

Didactic Knowledge of the Teacher Who Teaches Mathematics in Primary School

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Abstract: *In the framework of an ongoing doctoral study (Doctorate in Education, Faculty of Education, Universidad de Antioquia, Colombia.), the aim is to disseminate the progress of a systematic literature review that aims to know, identify and analyse some studies on the knowledge that a primary school teacher must have to interpret and manage the mathematics curriculum, plan educational practice and implement it in the classroom. The information analysed so far allows us to identify the existence of research that has focused on the teacher's knowledge and professional development, but we need to delve deeper into the didactic knowledge of primary school teachers in the process of (re)constructing the mathematics curriculum.*

Keywords: Mathematics Curriculum, Professional Knowledge, Didactic Knowledge, Professional Development

1. Introduction

Teaching mathematics implies, according to Ball, Thames and Phelps (2008), that the teacher knows the subject he is teaching and helps his students in learning it. This statement marks the beginning of an investigative interest, the result of the experience acquired as a mathematics teacher in public and private educational institutions in the city of Medellín Antioquia Colombia, and of the work currently carried out as a teaching director (Those who carry out the activities of direction, planning, coordination, administration, orientation and programming in educational institutions are called teaching directors. (Art. 6 Decree 1278 of 2002)).

From these experiences, it is considered that In general, primary school teachers are not specialized in an area of knowledge, which could be related to some difficulties they present with respect to mathematical knowledge.

A particular case is evidenced when in the practices of some teachers the teaching of number thinking, number systems, the exercise and setting of procedures prevails, a situation that partly leaves aside the teaching of other thoughts and systems that guide the Guidelines. Mathematics Curriculars (MEN, 1998), a problem that could be the product of the little or scarce training that primary school teachers have in the area of mathematics, which can be reflected in curricular aspects such as classroom planning and management.

In this regard, in the research by Carrillo and Climent (2006), it is considered that the curriculum is an obligatory reference for any approach to improving the education system and also reflects the need to relate it to professional development processes. Along the same lines, the United Nations Educational, Scientific and Cultural Organization (UNESCO, 2016) also highlights that the curriculum is a fundamental element for the achievement of high quality learning results and that is why it is done it is necessary to

look back at him; Furthermore, it states that teachers must be the essential protagonists because they are the ones who put the curriculum in the classroom on stage.

In the case of Colombia, the General Law of National Education (Law 115 of 1994) gives school autonomy to educational institutions to build and design the curriculum and is the Ministry of National Education (MEN) the person responsible for presenting general guidelines regarding it, as set out in articles 77 and 78 of this Law.

To respond to the above, the MEN in the last two decades hamade available to Educational Institutions some reference documents that guide the mathematics curriculum, such as: Curricular Guidelines, Basic Competency Standards, Basic Learning Rights and Learning Grids. And in the same way, he proposed, some pedagogical and curricular tools within the framework of the Integration Strategy of Curricular Components (EICC), so that Educational Institutions develop concrete actions that allow reviewing and updating the curricular processes with a view to strengthening the institutional and classroom practices (MEN, 2017).

From the above, it can be deduced that teachers then have a fundamental role in the design of the mathematics curriculum and its application in the classroom. However, in the light of Gómez and Velasco (2017), In order to put the reference documents of the MEN into practice, "teachers require disciplinary knowledge and mastery to interpret said documents" (p. 278). In other words, the teacher uses the curricular documents based on the interpretation they make of them and, this interpretation depends on how much they understand their content, a situation that is reflected in their classroom practices. In addition, different teachers have different ways of understanding and applying the curriculum, hence, teacher, curriculum development and professional development are closely linked elements (Carrillo and Climent, 2006).

ANDn words of Gómez and Velasco (2017), the solution is not a matter of explaining the content of the reference documents by means of short training sessions; In addition to the above, it is required to strengthen the initial and ongoing training of teachers. Therefore, it can be seen that this difficulty puts into consideration the need to analyze the didactic knowledge of primary school teachers in the interpretation process and management of the mathematics curriculum, the planning of its educational practice and its implementation in the classroom, through an accompaniment strategy that contributes to the knowledge and professional development of the teacher.

