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Exploring the Efficacy of Guided Discovery Learning in Improving Algebraic Thinking Skills

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Abstract: ***This study sought to explore how the use of guided discovery can enhance students' algebraic thinking skills in a grade 9 mathematics classroom. An action research with a quantitative design was employed where the students were taught various algebraic topics such as algebraic translation, algebraic expressions and algebraic equations. The study consisted of six students (n=6) between ages 14 and 15 years and was conducted at a popular high school in Kingston, Jamaica. Data was collected using observation schedule, field notes, questionnaire and also pre-test and post-test; and it was analyzed using descriptive statistics. 33% of the students had difficulties with algebraic problems since they would either get easily confused or were unable to remember the algebraic processes. The results revealed that there was a significant improvement in the algebraic skills and performances of the students as students were better able to work with algebraic problems after being introduced to the intervention. As this is so, a prime implication could be on the teaching and learning of algebra at the different grade levels at the secondary level that could eventually influence how students are prepared for CSEC examinations. The implication could also extend to the mathematics educational programs at the tertiary level.***

Keywords: Guided Discovery Learning, Algebraic Thinking, Algebra

1.Introduction and Literature Review

Over the years, while on teaching practice it has been noticeable that Algebra has been the most concern amongst the teachers as well as my peers. With this in mind, the researcher ventured out on her final year of teaching practicum in September 2021 performing an action research to see if this problem will be noticeable in the present classroom. It is noteworthy that the Caribbean Examination Council has been providing yearly reports of the Caribbean Secondary Education Certificate (CSEC) examinations taken by students in Jamaica and other CARICOM countries; and the areas of concern stated. In the 2018 Caribbean Examination Council annual report, it was reported that forty six per cent (46%) of the candidates who wrote the mathematics examinations that year achieved Grades I to III compared with 49 per cent in 2018. Additionally, a trend was observed in the sequence of the questions from the CSEC Mathematics External Examination paper where question 2 is usually an algebraic question **[1]**. The annual reports over a seven year period (2010-2017) showed that while these questions are generally attempted by the majority of the candidates (over 97% of candidates each year), there were less than 4% of candidates who actually earned full marks. The Ministry of Education, Youth and Information, Jamaica, in recent times, has implemented the National Standards Curriculum with the aim of developing students' knowledge and demonstrations of the 21st century learners skills (creativity, critical thinking, collaboration and communication) through the use of student centred teaching approaches. In this regard, the authors of the article "Developing higher level thinking" **[2]** stated that true understanding of concepts takes place when students are actively involved and are able to reflect on the reason behind particular concepts. A researcher who wrote an article in the journal Eastern Connecticut State University **[3]** also explored various ways of teaching algebra and found that when students find meaning behind the concepts they are introduced to in class; their interest levels will increase and thus, improve their performances in the area of mathematics. The researcher, while teaching and collecting data at an all-boys school in Kingston, Jamaica noticed particular trends in terms of the attitudes and skills that many of the Grade 9 students possess towards the Algebra strand. Some of these include, but are not limited to:

1) Students stating that algebra is "hard" even before being introduced to the topic.
2) Students who expressed a dislike towards the area of mathematics based on how it was taught by another teacher.
3) Students easily confuse the steps in the algebraic processes and then become confused and frustrated.

For our first session together, the researcher had a discussion with the students- with the focus being on the students' perception of the topic algebra and the issues they faced. This discussion was an open one where the students could volunteer to respond and share their concerns. Of the students who responded, almost 50% expressed a dislike for this strand. Further discussions revealed that the students had challenges understanding and applying the algebraic concepts such as the various processes to follow including the symbols and variables used in this specified area of maths. A study conducted in a district of Zambia **[4]** also made similar deductions based on an interview done with the students where they indicated their difficulties with grouping like terms and manipulating algebraic signs and symbols. These resulted in many misconceptions and errors when doing the algebraic processes. Other students expressed mixed feelings for algebra due to their previous experiences since some teachers make algebra seem easy and some make it look difficult. One major observation was made while students were giving responses to questions. Some would make mistakes by saying "a times a = 2a". Here, the students confused the repeated addition of algebraic variables with repeated multiplication. Additionally, this misconception was observed as they completed particular assignment tasks.

