}(\mathrm{x}) = \mathrm{x}$ then $\mathrm{Y}(\mathrm{k}) = \delta(\mathrm{k}-1)$

Theorem 10: If $\mathrm{y}(\mathrm{x}) = \mathrm{e}^{\mathrm{ax}}$ then $\mathrm{Y}(\mathrm{k}) = \frac{\mathrm{a}^{\mathrm{k}}}{\mathrm{k}!}$

Theorem 11: $If\ y(x) = (1+x)^m\ then\ Y(k) = \frac{m(m-1)(m-2)\ldots(m-k+1)}{k!}$

Theorem 12: $If\ y(x) = \sin(wx+\alpha)\ then\ Y(k) = \frac{w^k}{k!}sin(k\pi+\alpha)$

Theorem 13: $If\ y(x) = \cos(wx+\alpha)\ then\ Y(k) = \frac{w^k}{k!}cos(k\pi+\alpha)$ *where w and α are constants*

3. Applications

In this section, we apply the (DTM) to some ordinary differential equations

Problem 1: Consider the following initial value problem

$$\frac{d^2y}{dx^2} - 2p\frac{dy}{dx} + p^2y = e^{px}, p(\neq 0) is\ areal\ number, given\ that\ y(0) = 0, y(1) = 1/p \qquad (1)$$

Apply DTM to (1), we obtain

$(k+2)(k+1)Y-2p(k+1)Y(k+1)+p^2Y(k)=\frac{p^k}{k!}$

$put\ k=0\ then\ 2Y(2)-2pY(1)+p^2Y(0)=1$

$$2Y(2)-2p\left(\frac{1}{p}\right)+p^2(0)=1$$

$$Y(2)=\frac{1+2}{2}=\frac{3}{2}$$

Put k = 1, then $6Y(3)-2pY(2)+p^2Y(1)=p$

$$6Y(3)-2p\left(\frac{3}{2}\right)+p^2\left(\frac{1}{p}\right)=p$$

$$6Y(3)-p(3)+p=p$$

$$Y(3)=\frac{3p}{6}$$

And so on.

The solution is

$$y(x)=\sum_{k=0}^{\infty}Y(k)x^k$$

$$y(x)=Y(0)x^0+Y(1)x^1+Y(2)x^2+Y(3)x^3+\cdots$$

$$y(x)=0.1+\frac{1}{p}x+\frac{3}{2}x^2+\frac{3p}{6}x^3+\cdots.$$

$$y(x)=\frac{1}{p}x+\frac{3}{2}x^2+\frac{p}{2}x^3+\cdots$$

Problem 2: Consider the following initial value problem

$$\frac{d^2y}{dx^2}+l^2y=e^{lx}+sinlx+coslx,\ l(\neq 0)is\ areal\ number,$$
$given\ that\ y(0)=1, y(1)=l$ (2)

Apply DTM to (2), we obtain

$(k+1)(k+2)Y(k+2)+l^2Y(k)=\frac{l^k}{k!}+\frac{l^k}{k!}sin(k\pi)+\frac{l^k}{k!}cos(k\pi)$

Put k = 0, then $2Y(2)+l^2Y(0)=1+0+1$

$$2Y(2)+l^2.1=2$$

$$Y(2)=\frac{2-l^2}{2}$$

Put k = 1, then $6Y(3)+l^2Y(1)=l+l(0)+l(-1)$

$$6Y(3)+l^2\left(\frac{2-l^2}{2}\right)=l$$

$$Y(3)=\frac{l-l^2\left(\frac{2-l^2}{2}\right)}{2}=\frac{2l-2l^2+l^4}{4}$$

And so on.
The solution is

$$y(x)=\sum_{k=0}^{\infty}Y(k)x^k$$

$$y(x)=Y(0)x^0+Y(1)x^1+Y(2)x^2+Y(3)x^3+\cdots$$

$$y(x)=1.1+lx+\frac{2-l^2}{2}x^2+\frac{2l-2l^2+l^4}{4}x^3+\cdots.$$

Problem 2: Consider the following initial value problem

$$\frac{d^3y}{dx^3}+a^2\frac{dy}{dx}=sinax,\ a(\neq 0)is\ areal\ number,$$
$given\ that\ y(0)=1, y(1)=a, y(2)=a^2$ (3)

Apply DTM to (3), we obtain

$(k+1)(k+2)(k+3)Y(k+3)+a^2(k+1)Y(k+1)=\frac{a^k}{k!}sin(k\pi)$

Put k = 0, then $6Y(3)+a^2Y(1)=0$

$$6Y(3)+a^2.a=0$$

$$Y(3)=-a^3$$

Put k = 1, then $24Y(4)+a^2Y(2)=a.0$

$$24.Y(4)+a^2(-a^3)=0$$

$$Y(4)=\frac{a^5}{24}$$

And so on.