It can be inferred then, that it would be necessary to analyze the knowledge of primary school teachers, in order to recognize ways of thinking and promote improvements in their classroom practices, in the words of Rojas, Flores and Carrillo (2015) the performance of the teacher who teaches Mathematics in elementary school is influenced by professional knowledge, which allows you to design, apply or act on your educational practice. In addition, in the light of Santa and Jaramillo (2015), consolidating spaces for professional reflection, forming collectives, understood as groups composed of teachers in interaction with themselves, with peers, with students and with the media, can promote transformations in the knowledge that teachers have about teaching.

Regarding the professional knowledge of the mathematics teacher, it is perceived that the theories highlight the importance of the knowledge of the curriculum, which, according to Ball, Thames and Phelps (2008) refers to the knowledge of the objectives, aims, curricular orientations, materials and resources available to teaching, and are necessary in decision-making and ways of guiding teaching.

In this sense, the purpose of this article is to identify some aspects associated with the didactic knowledge of teachers who teach mathematics in primary school, through a systematic literature review. To achieve this, the following guiding question is posed: What aspects have been associated with the didactic knowledge of primary school teachers in the process of (re) construction of the mathematics curriculum?

The structure of this article is based on the phases proposed by Kitchenham (2007) and is given by the methodology of the review, which considers stages such as a systematic review of the literature, a formation of the corpus and a classification of the literature; The results are presented organized in three themes: an approach to the mathematics curriculum, professional knowledge of the mathematics teacher and professional development of the mathematics teacher; and finally, present some conclusions of the respective review.

2. Methodology of the Review

2.1. Systematic review of the literature

The literature review on a particular topic allows to have an overview of what is known about it, in light of

Kitchenham (2007), is the possibility to identify gaps in the literature that allow posing a research problem and present a reasonable evaluation of the subject by using a reliable, rigorous and auditable methodology (p.6).

Although there are different types of literature reviews, the present design corresponds to a systematic literature review, which unlike another type:

It is a reproducible, auditable and systematic methodology to formulate relevant research questions about a thematic area or phenomenon of interest and to search, select, analyze and synthesize all the relevant, empirical or theoretical research, necessary to answer said research questions (Velásquez, 2014, p. 2).

To carry it out, three phases are taken into account: literature search, classification of the same applying inclusion and exclusion criteria, and review report. These phases are supported by the stages proposed by Kitchenham (2007) and the steps (objective of the literature review, protocol and training, literature search, practical assessment, quality assessment, data extraction, study synthesis, writing the review) raised by Okoli and Schabram (2010), so that the rigor of the process can be kept and its replicability sought.

2.2. Conformation of the corpus

In order to guarantee that the studies found were relevant to the topic of interest, a plan was drawn up that would allow the review to be carried out and its relevance evaluated. It was then defined as Search categories (in Spanish, English or Portuguese): 'mathematics curriculum', 'mathematics curriculum in the context of didactic knowledge', 'primary teacher's curriculum' and 'knowledge and professional development of the mathematics teacher', and the period of time.

Articles published in the last ten years, that is, in the period between 2009 and 2019, were selected in databases such as Scopus, Springer, Scielo, Dialnet, JSTOR, DOAJ and Science Direct, and in Google Scholar. Taking into account criteria such as keywords, abstract, theoretical framework, methodology and conclusions of each document; 67 were preselected, of which 39 showed affinity with the proposed guiding question. Finally, the bibliographic references of the articles found were reviewed, which allowed obtaining a total of 42 documents, some of them from specialized journals.

2.2.1. Inclusion Criteria:

1. Types of studies: research articles (peer review), review articles, conference proceedings recognized on the subject, books or book chapters from publishers recognized by the academic community.
2. Time period: 2009 to 2019
3. Study participants: pre-service teachers, teachers who teach mathematics in primary school.
4. Languages: English, Spanish or Portuguese (most common languages in which information is published in mathematics education).

5. Methodologies: studies that specify the research methodology used (qualitative, quantitative or mixed).

Table 1 show the sources and the databases are specifically indicated.

