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Evaluation of Split-Step Fourier Transform for Tropospheric Electromagnetic Wave Propagation

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Abstract: ***In this letter, we adopt Kuttler and Dockery theoretical model using the split-step Fourier transform method which is a numerical solution of the parabolic solution to investigate the presence of rain drops in the troposphere. The parabolic equation (PE) model works by assuming that an outgoing EM wave dominates backscattered EM wave and factoring the operator in the frequency-domain wave equation to obtain an outgoing wave equation. Here, we study the interactions between propagating EM wave and rain drops in the troposphere. This procedure derives the scalar Helmholtz equation in spherical coordinates and uses conformal mapping to convert it to rectangular coordinate without requiring approximations. MATLAB programming software was used to simulate the numerical narrow angle split-step Fourier transform model and plots of field profile versus range and heights illustrate the effects of rain drops in the troposphere.***

Keywords: Split step Fourier transform, Troposphere, Electromagnetic wave, Propagation

1. Introduction

Scattering phenomena such as reflection, refraction and diffraction affects radio-wave propagation over the earth's surface and in heterogeneous atmosphere. To ensure reliable design of radar and communication systems, it is germane to understand the effects of varying conditions on radio wave propagation. For instance, tropospheric waves play significant role in communications as they can travel over the horizon and increase the coverage area and thus, distort the communication links due to interference that are doesn't exist in normal conditions.

Due to the abrupt change in refractive index in the troposphere, such waves are propagated by bending or refraction, and create abnormal propagation. In cases where the refractive gradient exceeds certain threshold, there may be trapped radio waves in a duct and guided over distances farthest from normal range.

Irregular terrain surfaces and tropospheric effects have great impact on radio-wave propagation due to their ability to reflect and diffract electromagnetic waves in a complex way. Consequently, there is need to design an effective radar or communication system using a model that can properly take into account the reflectivity and terrain factors. Researchers for many decades, are faced with the rigorous task of modelling radio-wave propagation in complex environment both analytically and numerically. The difficulty ranges from the vast variability of propagation medium properties, surfaces and obstacles that re-direct the propagating energy which leads to unpredictability of propagating radio-waves. It was initially reported that analytical techniques such as diffraction methods, ray tracing and waveguide mode theory were employed to predict radio-wave propagation [1-6]. These methods however, require their geometries to appear as a member of some set of canonical geometries and suffer from vertically-varying refractivity profile presence in the troposphere. The aforementioned difficulties were easily handled due to advances in computing by some numerical techniques. Parabolic equation (PE) method has wide range applications in modelling radio-wave propagation to predict its characteristics between a transmitter and receiver over the two-dimensional (2D) earth's surface, due to its high potential in modelling both horizontally and vertically varying atmospheric refraction (most importantly ducting) effects [7]. Helmholtz equation is obtained from the standard PE such that the rapidly varying phase term is neglected to obtain a reduced function having slow variation in range for propagating angles close to the paraxial direction. Two differential equations mainly forward and backward propagating waves are used to approximate the Helmholtz equation. Each of the differential equation is the form of a parabolic differential equation [8].

The standard PE is a one way forward scatter model and valid in the paraxial region. The split-step parabolic equation (SSPE) is an initial value problem starting from a reference range (from an antenna) and marching out in range by obtaining the field along the vertical direction at each range step, through the use of step-by-step Fourier transformation [9-14].

In this paper, we developed MATLABcodesfor the narrow angle PE method to model millimetre wave propagation in rain over a spherical earth at temperature $T = 306K$ and rain rate $0.7081mm/hr$ in Yenagoa city, Bayelsa State [16]. For both rain intensity and effective refractive index n that are uniform through the rainfall region, the simulation was implemented. We were able to predict multipath propagation effects due to electromagnetic wave propagation in the troposphere with rain drops presence by estimating the propagationloss on a range/height scale.

