

Finite Element Analysis of Concrete Beam using MATLAB

P. Raj Kumar

PG Student, KITS Warangal, Warangal, India
m20sc004[at]kitsw.ac.in

Abstract: Finite element analysis generally refers to the process of determining the behavior of the physical phenomena. In case of beams, the displacements and rotations that are determined by using the finite element analysis involves in creating the mesh of the very finite elements that are combined to form the shape or outline of the structure that is being under consideration. It involves in solving of differential equation generally of the fourth order. In case of beams, Euler Bernoulli theory is used to determine the behavior of the beams which involves in the determination of stiffness matrix and force matrix which are used to compute the unknown rotations and displacements. With increase in the number of elements or discretized parts the order of the global matrix gets increased (for example, when the number of elements considered are one the order of global matrix is 4×4 , when the number of elements considered are two the order of global matrix is 6×6 and so on...), and the calculations gets complicated. Hence for the easier computations MATLAB software is used which involves in use of loops like for loop, if loop etc. In the current work, the concrete beam of various dimensions is analyzed using the finite element method with the MATLAB software. The unknown displacements and the rotations are determined by forming the global stiffness and global force matrix with the help of element stiffness and element force matrix. To find the unknown displacements and the rotations using MATLAB an error free, executable program is coded using the command line present in the MATLAB. This executable program is developed using the loops. The obtained results are then compared with that of the manual method.

Keywords: Finite element analysis, concrete beam, deflections, rotations, MATLAB.

1. Introduction

Finite element analysis involves in creating the mesh of the very finite elements that are merged together to make the shape or outline of the structure that is being under consideration and analysed. Finite element generally involves in the solving of the differential equation [1].

In case of beams, Euler Bernoulli theory is used to determine the behaviour of the beams. It involves in the determination of the stiffness matrix and force matrix which are used to compute the unknown rotations and displacements [2]. With increase in the number of elements or discretized parts the order of the global matrix gets increased (i.e., for n elements the order of global matrix will be $2(n+1)$). As a result, the calculations get complicated while performing the manual method.

MATLAB (MATrixLABoratory) is a computerized programming software developed by 'Cleve Moler' which is used to perform the various numerical calculations of the engineering problems. It is most accurate and the well optimized software developed to run the scientific and the engineering calculations. It has many advantages like easy to use, independent of the platform, contains a library of functions, contains the plotting tool, easy to develop graphical user interface and contains versatile compiler but has some disadvantages like longer run time when compared to other compilers and the basic version costs much higher than other compilers. The file that are saved in the MATLAB is generally known as the M files and contains the extension of .m which is executable.

2. Research Significance

In order to counter the complexity in the manual method with increase in the number of elements and for the easier computations, the finite element analysis of concrete beam is carried out using the MATLAB software which greatly reduces the involvement of time and can be applied to similar problems having different geometry and dimensions.

1. Beam details

The details of the beam are:

- Width of beam (b) = 0.1 m,
- Depth of beam (D) = 0.15 m,
- Length of beam (L) = 1.2 m,
- Grade of concrete (f_{ck}) = 40 N/mm²,
- Grade of steel (f_y) = 415 N/mm²,
- Modulus of elasticity (E) = 5000 sqrt(f_{ck})=32000000 kN/m²,
- clear cover = 25mm,
- Diameter of main reinforcement = 10mm,
- Number of bars = 2,
- Ast = 157 mm².

A total of three cases are considered i.e., when the section is uncracked, at cracking and after cracking. The finite analysis in manual method is carried out by considering two elements only as increase in the number of elements will lead to complex calculations in manual method.