Table 1: Sources and numbers of selected articles

Source	Web address	Number of Items
Scopus	https://www.scopus.com/	16
Springer	http://www.springer.com/	9
Scielo	https://www.scielo.org/	12
Dialnet	https://dialnet.unirioja.es/	14
JSTOR	https://www.jstor.org/	one
DOAJ	https://doaj.org/	two
ScienceDirect	https://www.sciencedirect.com/	two
Google Sholar	https://scholar.google.com/	eleven

Own elaboration

2.3. Classification and organization of the literature

At this stage, the 67 documents were analyzed and the information was systematized, for this the summary (or abstract) was read and a matrix was designed in Microsoft Excel where the following information was included: references, keywords, ideas of the approach of the problem, objectives, theoretical framework, conclusions, questions that were left open, inclusion or exclusion criteria. Those who made explicit reference to the search term in the title, abstract or keywords were included; and those documents that did not include professional knowledge, or where no explicit reference was made to the teacher's curriculum, were excluded.

Once the documents were selected, taking into account the inclusion and exclusion criteria, we proceeded to prepare files, in a Microsoft Excel format, which would preserve the information from the matrix, but also included: date of review, results and possible important aspects. The synthesis of this information made it possible to identify common elements, which were later grouped into themes: mathematics curriculum, professional knowledge of the mathematics teacher and professional development of the mathematics teacher as a possibility to build knowledge. Table 2 shows the classification of the documents by subject, categories and author.

Table 2: Classification of documents by subject, categories and author

Thematic	Categories:	Author	Comments
Mathematics Curriculum	Perspectives	Osorio (2017), Gimeno (2010), Guacaneme; Obando, Garzón, Villa-Ochoa (2013), Carrillo and Climent (2006).	Alternative to think about the curriculum and diversity in teacher training.
	Curriculum reference documents	Gómez and Velasco (2017), Osorio (2016)	The teacher uses the curricular documents based on his interpretation of them.
	Curriculum management	Ponte (2005), Orobio and Zapata (2017), Rico (2010), (Climent et. Al, 2014)	The teacher necessarily requires planning, interpreting, putting into practice and evaluating their curricular options.
Professional knowledge of the mathematics teacher.	Teacher's role	Canavarró and Ponte (2005), Ponte, Matos and Abrantes (1998)	Of all the curricular decision-makers, the teacher is undoubtedly the most important in the development of the curriculum.
	Pedagogical Content Knowledge (PCK)	Shulman (1996 and 1987), Bromme (1994), Ball, Thames and Phelps (2008)	It seeks to highlight the importance of content knowledge for teaching and differentiate it from the content knowledge that other professionals have.
	Mathematical Knowledge for Teaching (MKT)	Hill, Ball and Schilling (2008) Ball, Thames and Phelps (2008)	They establish a practical foundation based on what is called mathematical knowledge for teaching.
	Mathematics Teacher's Specialized Knowledge (MTSK)	(Carrillo et al, 2013) (Carrillo et al, 2015)	The MTSK model starts, like the MKT model, from two large knowledge domains: (a) mathematical content knowledge (MK) and (b) didactic content knowledge (PCK).
	Semiotic Logical Approach (ELOS)	Socas (2001 and 2007), Socas (2010)	Consideration of the relationships that exist between the three elements: curricular mathematical content, mathematics students and teachers.
	Didactic Knowledge	Ponte (2012)	It has knowledge of teaching practice at its center
	Didactic-Mathematical Knowledge	Godino, Batanero and Font (2007) Pino-Fan, Godino and Font (2015)	The CDM model, which proposes three large dimensions for the analysis of the teacher's knowledge.

		Assis and Godino (2015) Pino-Fan, Assis and Castro (2015) (Castro, Pino-Fan and Parra-Urrea, 2018)	
Professional Development of the mathematics teacher.	Perspectives	Hernández and Pérez (2017) Rodríguez and Pérez (2016) (Climent et. Al, 2014)	It is not synonymous with domain of certain content.
	Continuous training	Imbernon (2011 and 2013), Vaillant and Marcelo (2015), Vaillant (2016), (Muñoz et. Al, 2013)	Any systematic attempt to improve work practice, professional knowledge and beliefs.
	Process	Bautista and Ortega Process (2015), Avalos (2011), Lima (2017)	How do they learn to learn and transform their professional knowledge into practice to benefit the development of their students?
	Realization of the teaching staff	Ponte (2012), (Climent et. Al, 2014)	Progressive development of potentialities and the construction of new knowledge.