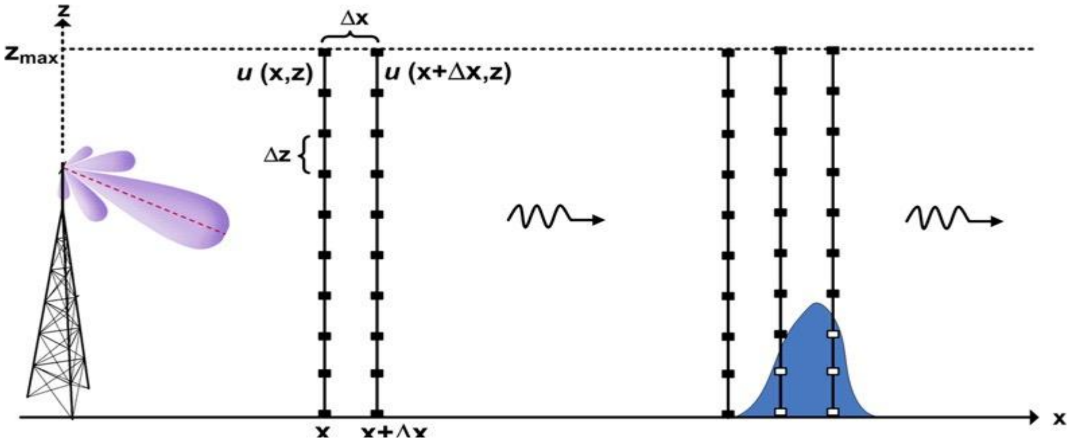

Figure 1: One-way SSPE framework with forward propagating waves. (White nodes represent zero fields.) [15]

1.1 Tropospheric Propagation of EM waves by the Fourier Split-step Method

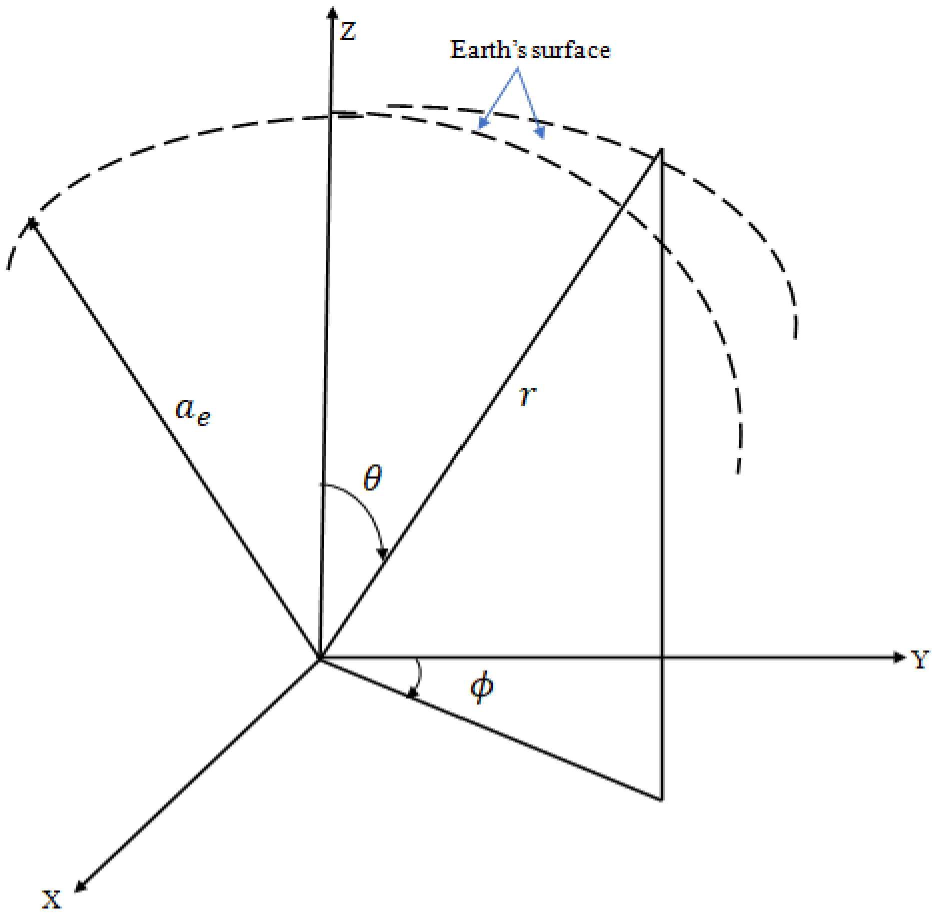

Figure 2: Earth Geometry of radiated fields by Gaussian source [11]

Theoretical development of Paraxial approximation based on tropospheric propagation of EM waves using the Split-step Fourier transform was reported by Kuttler and Dockery [11]. It was earlier stated that for a given polarization, antenna radiation patterns can be modelled using the Split-step Fourier transform [11]. Figure 2 illustrate an earth geometry describing tropospheric propagation from a source located at $\theta = 0$ and radial distance $r > a_e$ where $a_e$denotes the earth radius and located at the Fraunhofer region of the antenna source. For vertically and horizontally polarized electric dipole with Gaussian antenna radiation pattern the scalar PE associated with EM tropospheric propagation can be used to describe acoustic propagations in the ocean.

