The Effect of Fluctuating Economic Variables on the Stock Exchange Stability: Implications for Sustainable Development and Investment Planning

Nafo N. M.

Department of Mathematics, Rivers State University, Nkpolu-Oroworukwo Port Harcourt, Rivers State, Nigeria
E-mail: *nafongia[at]gmail.com*

Abstract: ***A basic technique of studying the interaction between competing investors for wealth creation is the use of the method of mathematical modelling and numerical simulation. In this paper, we are proposing to utilize the method of numerical simulation to examine the effect of fluctuating induced-interest rate and inflation factors on the stability of two competing investors. At different instances, we have observed that when the growth rates of the first and second populations of investors as well as their carrying capacities are varied, the co-existence steady state solutions are dominantly stable having two negative eigenvalues. The results of our present analysis have some vital roles to play in the functioning of sustainable development and investment planning. These results have not been seen elsewhere they will presented and discussed in this talk.***

Keywords: Competing Investors, Numerical Simulation, Induced Interest Rate, Stability, Inflation Factors, Sustainable Development

1. Introduction

The aim of this research is to investigate the effect of fluctuating economic variables on the stock exchange stability. Fluctuating economic variables and the notion of a stock exchange concern financial engineering while the notion of stability is a mathematical idea. In the scenario of stock exchange Lotka-Volterra interaction, the effect of growth rates of the investors' dividends over the length of the trading period in the unit of days in combination with the intra-competition and inter-competition coefficients especially when two investors interact mutualistically on stability can be quantified using a mathematical idea of repeated numerical simulations. Stock exchange modelling is a challenging process; hence it would be a good practice to find out the regimes of parameter variations that can permit either valid stability of the co-existence steady-state solution or the instance of degeneracy of the steady-state solution. The variations of model parameter values that produce stability can be useful on some aspect of sustainable development and investment planning whereas the variations of model parameter values that produce degeneracy behaviour can provide vital control insight to avoid in the event of stock exchange crash which is inevitable because the stock exchange system is dominantly economically driven,

2. Model Formulation

In this work, we have used a system of continuous non-linear first order ordinary differential equations of Lotka Volterra type given by:

$$\frac{dx_1(t)}{dt} = x_1(\alpha_1 - \beta_1 x_1 - y_1 x_2) \qquad (1.1)$$

$$\frac{dx_2(t)}{dt} = x_2(\alpha_2 - \beta_2 x_2 - y_2 x_1) \qquad (1.2)$$

$$x_1(0) = x_2(0) = 0$$

Where

x_1 and x_2 represent the first and second populations of investors respectively.

α_1 and α_2 represent the intrinsic growth rates of the first and second population investors respectively.

β_1 and β_2 represent the intra-specific coefficients of the first and second populations investors respectively.

y_1 and y_2 represent the inter-specific coefficients of the first and second populations of investors respectively.

$x_1(0)$ and $x_2(0)$ are the initial investments of the first and second populations of investors respectively.

Following Solomon and Richmond (2002), Solomon (2000), Blank and Solomon (2000) and Richmond (2001) who are famous experts in financial mathematics modelling, Lotka-Volterra characterization in economics and finance is a popular idea. In this pioneering research, we have derived the model parameter values which describe the dynamics of two mutualistically interacting investors such as the intrinsic growth rate parameter values of 0.037 and 0.03 followed by the intra-competition coefficients of 0.0014 and the inter-competition coefficients of 0.0012 (Okorafor and Osu, 2009).

3. Method of Analysis

From the point of view of a mathematical stability for a system of two interacting populations in the form of a continuous non-linear first order ordinary differential equations whose interacting functions are assumed to be continuous and partially differentiable, a unique positive steady-state solution can be classified as being stable if the eigenvalues that produce this qualitative behaviour have negative signs (or unstable if the eigenvalues have opposite signs). On the other hand, if the steady-state solution has a negative co-ordinate, then it is classified as degenerate in which case the signs of the eigenvalues do not provide any meaningful stability inference. Using the notion of a numerical simulation on the basis of a MATLAB programming formalism, we have identified a dominant scenario of stability due to the simultaneous variations of the growth rate parameter value of the first population of investors in combination with the growth rate parameter value of the second population of investors. Next, we have identified a dominant scenario of stability due to the simultaneous variations of the growth rate parameter value of the first population of investors in combination with the growth rate parameter value of the second population of investors. However, we have identified a dominant scenario of degeneracy due to the simultaneous variations of the growth rate parameter value of the first population of investors in combination with the intra-competition parameter value of the first population of investors.