Own elaboration

3.Results

In accordance with what was stated in the previous section, the review of the systematized literature on topics such as: mathematics curriculum, professional knowledge of the mathematics teacher and professional development of the mathematics teacher is presented below. The reflections that originated from a careful reading allowed an approach to a possible answer for the question that guided the review; in this sense, the proposed topics and the analyzes that are specified in each of them are presented below.

3.1. An approach to the mathematics curriculum

This section initially presents a look at the mathematics curriculum through the technical standards of the MEN in order to contextualize the reader, later some trends of national and international authors are presented regarding the subject in question.

The Ministry of National Education has designed some reference documents with the purpose of providing national guidelines and criteria on curricula, an example of this are the Curricular Guidelines (MEN, 1998), Basic Competency Standards (MEN, 2006), Basic Rights of Learning (MEN, 2017) and Learning Mesh (MEN, 2017).

The Curricular Guidelines in Mathematics are intended to offer conceptual, pedagogical and didactic orientations for the design and development of the curriculum in the area. They show in part the philosophical and didactic principles establishing relationships between the basic knowledge (numerical and number systems, spatial and geometric systems, metric and measurement systems, variational and algebraic and analytical systems, and random and data systems), the processes (reasoning, communication, modeling, problem solving and posing, elaboration, comparison and exercise of procedures) and the contexts (mathematical, everyday life and other sciences); mediated by problem situations and evaluation as components that can contribute to orient, to a large extent.

In this regard, the organization of school work framed in basic knowledge, processes and contexts, particularly in thoughts, presupposes, according to Zapata and Jaramillo (2018), that at the levels of basic and secondary education,

mathematical thinking permeates the curriculum with its five subdivisions, and that, to achieve this purpose, teachers should know the characteristics and structure of each one of them (p.195).

Faced with the Basic Competency Standards (EBC), according to (MEN, 2006), these are proposed to provide parameters of what every student must know and know how to do to achieve the level of academic quality expected by grades of schooling: (1st-3°), (4°-5°), (7° -9°) and (10°-11°). For its part, the Basic Learning Rights (DBA) present a group of Structuring Learning that promote the integral development of those who learn and make explicit learning that is recommended to be the object of reflection for the construction of curricula in educational institutions (MEN, 2017).

According to the MEN, the DBAs can be understood as a set of knowledge that can be mobilized from one grade to another and that these by themselves do not constitute a curricular proposal but rather a reference for the planning of the area and the classroom. In the same way, the Learning Mesh retakes the structuring learning defined in the DBA and puts them in dialogue with the Curricular Guidelines and the Basic Learning Standards, according to the MEN (2017), with the purpose of providing educational institutions already the teachers clarify about the learning that students are expected to develop grade by grade, offering didactic suggestions that guide the processes of strengthening and updating the curriculum, especially in the classroom.

Apparently, the national reference documents imply challenges of a curricular nature and updating processes. These documents by themselves do not constitute a curricular proposal, they must be articulated with the approaches, methodologies and strategies defined in each institution within the framework of the PEI; neither the learning grids nor the DBAs substitute the curricular grids of each educational institution (MEN, 2017).The MEN hopes that these documents will be taken up by groups of teachers in educational institutions, so that they are the object of analysis and starting point in processes of re-meaning of their pedagogical practices.

In that order of ideas, the curriculum in some way continues to be the object of analysis. In this regard, Orobio and Zapata (2012) present two types of mathematics curriculum, the prescribed curriculum and the applied curriculum; the first as a set of actions and claims of the institution, which from the academic component of the Institutional Educational Projects orients the study plans and area plans, and the second as the set of educational practices carried out by the teachers. According to the authors, the prescribed curriculum accounts for the educational institution's interpretation of the policies formulated by the MEN in terms of curriculum management and it is important that it is in line with the applied curriculum.

Furthermore, the concept of curriculum seems to adopt different meanings. Gimeno (2000) presents a vision of curriculum as a process and considers different curricula resulting from different interest groups: prescribed curriculum, guided by political-administrative bodies; designed curriculum, guided by the norms that come to the teacher; organized curriculum, which results from the teacher's interpretation; Curriculum in action, refers to the one practiced in the school reality, in the classroom; and evaluated curriculum, that evaluated in standardized tests, considering what is worth learning. The curricular role of the teacher is recognized, individually or in conjunction with their colleagues, it is the teacher who is responsible for adapting the prescribed curriculum to the needs of their students and the context (Gimeno, 2010).