One example may be seen in **Figure 1**below as a student tried to simplify after correctly applying the distributive law.

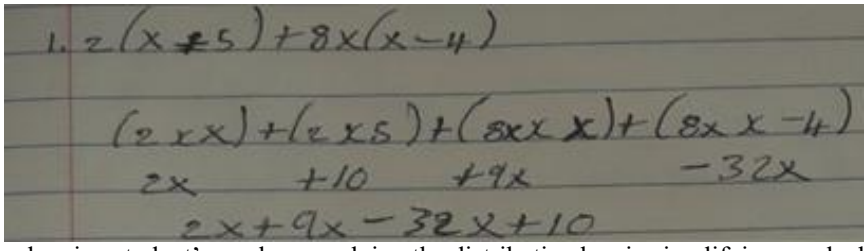

Figure 1: Image showing student's work on applying the distributive law in simplifying an algebraic expression

These misconceptions may be due to students not understanding the difference between these two processes since they were now using algebraic terms, instead of normal numbers. It was evident that the students did not see the connection between these two processes in order to effectively apply it to solving algebraic problems. Additionally, while most students were able to effectively group simple terms in order to simplify, there were particular students who were not able to remember the algebraic processes; such as application of the distributive property to simplify algebraic expressions (that is, expanding before simplifying). This was so as the students were not paying attention to the parentheses in the algebraic equations and expressions. One example may be seen below in Figure 2.

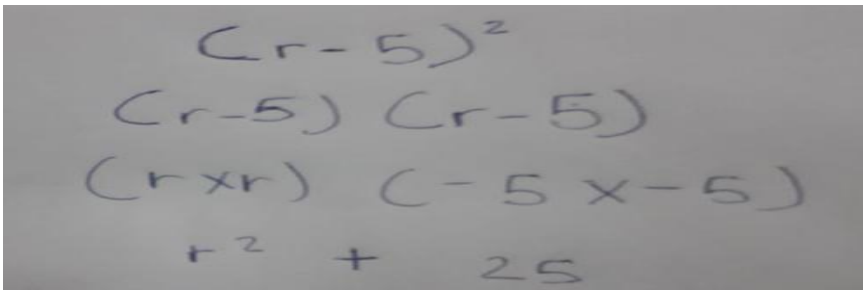

Figure 2: Image showing a student working with the parenthesis in expanding and simplifying an algebraic expression

Similar observations were made in a journal titled "Errors and Misconceptions of eight-grade students regarding operations with algebraic expressions" **[5]** where students would make an error by ignoring the brackets and either multiplying everything or by doing half. For example, given 4 + 5 (x-2), students would get 4 + 5x -2 to give 2+5x. The teacher- researcher also noticed that some students would remain silent and not respond when random questions were asked to probe and follow-up based on their responses given. Whenever they were called individually, it would be an incorrect answer given by the same students. Other times, they would remain silent when they are called at any instance for a response. It was seen that students may respond to repeated failure with withdrawal of effort and avoidance behaviours **[6]**. When this occurs, the teacher may find it difficult to get the students' attention and also to keep them tuned in to the lesson. For the students of my Grade 9 classroom, similar attitudes were observed and this may be because of the low grades they received from their inability to solve the algebraic questions effectively. The existence of this problem negatively impacted the teaching and learning process in the sense that the teacher-researcher had to spend more time addressing the behaviours and repeating what was said, when compared to time spent completing the content of the lesson. Thus, the purpose of this research was to explore the use of guided discovery to enhance students' algebraic thinking skills in a grade 9 mathematics class. It has been stated **[7]** that Mathematics algebraic-thinking skills greatly rely on students' visualization abilities. On the other hand, it has been shared that high anxiety levels may affect this process as it reduces their storage capacity for visual or spatial information (critical skill needed to effectively solve algebraic problems) **[8]**. This may also be linked to the teaching methods that are employed by educators as they seek to bring algebraic concepts across to students. A study done titled "Understanding Inverse Functions: The relationship between Teaching Practices and Student Learning" **[9]** deduced that students who were taught using methods that were focused on developing conceptual understanding performed better in algebra than the students who were taught using strategies that focused on developing their procedural understanding of concepts. Thus, the following research questions were determined:

1) What are some of the challenges that students experience solving algebraic problems in a Grade 9 class?
2) How effective is it to use Guided Discovery to improve the problem-solving skills of Grade 9 students?

Study Design

This study is an action research using the survey design. The survey design enables the numerical and qualitative description of data in a target population where a sample was taken from within this population **[10]**.

Participants

The research was done with six (6) Grade 9 male students' ages of 14-15 years from an all-boys secondary school in Kingston.

Instruments, Reliability, and Data Analysis

To account for the reliability of the study, the 'Triangulation of Instruments' method was utilised. The instruments that were used throughout the study were: observation schedule and field notes, questionnaire, and pre-test and post-test. Data was analysed using descriptive statistics and Microsoft Excel.

2.Results

The results of this study are presented and discussed based on the research questions.

Results Based on Research Question 1

The first research question speaks to some of the challenges Grade 9 students' experienced while solving algebraic problems.

Figure 3 shows that 33% of the respondents experience some level of confusion when they struggle with understanding the symbols and variables used in algebra. Another 33% of the respondents admitted to over thinking questions, resulting in small mistakes. None of the respondents indicated that they had difficulties practising maths on their own or expressed that they had no challenges working algebraic problems. However, 17% of the respondents did state that they had difficulties remembering the algebraic processes to follow when solving these types of questions. Similarly, 17% of the respondents indicated that they had difficulties applying the concepts that were taught in class, in order to complete algebraic tasks.

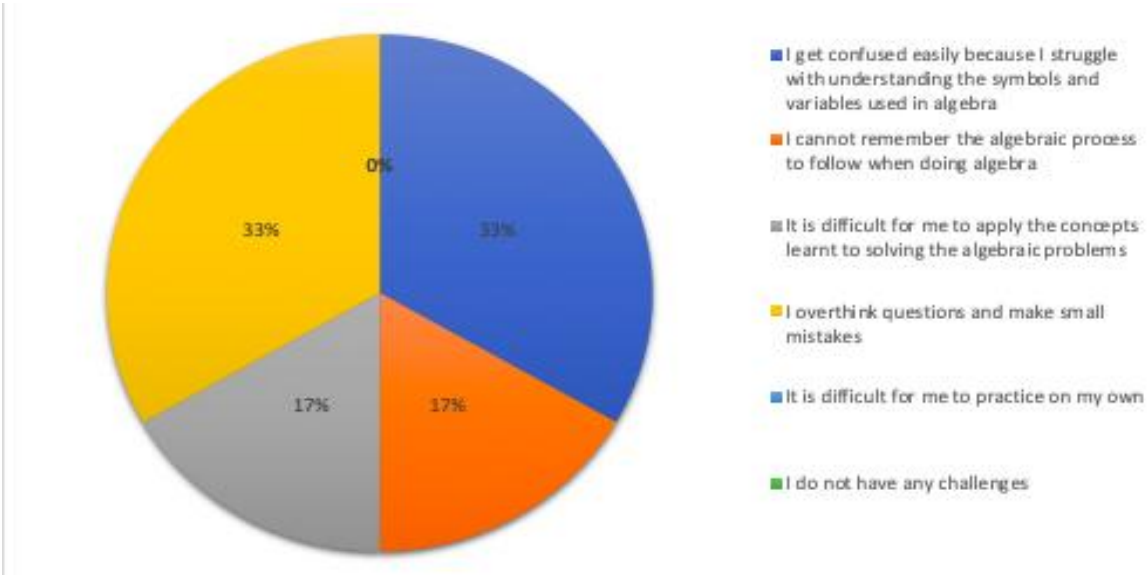

Figure 3: Pie chart showing responses to question: What challenges do you face when solving algebraic problems?