In this study, the notations α_1 and α_2 represent the intrinsic growth rate parameter values of the first and second populations of investors as indicated earlier, while the notations $K_1$and K_2 represent the carrying capacities of the first and second populations of investors. The point (P_{1e}, P_{2e}), is called the unique positive co-existence steady-state solution.The detail of our simulation analysis will be presented in the following section.

4. Results and Discussion

Since the carrying capacities of two interacting investors depend on the fraction of the growth rate parameter value and its intra-competition parameter value, we have systematically investigated the effect of the growth rate of the first population of investors in combination with the growth rate of the second population and with the intra-competition and inter-competition coefficients of the first population of investors. The results that we have obtained are classified into three groups. The first group concerns the stability characterization due to a simultaneous variation of the growth rate parameter values of the first and second populations of investors. When the growth rates α_1 and α_2 as well as the carrying capacities K_1 and K_2 are varied such that $0.0004 \leq \alpha_1 \leq 0.0592$; $0.0003 \leq \alpha_2 \leq 0.0480$; $0.2643 \leq K_1 \leq 42.2857$; $0.2143 \leq K_2 \leq 34.2857$; the co-existence steady-state solutions within the range $0.85 \leq P_{1e} \leq 136.33$ and $0.82 \leq P_{2e} \leq 131.67$ are dominantly stable having two negative eigenvalues.

When the growth rate of the first population of investors is varied in combination with the inter-competition coefficient of the second population of investors denoted by γ to inhibit the growth of the first population of investors for the close intervals $0.00037 \leq \alpha_1 \leq 0.0592$ and $0.00001 \leq \gamma_1 \leq 0.0016$ with $0.2643 \leq K_1 \leq 42.2857$ and $K_2 = 21.4286$, the co-existence steady-state solutions within the range $0.42 \leq P_{1e} \leq 363.56$ and $21.73 \leq P_{2e} \leq 281.11$ are dominantly stable having two negative eigenvalues.

When the growth rate of the first population of investors is varied in combination with the intra-competition coefficient of the first population of investors which is, here, denoted by β_1 to inhibit the growth of the first population of investors for the close intervals $0.00037 \leq \alpha_1 \leq 0.01887$ and $0.000014 \leq \beta_1 \leq 0.000714$ with $K_1 = 26.4286$ and $K_2 = 21.4286$, the co-existence steady-state solutions within the range $-31.13 \leq P_{1e} \leq -141045$ and $-0.81 \leq P_{2e} \leq -100725$ are dominantly degenerate. These instances of degeneracy are lost and stability is re-gained for the following ranges of parameter values: $0.01924 \leq \alpha_1 \leq 0.0592$, $0.000728 \leq \beta_1 \leq 0.00224$ with $K_1 = 26.4286$ and $K_2 = 21.4286$, the unique positive co-existence steady-state solutions within the range $52.85 \leq P_{1e} \leq 2965.12$ and $59.18 \leq P_{2e} \leq 2139.58$ are dominantly stable having two negative eigenvalues.

5. Summary and Conclusion

Due to the effect of fluctuating interest rate and growth rate on a mutualistic interaction between two populations of investors, a variation of the growth rate parameter value of the first population of investors in combination with the growth rate parameter value of the second population of investors as well as a variation of the growth rate parameter value of the first population of investors in combination with the inter-competition parameter value of the second population of investors enhance dominant scenario of stability. This insight is capable to sustain effective investment planning stabilization with its sustainable development contribution. The interaction of the first population of investors with itself otherwise called self-interaction phenomenon has produced a first regime of dominant degeneracy within a few instances of mathematically tractable conditions which can be used as some sort of mitigation control in financial education, financial business and stock exchange trading in the un-expected event of a crash. Suddenly the loss of degeneracy to a dominant stability is another interesting contribution due to the significance of the region of bifurcation from degeneracy to stability and other fluctuating economic variables such as political instability and sudden increase in the interest rate that can introduce some random noise into the interacting system.

References

[1] Blank, A. and Solomon, S. (2000). *Physica A* 287, 279.
[2] Okorafor, A.C. and Osu, B.O. (2009). An empirical optimal portfolio selection model. *African Journal of Mathematics and Computer Science Research*, 2(1), 001-005.
[3] Richmond, P. (2001). *Eur. J. Physica B* 4, 523.
[4] Solomon, S. (2000). Generalized Lotka-Volterra (GLV) models and generic emergence of scaling laws in stock markets, in Application of Simulation to Social Sciences, edited by G. Ballot, G. Weisbuch (Hermes Science Publications).
[5] Solomon, S. and Richmond, P. (2002). Stable power laws in variable economies: Lotka-Volterra implies Pareto-Zipf. *The European Physical Journal B*, 27, 257-261.