It is worth noting that for the VED (vertical electric dipole), the electric field is perpendicular to the earth's surface and the electric field is parallel to the earth's surface for HED (horizontal electric dipole). Considering a source-free medium with $exp(-j\omega t)$ time dependence and slowly varying ε in the direction of φ. The electric field is a function of (r, θ, φ) for spherical coordinates. The vector wave equation for the vertically polarized antenna can be easily obtained using Maxwell's curl equations of the electric field and magnetic field. The vector wave equation is reduced to a scalar wave equation by neglecting φ dependencies and the nonzero components of the fields are E_r, E_θ and $H_\varphi, H = H_\varphi \hat{\varphi}$.

1.2 Maxwell's Curl Equations in differential form for source-free medium

$$\nabla \times H = \frac{\partial D}{\partial t} \quad (1)$$

$$\nabla \times H = -\frac{\partial B}{\partial t} \quad (2)$$

Recall that the phasor form of the E-field is expressed as $\hat{E}(r) = \hat{E}(r)e^{-j\omega t}$ and the electric flux density $D = \varepsilon E$. Equation (1) becomes

$$\nabla \times H = j\omega D \quad (3)$$

and

$$\nabla \times H = j\omega\varepsilon E \quad (4)$$

Equating (1) and (4) yields

$$j\omega E = \frac{1}{\varepsilon}\frac{\partial D}{\partial t} = \frac{1}{\varepsilon}(\nabla \times H) \quad (5)$$

Taking the curl of (1) and substituting $D = \varepsilon E$ into (3) yields

$$\nabla \times \nabla \times H = j\omega(\nabla \times (\varepsilon E)) \quad (6)$$

Where $\nabla \times D = (\nabla \times (\varepsilon E)$, which was obtained from

$$\nabla \times \nabla \times H = \nabla \times (j\omega D) = j\omega(\nabla \times D) \quad (7)$$

Applying the product of a scalar and a vector

$$\nabla \times (\psi A) = \psi(\nabla \times A) + (\nabla\psi) \times A \quad (8)$$

$$\nabla \times (\varepsilon E) = \varepsilon(\nabla \times E) + (\nabla\varepsilon) \times E \quad (9)$$

Substituting of equation (9)into (6) gives

$$\nabla \times \nabla \times H = j\omega(\varepsilon(\nabla \times E) + (\nabla\varepsilon) \times E) \quad (10)$$

$$\nabla \times \nabla \times H = j\omega(\varepsilon\nabla \times E - j\varepsilon\mu\omega E) \quad (11)$$

This equation can be further expressed as

$$\nabla \times \nabla \times H = \varepsilon\nabla \times j\omega E + j\varepsilon\mu\omega E) \quad (12)$$

Substituting of equation (5) into (12) gives

$$\nabla \times \nabla \times H = \varepsilon\nabla + \frac{1}{\varepsilon}(\nabla \times H) + \varepsilon\mu\omega^2 E \quad (13)$$

$$\nabla \times \nabla \times H = \frac{\varepsilon\nabla}{\varepsilon} \times \nabla \times H + \varepsilon\mu\omega^2 E \quad (14)$$

$$\nabla \times \nabla \times H - \frac{\varepsilon\nabla}{\varepsilon} \times \nabla \times H - \varepsilon\mu\omega^2 E = 0 \quad (15)$$

The scalar wave equation can be expressed as

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{\frac{1}{\varepsilon}r\frac{\partial}{\partial r}H_\varphi\right\} + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left\{\frac{1}{\varepsilon sin\theta}\frac{\partial}{\partial\theta}(sin\theta H_\varphi)\right\} + \mu\omega^2 H_\varphi = 0 \quad (16)$$

Similarly, the non-zero field components for the parallel polarized E-field are H_r, E_θ and E_φ and expressed as

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rE_\varphi) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left\{\frac{1}{sin\theta}\frac{\partial}{\partial\theta}(sin\theta E_\varphi\right\} + \mu\omega^2 E_\varphi = 0 \quad (17)$$