2. MATLAB Programming code

The finite element analysis of beam is carried out in MATLAB is carried out by considering 80 elements. The MATLAB programming code is:

```
clc;
clear all;
```

```

% Beam Details
fck=40 % fck in N/mm^2
fy=415 % fy in N/mm^2
Ec=32*10^6 % Youngs Modulus of Concrete in kN/m^2;
b=0.10; % Width of beam in m
D=0.15; % Depth of beam in m
n_b=2; % Number of bars
dia=10; % Diameter of bar in mm
A=(n_b*(3.14/4)*dia*dia);
c_c=25 % Clear cover in mm
d=D-(c_c*10^-3)-((dia*10^-3)/2); % effective depth in m
Igr=b*D^3/12 % Ieff in m^4
DL=25*b*D;
LL=1.0; % LL in kN-m^2
L=input('Length of Beam (m)=');
Le=1.0 % Effective length in m
P=-1.5*(DL+LL) % Load on Beam (kN/m);
M=(-P*Le^2)/8; % M in kNm
fr=0.7*sqrt(fck);
Mcr=((fr*Igr*10^12)/((D*10^3)/2))*10^-6 % Mcr in
kNm
if Mcr>M
Ief=Igr;
else
m=(2*10^5)/(Ec*10^-3);
D_N=((-
m*A)+(sqrt((2*(b*10^3)*m*A*(d*10^3))+((m*A)^2))))/(b
*10^3);
Z=((d*10^3)-(D_N/3));
Icr((((b*10^3)*(D_N^3))/3)+(m*A*((d*10^3)-
D_N)*((d*10^3)-D_N)))10^-12; % Icr in m4
Ief=((Icr)/(1.2-((Mcr/M)*(Z/(d*10^3))*(1-
(D_NA/(d*10^3))))));
end
EI=Ec*Ief % EI in kN-m^2
% Discretization
ne=80; % Number of elements
n_c=linspace(0,L,ne+1)'; % Node Coordinates
x=n_c;
L=max(n_c); % Check
n_n=size(n_c,1); % Number of nodes
x=n_c(:,1);
% Element Nodes
for j=1:ne
en(j,1)=j;
en(j,2)=j+1;
end
g_dof=2*n_n; % Global Degree of Freedom
% Initialising Matrices
% Unknown rotations and displacements matrix
Un=zeros(g_dof,1);
% Force matrix
Force=zeros(g_dof,1);
% Stiffness matrix
Stiffness=zeros(g_dof);
% Rotation matrix
Theta=zeros(g_dof,1);
% Displacement matrix
Y=zeros(g_dof,1);
% Stiffness Matrix and Force Matrix
for e=1:ne
in=en(e,:);
edof=[2*(in(1)-1)+1 2*(in(2)-1)...

```

```

2*(in(2)-1)+1 2*(in(2)-1)+2];
% Length of each element
a=x(in(2))-x(in(1));
% Stiffness matrix
K=(EI/(a^3))*[12 6*a -12 6*a ;
6*a 4*a^2 -6*a 2*a^2;
-12 -6*a 12 -6*a;
6*a 2*a^2 -6*a 4*a^2];
% Force matrix
F=[P*a/2;
P*a*a/12;
P*a/2;
-P*a*a/12];
% Global Stiffness Matrix
Stiffness(edof,edof)=Stiffness(edof,edof)+K;
% Global Force Matrix
Force(edof)=Force(edof)+F;
end
% Boundary conditions
u=[1 2*ne+1]';
v=[];
pdof=[u;v];
% Determining unknown rotations and deflections
adof=setdiff([1:g_dof]',[pdof]); % Active Degree of
Freedom
Un=(Stiffness(adof,adof)\Force(adof))*10^3;
% Grouping all rotations
Theta=Un(1:2:2*ne) % Rotations in radians
Theta_max=max(abs(Theta))
% Grouping all deflections
Y=Un(2:2:2*ne-1) % Deflection in mm
Y_max=max(abs(Y))
figure(1)
plot(Y);
ylabel('Deflection in mm');
xlabel('Number of Elements');
figure(2)
plot(Theta);
ylabel('Rotations in radians');
xlabel('Number of Elements');

```

3. Results

The finite element analysis of beam is carried out in manual method and MATLAB software. The deflections and rotations are determined.

3.1 Deflections

From MATLAB, the unknown deflections are found out by considering 80 number of elements. It has found that the maximum deflection from MATLAB is equal to that of maximum deflection obtained in manual method for the various cases considered and occurs at the centre of the mid span for simply supported beam. The Table 1 shows the deflections values from both the manual method and the MATLAB software for different cases considered.