The pre-test had 15 items and sought to test a few objectives with the intention of knowing students' competencies of each. Five (5) of the items focused on the algebraic thinking skills of the students in Algebraic translation, five (5) focused on the algebraic thinking skills of the students in Simplifying Algebraic Expressions (Distributive, Associative and Commutative Properties), and the remaining five (5) focused on the algebraic thinking skills of the students in Solving Simple Algebraic Equations (Requiring Transposition). **Table 1** focused on the objective "*Solving Simple Algebraic Equations (Requiring Transposition)*" which showed that 66% of the respondents were able to apply the one step method accurately in order to solve the given algebraic equations. On the other hand, 33% of the respondents were able to correctly use two or more steps in order to solve algebraic equations (applying distributive law then simplifying).

Table 1: Table showing results of items on the pre-test that measured students' ability to use transportation (one or more than one steps) to solve algebraic equations

Algebraic Skills Observed	Number of respondents who responded correctly(N)	Total (%) of respondents who responded correctly
Correctly conduct one step transposition in order to solve simple algebraic equations	4	66
Apply distributive law to expand then using transposition to solve for unknown (more than one steps)	2	33

Results Based on Research Question 2

The second research question speaks to the effectiveness of using Guided discovery to improve the algebraic thinking skills of the students.

The post-test sought to test the same objectives as the pre-test, containing the same number of items, with the intention

of knowing whether or not students' competencies of each improved after using the guided discovery method. The same objective was focused on from the post-test as the pre-test "*Solving Simple Algebraic Equations (Requiring Transposition)*". **Table 2** showed that all (100%) the respondents were able to apply the one step method accurately in order to solve the algebraic equations. On the other hand, most (66%) of the respondents were able to correctly use two or more steps in order to solve algebraic equations (applying distributive law then simplifying).

Table 2: Table showing results of items on the post-test that tests the students' ability to use transportation (one or more than one steps) to solve algebraic equations

Algebraic Skills Observed	Number of respondents who responded correctly (N)	Total (%) of respondents who responded correctly
Correctly conduct one step transposition in order to solve simple algebraic equations	6	100
Apply distributive law to expand then using transposition to solve for unknown (more than one steps)	4	66

Analysis of a Specific Item Relating to Research Question 1

The study conducted a further analysis with items on research question 1: What are some of the challenges that students experience solving algebraic problems in a Grade 9 class?

The data presented in Figure 4 shows that most students (66%) were confused when solving algebraic problems.

A follow up question in the questionnaire, that asks students to state the reason for their response, further indicated that this occurs when they struggled with remembering the algebraic concepts and how to apply the same. Another reason highlighted was because this is not an area they enjoyed doing. In addition, three (3) respondents were not motivated to complete any algebraic task, and this was affiliated with a struggle to remember and also apply the algebraic concepts and processes, and also due to their struggle in understanding the algebraic symbols and variables. Only two (2) students indicated that they were determined to complete the algebraic tasks and three (3) of the respondents stated that they were confident when doing algebraic tasks since, whenever they fully understood the related algebraic concepts.