Equations (16) and (17) are the Helmholtz equations in spherical coordinates. There is need to transform equations (16) and (17) from spherical to polar or rectangular coordinate system where the axis represents the earth's surface and its origin lies directly beneath the antenna on the surface [11]. This transformation is done by conformal mapping. This process could be referred to earth flattening transformation when the term $\frac{2z}{a_e}$ which accounts for spherical earth is neglected. Equations (18), (19) and (20) shows conformal mapping

$$\xi = 2^{a_e}\frac{a_e + i\zeta}{\zeta + ia_e} \quad (18)$$

The term a_e is the radius of the earth and

$$\zeta = rsin\theta + ircos\theta \quad (19)$$

$$\zeta = x + iz \quad (20)$$

For earth flattening approximation,

$$z = r - a_e \quad (21)$$

$$x = a_e\theta \quad (22)$$

Where x and $z$denotes altitude and range respectively.

The Split-step Fourier transform modelling tropospheric propagation in flat earth surface is given as

$$u(x + \Delta x, z) = u(x,z)e^{ik\left(\frac{n^2-1}{2}\right)\Delta x}F^{-1}\left\{e^{i\left(\frac{p^2}{2k}\right)\Delta x}F\big(u(0,z)\big)\right\} \quad (23)$$

$$u(x + \Delta x, z) = u(x,z)e^{ik\left(\frac{m}{2}\right)\Delta x}F^{-1}\left\{e^{i\left(\frac{p^2}{2k}\right)\Delta x}F\big(u(0,z)\big)\right\} \quad (24)$$

The variable m denotes the modified refractive index. For tropospheric propagation on spherical earth surface m is expressed as

$$m = n^2 + {}^{2z}/_{a_e} \quad (25)$$

And the modified refractivity is

$$M = (m - 1) \times 10^{-6} \quad (26)$$

The unit of M is N-units while the modified refractivity gradient index is M-units per km. The expressions of the modified refractive in a well-mixed atmosphere and reference atmosphere can be seen in [12]. $u(0,z) = u_0(z) = f(z)$, which may represent a complex array of length N used to model the reduced aperture field function with beam-width in degrees. The antenna aperture field can be obtained from the beam pattern. Code level implementation of the numerical solution is in advanced stage and evaluation of the modified refractivity profile empirical equations adopted from [12] were reported.

2. Narrow Angle SSFT Algorithm

1) Compute the initial field profile using the inverse Fourier transform (IFFT) of the Gaussian antenna

radiation pattern with specific height z_0, beamwidth θ_{bw}, and tilt or elevation angle θ_{tilt}.

2) Include boundary conditions in the initial field expression. Here, we use Dirichlet or Neumann boundary conditions.
3) Compute the refractive index of rain medium with refractivity profile.
4) Apply forward Fourier transform (FFT) on initial field profile.

5) Multiply the FFT on field profile by p-space propagator $(e^{i\left(\frac{p^2}{2k}\right)\Delta x})$.
6) Apply inverse Fourier transform (IFFT) on the product of p-space propagator and (FFT) on initial field profile.
7) Multiply (IFFT) on the product of p-space propagator and (FFT) on initial field profile by z-space propagator $(e^{ik\left(n^2+\frac{z}{a_e}\right)\Delta x})$.
8) Check if final range x is reached else set new old field profile and new initial field profile and repeat steps 1 to 7 until final range is obtained.

3. Results and Discussions

We apply the narrow angle PE method to model millimetre wave propagation in rain over a spherical earth at temperature $T = 306K$ and rain rate $0.7081 mm/hr$ in Yenagoa city, Bayelsa State. For both rain intensity and effective refractive index n that are uniform through the rainfall region, the simulation was implemented.

Figure 3 and 4 illustrate variation of field profile over range as the field travels through flat earth with rain drops with rain rate $0.7081 mm/hr$, temperature $T = 306K$ and effective refractive index $n = 2.58125 + 1.1304i$ with maximum heights $5\ km$ and $10\ km$, earth radius $r_e = 6371\ km$. A Gaussian source field that is horizontally and vertically polarized at $z_s = 50\ m$, with a beamwidth of 1°, elevation or tilt angle 1°, and frequency $60\ GHz$. Here we consider one obstacle that is modelled over the spherical earth using trigonometric functions. The location of the obstacle is between $3-5\ km$ with height $50\ km$. The figures below show diffraction and reflection effects through the range $0-1\ km$ for spherical earth surfaces with maximum heights $5\ km$ and $10\ km$ respectively [16]. We can see that diffuse reflections increases with increase in height of the earth surface.