Table 1: Deflections

Case	Maximum deflection (mm)	
	Manual method	MATLAB
Section is uncracked	-0.06	-0.061875
Section is at cracking	-0.80	-0.805477
After cracking	-0.89	-0.892130

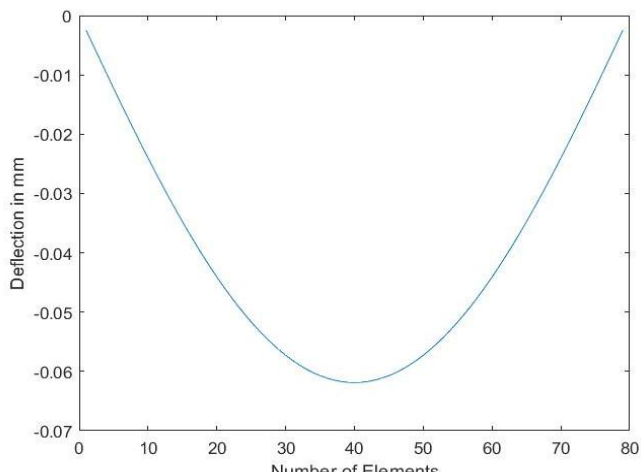


Figure 1 (a): Deflections vs Number of elements when section is uncracked

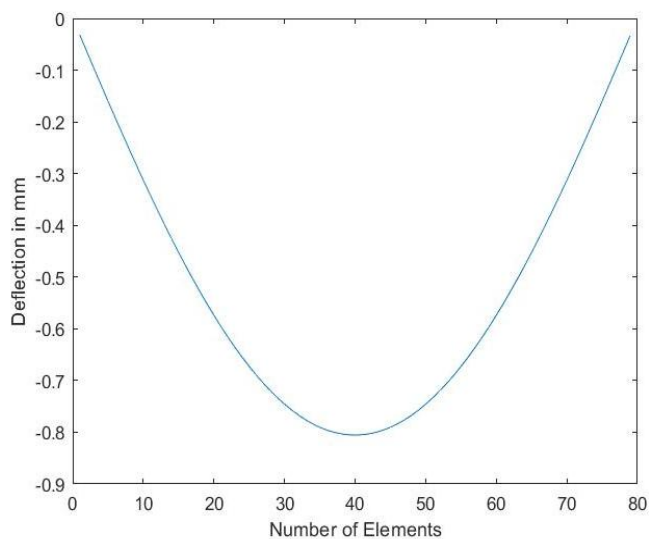


Figure 1 (b): Deflections vs Number of elements when section is at cracking

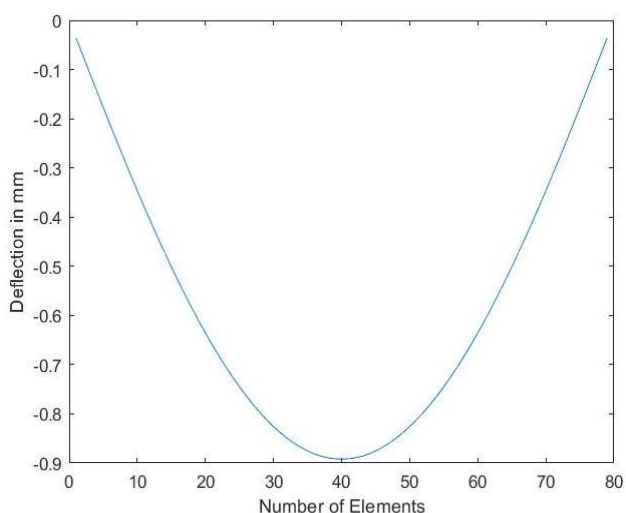


Figure 1 (c): Deflections vs Number of elements after cracking

3.2 Rotations

From MATLAB, the unknown rotations are found out by considering 80 number of elements for the various cases considered. The Table 2 shows the rotations values from

both the manual method and the MATLAB software for different cases considered.

Table 2: Rotations

Case	Maximum rotation (radians)	
	Manual method	MATLAB
Section is uncracked	-0.16	-0.165
Section is at cracking	-2.15	-2.147939
After cracking	-2.38	-2.379013

4. Conclusions

The current work is mainly focussed on usage of MATLAB to perform finite element analysis of concrete beam.

- MATLAB is most accurate, precise and the well optimized software developed to run the scientific and the engineering calculations.
- The maximum deflection obtained in the manual method is equal to the maximum deflection obtained in the MATLAB and it is found at mid span for the simply supported beam.
- The maximum rotation obtained in the manual method is equal to the maximum rotation obtained in the MATLAB and it is found at the supports for the simply supported beam.
- The accuracy and precision of the results have increased with the increase in the number of elements.

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