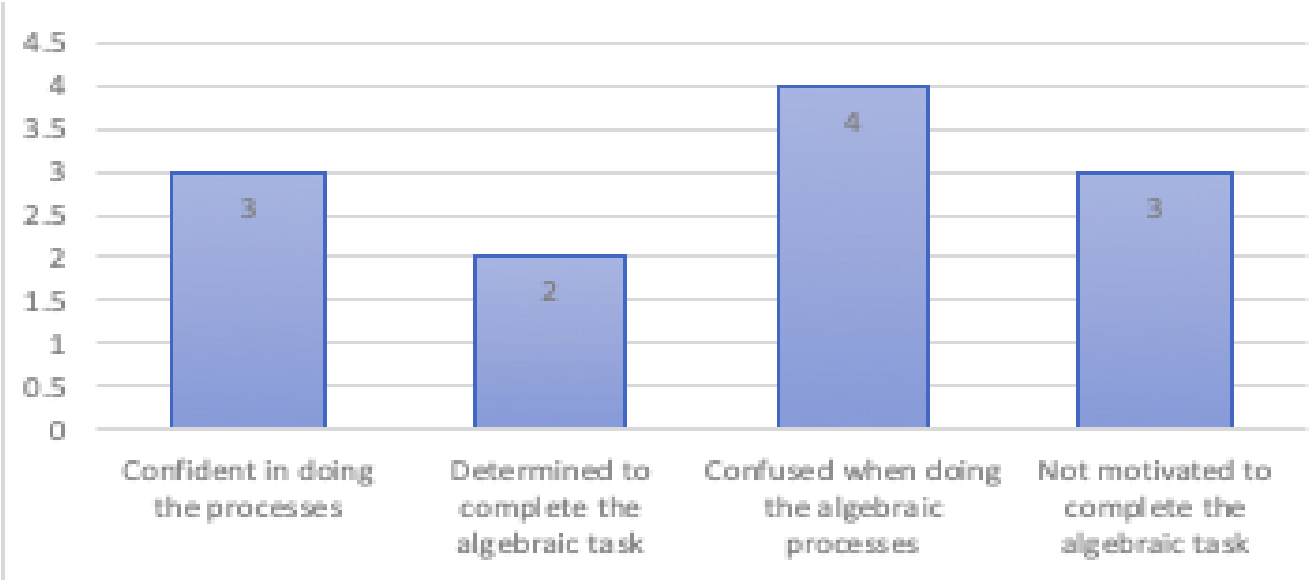

Figure 4: Bar Graph showing students' responses to question: How do you feel when solving any algebraic task?

The results from Table 3 were obtained from two questions that tested the Algebraic translation skills of the students. These questions were focused on testing the respondents' ability to translate from algebraic words to symbols and vice versa. 50% of the students were able to use the appropriate variables to represent given quantities from the given items. 33% of the respondents were able to select and use the correct mathematical operations based on statements given. Only one student (17%) was able to correctly use the variables and symbols in order to translate the algebraic phrase from worded to symbolic form. 50% of the respondents were able to select and use the most appropriate word phrases to represent the algebraic expressions.

Table 3: Table showing the pre-test results from items that focused on algebraic translation

Algebraic Skills Observed	Number of respondents who responded correctly (N)	Total (%) of respondents who responded correctly
Use the appropriate variables to represent given quantities	3	50
Use the correct mathematical operations based on statements given	2	33
Formulate the correct expression to represent an algebraic statement	1	17
Select and use the most appropriate word phrases to represent the algebraic expressions.	3	50

The items in **Table 4** were focused on testing the students' ability to use the various laws (Distributive, Associative and Commutative) to simplify algebraic expressions. This table shows that a few (17%) respondents were able to use the distributive law in order to expand algebraic expressions. At the same time, half (50%) of them were able to group like terms by applying the commutative property. However, only a few (17%) were able to correctly accurately simplify algebraic expressions by applying the associative law (or by other means).