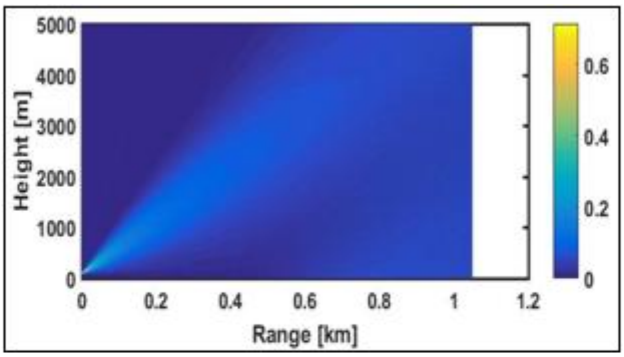

Figure 3: Field profile over the troposphere at maximum height $5\ km$

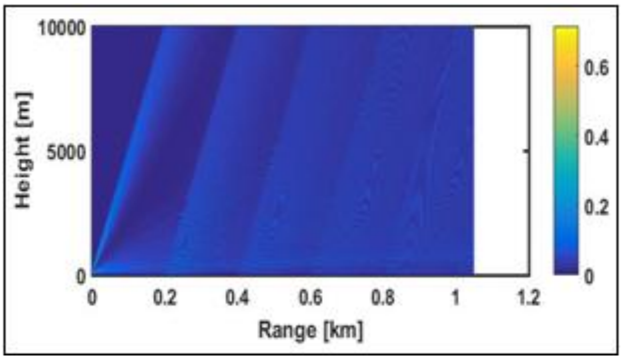

Figure 4: Field profile over the troposphere at maximum height $10\ km$

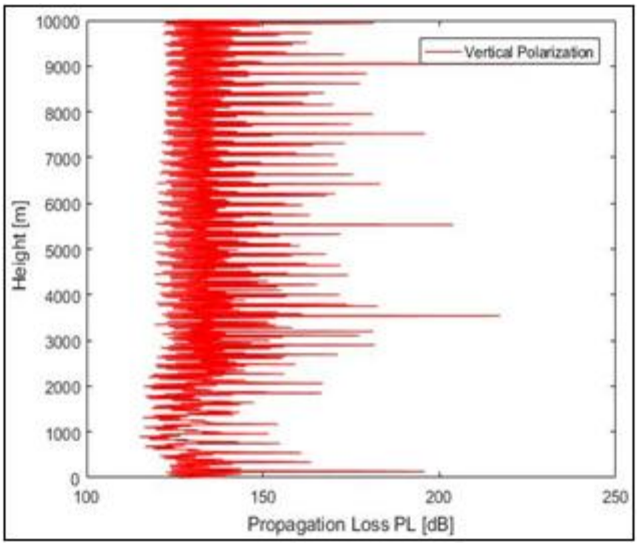

Figure 5: Propagation loss (dB) versus height(m) for vertical polarization

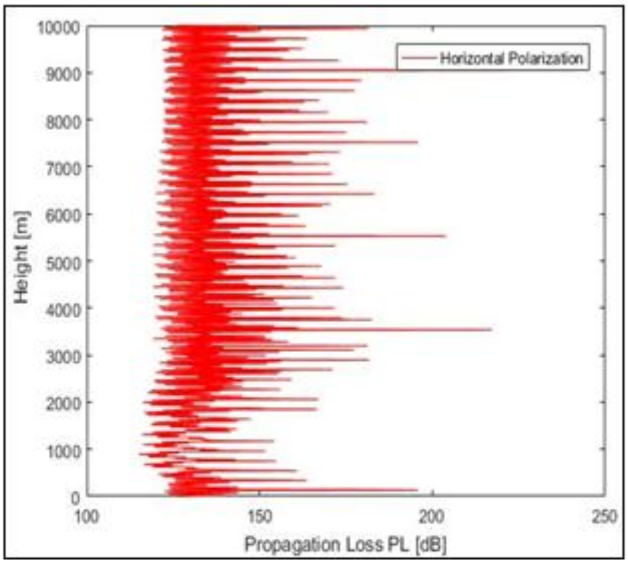

Figure 6: Propagation loss (dB) versus height(m) for horizontal polarization

Figure 5 and 6 illustrate the propagation loss (PL) as a function of height in rain medium with maximum height $z = 10\ km$ for both horizontal and vertical polarizations. The field profiles vary similarly in both cases for maximum range $x = 1200\ km$. The spherical earth surfaces show more diffuse reflections than flat earth surfaces assumed to

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be wet as shown in both cases of horizontal and vertical polarizations [16].