However, apparently there is a great distance between the curriculum and what happens in the classroom (Ponte and Canavarro, 2005), for these authors there are several elements that influence between the promulgation of the official curriculum and its practical experience on a day-to-day basis. of the school, made up of diverse actors, teachers and students. These authors return to Gimeno (1989) to consider the curriculum as a conflict of practices that imply diverse determinations, which naturally can vary from one country to another; For the author, it implies decision-making from the political-administrative system, parents' associations, unions, among others, which in one way or another influence its construction. They also state that each context and each group of authors have their version of the curriculum.

In this sense, it is important to focus on classroom practices, because teachers are an important element, since they are the ones who, in the end, must design and implement the most appropriate teaching strategies in the classroom, interests and needs of their students in order to promote the construction of better learning (Rico, 2010).

In any case, talking about the curriculum designates, according to Ponte, Matos and Abrantes (1998), the set of planned educational actions, even if they are carried out partially or totally outside the classes. However, as these authors indicate, the curriculum can be identified with everything that the student learns, either as a result of teaching by teachers or as a result of unforeseen processes. Likewise, they highlight that the terms curriculum and program are confused in some cases, and clarify that when

speaking of program, it refers above all to the sequence of content that must be given, being an element of the curriculum, but not the curriculum itself.

Otherwise, Osorio (2017) addresses the study of the curriculum through the contributions made by certain highly recognized authors and theorists in the current academic community, such as Magendzo (1996), Gimeno (1991), Kemmis (1988), Stenhouse (1984); identifies some perspectives or points of view for the construction of a curriculum idea, which are presented below: the curriculum understood as a project and as a process, rather than the planning of instructional purposes to be achieved; the curriculum as a social practice composed of behaviors, beliefs, values, didactics, ideologies and administrative-economic policies; curriculum as an instrument of social intervention through processes of selection, organization and transmission of culture in the school environment; the curriculum as an educational problem where knowledge, skills and values must be selected and organized; and the curriculum as a configurator of educational practice, which plans and organizes it. Perspective that will be assumed from the curriculum in the present investigation.

It should be noted that the reference documents that guide the curriculum are fundamental and cannot be ignored, however, in the construction of the mathematics curriculum various actors contribute, including the teacher, who performs it at different levels, moments and contexts (Canavarro and Ponte, 2003). In the light of these authors, to take advantage of the curricular role of the mathematics teacher, it would be important to (re) construct the curriculum; that has to do, with the way in which the teacher interprets and manages the mathematics curriculum, implies the planning of educational practice, its implementation in the classroom and the evaluation of their curricular options. Therefore, it is necessary to pay attention to the curriculum construction work in which the teacher is involved.

3.2. Professional knowledge of the mathematics teacher

In this second theme, there are works that pose a link between the teacher's knowledge and the mathematics curriculum. Under the assumption that a better quality of teacher knowledge will improve student learning, numerous investigations have been developed focused on the professional knowledge of mathematics teachers over the last decades (Ponte and Chapman, 2006). From these works theoretical models arise that attempt to describe the knowledge of the mathematics teacher.

Shulman (1986) stated that the hidden paradigm in training has been precisely the pedagogical content knowledge (PCK, in English Pedagogical Content Knowledge). This goes beyond the knowledge of the discipline itself and points to the knowledge of the discipline for teaching. It is not a conjunction of pedagogy and content, nor an intersection of both. According to this North American psychologist, within this category should be included, in relation to the topics that are taught more regularly in a subject, the most useful forms and representations of the

concept or procedure, the most powerful analogies, illustrations, examples, explanations and demonstrations.

Based on the work of Shulman (1986, 1987), Ball, Thames and Phelps (2008) introduce new categories of knowledge, devoting special interest to didactic content knowledge (Pedagogical Content Knowledge- PCK). Mathematical knowledge for teaching (Mathematical Knowledge for Teaching - MKT) According to these authors, it includes four main categories: knowledge of common content, with respect to the knowledge that teachers need to know about concepts and notations; specialized knowledge of content, which is the knowledge necessary to interpret concepts in different ways; the knowledge of the contents and the students that the teacher assumes to anticipate problems and their reasoning when carrying out tasks; and content and teaching knowledge, which relates to the knowledge necessary to set examples, present representations, and conduct mathematical discussions.