Table 4: Table showing the pre-test results for items that tested the respondents' ability to simplify algebraic expressions

Algebraic Skills Observed	Number of respondents who responded correctly (N)	Total (%) of respondents who responded correctly
Correctly apply the distributive law to expand algebraic expressions	1	17
Apply the commutative property in order to identify and group like terms	3	50
Accurately simplify algebraic expressions by applying associative law (or by other means)	1	17

Analysis of a Specific Item Relating to Research Question 2

The study conducted a further analysis with items on research question 2: How effective is it to use Guided Discovery to improve the problem-solving skills of Grade 9 students?

Based on Table 5 half of the respondents (50%) preferred direct explanations rather than discovery learning approaches. Responses from a follow up question showed that this is true because they preferred to just get straight to the point, rather than "beating around the bushes". Less than half (33%) of the respondents also enjoyed when they were asked the necessary guided questions for them to discover the concepts and the main reason was because they get a better understanding of why the concepts are what they are and not just how to do it. The least number of respondents (17%) preferred getting pictorial representations to help bring across the concepts, for the same reason as stated for the guided questions.

Table 5: Table showing responses for the question: Which is the most effective strategy used in class to help improve your skills in algebra?

Strategies Used in Class	Number of respondents who responded correctly (N)	Total (%) of respondents who responded correctly
When the concepts are explained by a more knowledgeable person without giving you a chance to take part in discovering concepts	3	50
When asked the necessary guided questions in order to discover the concepts	2	33
Getting pictorial demonstrations of the concepts you do not understand as a guide	1	17

Table 6 records the responses that focus on the objective *"Translate between worded problems and algebraic expressions (and/ or equations)"*. The results showed that a little more than half (66%) of the respondents were able to

select and use the correct mathematical operations based on statements given. The same percentage of respondents (66%) were able to correctly use the variables and symbols in order to translate the algebraic phrase from worded to symbolic form. Similarly, 66% of the respondents were able to select and use the most appropriate word phrases to represent the algebraic expressions. On the other hand, most (83%) of the participants were able to use the appropriate variables to represent given quantities from the given items. For example, given a worded problem such as "John's age is 4 years more than thrice June's age. Write algebraic expressions for each of them". The expected answer would be 'x' for June's age and '4+3x' for John's age rather than 'x' for June's age and 'y' for John's age. Relationship between June's and John's ages must be evident.

Table 6: Table showing post test results from items that focused on algebraic translation

Algebraic Skills Observed	Number of respondents who responded correctly (N)	Total (%) of respondents who responded correctly
Use the appropriate variables to represent given quantities	5	83
Use the correct mathematical operations based on statements given	4	66
Formulate the correct expression to represent an algebraic statement	4	66
Select and use the most appropriate word phrases to represent the algebraic expressions.	4	66

The teacher-researcher sought to find out if the students' ability has improved in applying the various laws (Distributive, Associative and Commutative) to simplify algebraic expressions after the guided discovery strategy was implemented. The responses showed that half (50%) of the respondents were able to use the distributive law in order to expand algebraic expressions. At the same time, half (50%) were able to group like terms by applying the commutative property. While a little more than half (66%) was able to correctly accurately simplify algebraic expressions by applying associative law (or by other means).

Table 7: Table showing the post-test results for items that tested the respondents' ability to simplify algebraic expressions

Algebraic Skills Observed	Number of respondents who responded correctly (N)	Total (%) of respondents who responded correctly
Correctly apply the distributive law to expand algebraic expressions	3	50
Apply the commutative property in order to identify and group like terms	3	50
Accurately simplify algebraic expressions by applying associative law (or by other means)	4	66

3.Further Discussion and Implications

A study conducted which involved a quantitative-qualitative analysis **[11]** indicated that students usually attribute algebraic concepts as being difficult due to a lack of understanding of the variables used in the algebraic expressions. The findings from Figure 4 showed that 50% of the respondents were not motivated to complete any algebraic task and this was affiliated with a struggle to remember and apply the algebraic concepts and processes, and also due to their struggle in understanding the algebraic symbols and variables. Some respondents also reported feeling a sense of confusion when this occurs. This could be as a result of having mathematics anxiety **[12]** that interferes with an individual's ability to solve algebraic problems as their ability to manipulate numbers and letters are greatly hindered; which can account for a reason shared by the students. Additionally, Dewey's Theory stipulates that students' critical thinking skills are positively impacted as they discover and see the relevance in particular concepts-making application of these concepts much easier. This was true for the students as the students indicated, as expressed in Figure 4, that they would feel confident and determined to complete algebraic tasks due to having a good understanding of the concepts.