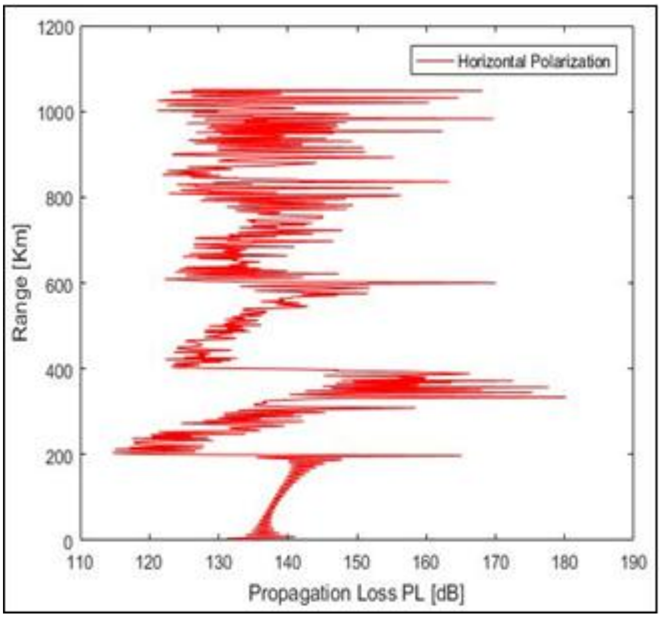

Figure 7: Propagation loss (dB) versus range (km) for horizontal polarization

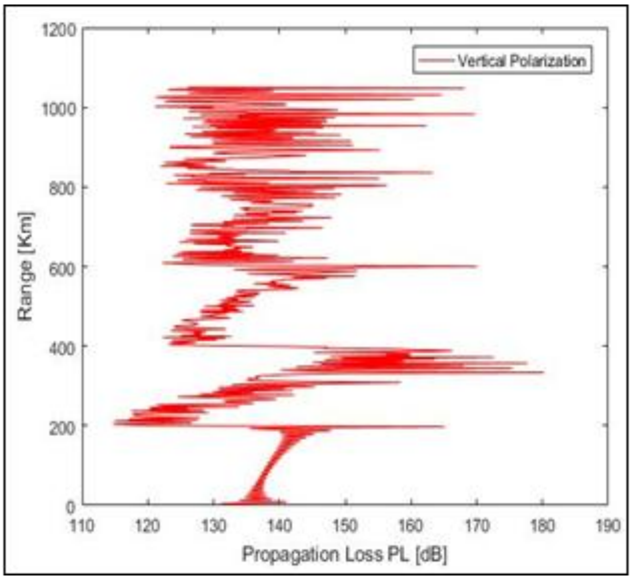

Figure 8: Propagation loss (dB) versus range (km) for vertical polarization

Figure 7 and 8 illustrate the propagation loss (PL) as a function of range in rain medium with maximum height $z = 10\ km$ for both horizontal and vertical polarizations. The field profiles vary similarly in both cases for maximum height $z\ = 10\ km$. The flat earth surfaces show less diffuse reflections than irregular terrains assumed to be wet as shown in both cases of horizontal and vertical polarizations. The propagation loss is highest at range $x = 250\ km$ and lowest at $= 0\ km$. This explains that low altitudes, multipath propagation effects are not significant [16].

4. Conclusion

MATLAB computing software was used to simulate the one-way split-step narrow angle parabolic equation model which has been introduced and discussed systematically for a radio-wave propagation problem over the troposphere through variable and inhomogeneous atmosphere. It has been concluded that the standard PE model has certain drawbacks in terms of handling backward propagation, as well as large propagation angles. These issues may be predominantly observed in problems involving arbitrary obstacles on the propagation path or in short-range problems, or combination of two (for example, in urban propagation scenarios). They may also be observed in propagation scenarios where abrupt, severe changes in atmospheric conditions occur. The wide-angle version of the two-way SSPE overcomes these limitations up to a great extent and serves as an important research interest for radar communications purposes.

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