The solution is

$$y(x)=\sum_{k=0}^{\infty}Y(k)x^k$$

$$y(x)=Y(0)x^0+Y(1)x^1+Y(2)x^2+Y(3)x^3+Y(4)x^4+\cdots$$

$$y(x)=1.1+ax+a^2x^2-a^3x^3+\frac{a^5}{24}x^4+\cdots.$$

4. Conclusion

The observations of the present study have shown that the (DTM) is easy to apply and effective. As a result, the conclusion comes through this work, is that the Differential Transform Method can be applied to a wide class of differential equations, due to the efficiency in the application to get the possible results.

References

[1] Ayaz.F, Solutions of system of differential equation by Differential Transform Method.Appl.Math.comput 2004.pp147-547-567.
[2] G.Adoman.R.Rack, On linear and nonlinear integro-differential equations. J. Math. Anal. Appl.113(1)(1986), pp. 117-120.
[3] Kaya.N.T. Comparing numerical methods for solutions of ordinary differential equations, Appl.Math. Lett, 17(2004)323-328.
[4] Liu.H and Y.Song, Differential Transform Method applied to higher index differential- algebraic equations. Appl.Math. Comput,2007,184-748-753.

[5] Zhou.J.K. Differential transformation Method and applications for electrical circuits, Huazhong University press, Wuhan, China, (1986)
[6] Onur Kiymaz, An Algorithm for Solving Initial Value Problems, using Laplace Adomian Decomposition Method. Applied Mathematical Sciences, Vol. 3, 2009, no. 30, 1453 - 1459
[7] E. Hesameddini, H. Latifizadeh. A new vision of the He's homotopy perturbation method. International Journal of Nonlinear Sciences and Numerical Simulation. 2009.
[8] E. Hesameddini, H. Latifizadeh. Reconstruction of variational iteration algorithms using the Laplace transform. International Journal of Nonlinear Sciences and Numerical Simulation. 2009.
[9] S.T. Mohyud-Din, M.A. Noor, K.I. Noor. Some relatively new techniques for nonlinear problems. Math. Porb.Eng. Article ID 234849, 25 pages, doi: 10.1155/2009/234849, 2009.
[10] S.T. Mohyud-Din, A. Yildirim. Variational iteration method for solving Klein- Gordon equations. Journal of Applied Mathematics, Statistics and Informatics.2010.
[11] Jagdev Singh, Devendra Kumar and Sushila Rathore, Application of Homotopy Perturbation Transform Method for Solving Linear and Nonlinear
[12] Klein- Gordon Equations, Journal of Information and Computing Science,ISSN 1746-765 England, UK,Vol. 7, No. 2, 2012, pp. 131-139.
[13] A. Yildirim. An Algorithm for Solving the Fractional Nonlinear Schröndinger Equation by Means of the Homotopy Perturbation Method. International Journal of Nonlinear Science and Numerical Simulation. 2009.
[14] Abbasbandy. S., 2006, "Homotopy perturbation method for quadratic Riccati differential equation and comparison with Adomian's decomposition method," Applied Mathematics and Computation, **172** (1), pp.485–490.
[15] Adomian, G., and Rach, R., 1986, "Solving nonlinear differential equations with decimal power nonlinearities," Journal of Mathematical Analysis and Applications, **114** (2), pp.423–425.
[16] Ahmadian, M., Mojahedi, M., and Moeenfard, H., 2009, "Free vibration analysis of a nonlinear beam using homotopy and modified lindstedt-poincare methods," Journal of Solid Mechanics, **1** (1), pp.29–36.
[17] AL-Jawary, M. A., and Al-Razaq, S. G., 2016, "A semi analytical iterative technique for solving Duffing equations," International Journal of Pure and Applied Mathematics, 108 (4), pp. 871–885.
[18] AL-Jawary, M. A., and Raham, R. K., 2016, "A semi-analytical iterative technique for solving chemistry problems," Journal of King Saud University, (In press).
[19] AL-Jawary, M. A., 2017, "A semi-analytical iterative method for solving nonlinear thin film flow problems," Chaos, Solitons and Fractals, **99**, pp.52–56.
[20] Aminikhah, H., 2013, "Approximate analytical solution for quadratic Riccati differential equation," Iranian Journal of Numerical Analysis and Optimization, **3** (2), pp.21–31.
[21] Anderson, B. D., and Moore, J. B., 1999, "Optimal control-linear quadratic methods," Prentice-Hall, New Jersey.
[22] Banach, S., 1922, "Sur les opérations dans les ensembles abstraits et leur application aux equations integrals," Fundamenta Mathematicae, **3**, pp. 133–181.
[23] Barari, A., Ganjavi, B., Jeloudar, M. G., and Domairry, G., 2010, "Assessment of two analytical methods in solving the linear and nonlinear elastic beam deformation problems," Journal of Engineering, Design and Technology, **8**, (2), pp.127–145.
[24] Barari, A., Omidvar, M., Ganji, D. D., and Tahmasebi, A. P., 2008, "An Approximate Solution for Boundary Value Problems in Structural Engineering and Fluid Mechanics**,"** Journal of Mathematical Problems in Engineering, Vol. 2008, pp.1–13.
[25] Lasiecka, I., and Tuffaha, A., 2007, "Riccati equations arising in boundary control of fluid structure interactions," International Journal of Computing Science and Mathematics, **1** (1), pp.128–146.