While this is so, based on Figure 3, 66% of the respondents indicated that they would get confused whenever they

struggled to remember the algebraic processes and also when they struggled to determine how to apply the particular algebraic processes in solving algebraic problems. This may be due to the students not being able to understand the particular algebraic processes and as such, may find it difficult to apply this same knowledge in solving new problems (leading to a sense of confusion). This is another challenge that students may experience when doing algebraic problems. It has been shared **[13]** that a teacher-centred classroom is more organized, since the teacher is in charge, which makes it easier for students to know where to channel their focus during their lessons while a student-centred classroom is one in which the students are responsible for their own learning, with the teacher as a guide to ensuring the necessary objectives are achieved. The researcher wanted to know if the students had a preference for a student-centred teaching approach when compared to the teacher-centred approach which they were initially introduced to by the cooperating teacher. Based on Table 5, 50% of the respondents preferred when the teacher explained concepts, without being involved in the process, while another 50% preferred a more guided approach. It was found that the respondents preferred a teacher-centred approach because they will get straight to the point instead of "beating around the bushes". This may be because for a student-centred approach, the students get to explore with content, making mistakes, asking questions, among others, as they are guided through the process of discovering new concepts. While this may result in some students being better able to recall and apply algebraic concepts (as indicated by students who preferred the guided discovery approach) it may be somewhat of a stress to other students as they want to get straight to the point.

The teacher-researcher focused on the algebraic thinking skills of the students in three (3) different algebraic areas: Algebraic Translation, Algebraic Expressions and also Algebraic Equations. It was observed that even though the students would be able to state a variable that they wanted to use to represent a quantity (after being asked), discovering that a variable was to be used and also developing the algebraic expression (using these variables) to represent a situation posed much of a challenge for some. Also, based on Table 3, 50% of the respondents were able to use the appropriate variables to represent given quantities and 33% were able to select the correct operations ($+$, $-$, $\div$ and $\times$) for use. Only 17% of the students were able to formulate the correct expression to represent an algebraic statement. This meant that there was a challenge for the students to connect the dots in an effort to develop the most appropriate algebraic expression to represent a particular worded expression. On the other hand, 50% of the participants were able to select and use the most appropriate word phrases to represent the algebraic expressions. The main misconception observed by the researcher was translating the symbolic representations to worded problems; and correctly referring to the variables and the powers. For example, if the students were asked to translate $(a + b)^2$ to words, it was observed that some students would respond by saying "a plus b square", whose symbolic translation is "$a + b^2$". This means that the students were not considering the brackets involved. One suitable translation of this expression would be "the sum of a and b all squared". After being introduced to the strategy for the research period, more students were able to use the appropriate variables to represent given quantities (83%) and also to use the correct mathematical operations based on statements given (66%), thus formulating the correct algebraic expressions to represent an algebraic statement- as indicated by **Table 6**.

The researcher also made note of some reasons they decided to use a particular expression to represent the worded expression (during class discussions) and one response that stood out was "because we need variables to represent the number of apples used so 'a' will be for apples. Twice the amount means multiply by 2 and 5 more means addition so that's how I got 2a+5". This response shows that the student was reasoning with the expression as he/she tried to translate. An additional student (making it 66%) was able to select the most appropriate word phrases to represent the variables in order to represent the algebraic expression. This may be due to the students being guided by being asked the necessary questions whenever they explained that they did not understand, making it easier to make the necessary discoveries needed to connect the dots between all three areas (determining variables to be used, mathematical operations and putting those together to develop the mathematical expression). From **Table 4**, only 17% of the respondents were able to correctly apply the distributive law in order to expand an algebraic expression. The researcher is aware that this is a process that may require some amount of focus and technique in order to eliminate any mistake at this level. During the observation period, the researcher made particular observations. One major error observed was that some students would tend to not multiply through the entire bracket when doing the distributive property. For example, it was observed that if given 5(2x+4), some students would multiply the 5 by the 2x but will not follow through by also multiplying the 5 by the 4. Therefore, the answer that would be achieved would be 10x +4, instead of 10x + 20. While this is so, **Table 4** also shows that 50% of the students were able to group particular like terms after they have expanded the brackets (even if done incorrectly). This showed that the students were still unclear of how the distributive property is to be applied.

The researcher also noticed that even though some students were able to identify and group like terms, only one out of the 6 participants (17%) was able to correctly simplify the algebraic expression. This was so as they struggled with adding or subtracting the coefficients of these like terms and the cause of this may be due to the students not identifying a term with the coefficient of 1. As stipulated by the educational philosopher, John Dewey, experiential learning incorporates students' active engagement and aids in use of their critical thinking skills. As such, the teacher-researcher utilized visual representations in an effort to provide some level of experimenting and guide for the students to better understand the distributive property. Here, two rectangles were used to represent an algebraic expression that needed to be expended. The students then experimented with finding its area, after which they were introduced to the theoretical process that outlines the distributive law (F.O.I.L Method). This method made it clearer for the students to see the logic behind the distributive law- this took more than two 1-hour classes for the students to demonstrate an

acceptable level of understanding of the distributive process. In the end, as displayed in Table 7, 50% of the students were able to correctly apply the distributive law to expand algebraic expressions, 50% of the students were able to apply the commutative property in order to identify and group like terms, and 66% were able to accurately simplify algebraic expressions by applying the associative law (or by other means). The respondents were able to provide logical explanations to why each response was given. This is so as they identified that the distributive properties will be needed then stated the like terms before going on to grouping.

Finally, the researcher also tested the students' skills in working with simple algebraic equations. The results from **Table 1** showed that 66% of the respondents were able to conduct the one step transposition process in solving simple one step algebraic equations. The post-test results showed that all the respondents were able to solve the questions of this nature correctly, as seen in **Table 2**. The researcher used real-life balanced scale simulations in order to bring across the concept of solving algebraic equations using a one-step transposition. This may have resulted in a better understanding of the processes among the students as they were able to see how the transposition concepts work. Additionally, **Table 4** showed that only 33% of the respondents were able to apply the distributive law to expand, followed by the use of transposition to solve for the unknown (using more than one step to solve). The post-test results (seen in **Table 2**) also showed an improvement as now 66% of the respondents were able to do a mixture of processes, leading up to transposition, in order to solve for an unknown in an algebraic equation. The researcher utilized the flow chart method in order to bring across this concept to the students. Even though it took a while for the students to understand this concept, the results do show that it was effective in most cases.

The findings of this study could have the following implications: (a) it provides suggestions of teaching methods that could be used to shape the teaching and learning of algebra at the different grade levels at the secondary level; and (b) identifies the challenges that students face with the strand of Algebra may also help to shape how mathematics educational programmes at the tertiary level are designed to train teachers for our classroom in Jamaica and beyond.

4.Conclusions

The following conclusions were made based on the findings of the study:

1) The algebraic skills that the students had most issues with were:
   - Application of the distributive law;
   - Deducing the correct algebraic expressions for a worded expression;
   - Applying all rules to simplify an algebraic expression correctly.
2) The results showed that guided discovery learning is effective in improving students' algebraic thinking skills.

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