Otherwise, the research group at the University of Michigan managed to characterize mathematical knowledge for teaching, based on the components of the PCK; They propose a model of mathematical knowledge for teaching in which they distinguish two large domains: knowledge of mathematical content and didactic knowledge of content. Knowledge of mathematical content is made up of three subdomains. Common knowledge of content, which refers to "mathematical knowledge and skills that are used in situations that are not exclusive to teaching" (Ball; Thames; Phelps, 2008, p.399). Knowledge of the mathematical horizon is "the teacher's knowledge of how the mathematical topics included in the curriculum are related" (Ball; Thames; Phelps, 2008, p.403). AND, Didactic content knowledge is also made up of three subdomains. Knowledge of the content and of the students, which implies the "knowledge of the content that is intertwined with the knowledge of how the students think, know or learn a particular content" (Hill et al., 2008, p.375); knowledge of content and teaching as one that "combines knowledge about teaching with mathematics" (Ball; Thames; Phelps, 2008, p.401) and knowledge of the curriculum as one that allows the teacher to guide their practice and select the appropriate tasks for the learning of their students (Ball; Thames; Phelps, 2008).

In contrast to this, at the University of Huelva, the Research Seminar in Mathematics Didactics (SIDM), has specified some aspects of mathematical knowledge for teaching, considering the specialized nature of the complete model, not exclusively of one of its subdomains, which leads to redefine the MKT model and give rise to the mathematics teacher's specialized knowledge model (Mathematics Teacher's Specialized Knowledge - MTSK). The MTSK model focuses on the specificity of the mathematics teacher's knowledge regarding the teaching of content, in addition, according to (Carrillo et al., 2013) it considers the beliefs of the teachers related to mathematics and the teaching of mathematics.

For its part, the Didactic-Mathematical Knowledge (CDM) model has been the subject of study by the community of researchers in Mathematics Education (Pino-Fan, Font and

Godino, 2014; Pino-Fan, Assis and Castro, 2015; Pino-Fan and Godino, 2015); This model tries to broaden the discussion that other models, with the same nature, have tried to describe, among them the PCK and the MKT. Models that have had the purpose of "determining and characterizing the components of the complex of knowledge that a mathematics teacher should have in order to effectively carry out their teaching practice and facilitate the learning of their students on specific topics of mathematics" (Pino-Fan, Font and Godino, 2014, p. 138).

Specifically, the Didactic-Mathematical Model is based on the application of didactic analysis tools, proposed by the Ontosemiotic Approach (Pino-Fan and Godino, 2015) and seeks to answer questions that other models have not considered, including the relationship between the different components of the teacher's mathematical knowledge (Pino-Fan, Font and Godino, 2014). Pino-Fan and Godino (2014) and Pino-Fan, Assis and Castro (2015) characterize three dimensions of the Mathematical Didactic Knowledge model: the mathematical dimension, the didactic dimension and the meta-didactic-mathematical dimension; four levels of analysis: problems, practices, objects and processes; and four phases of the didactic design: preliminary study, design, implementation and evaluation.

The CDM model proposes dimensions that provide an overview of the teaching and learning process through categories and subcategories of teacher knowledge that offer a good approximation to the teacher's professional performance panorama (Castro, Pino-Fan and Parra-Urrea, 2018) , given that it recognizes the complexity of the educational act and provides analysis tools for the various categories of knowledge. For these authors, the operationalization of the Mathematical, Didactic and Meta Didactic-Mathematical dimensions, together with the relationships between the phases of didactic design and the levels of analysis; they not only establish the various types of knowledge required by teachers, but also provide guidelines and tools to design and assess their study process.

This model allows a detailed analysis of the different types of knowledge that the teacher must possess in order to achieve an ideal teaching of mathematics. More specifically, this model interprets and characterizes the teacher's knowledge, offering specific tools that allow a more detailed analysis of the teacher's didactic-mathematical knowledge, considering the facets (epistemic, cognitive, affective, interactional, mediational and ecological). involved in the teaching and learning processes that a teacher must put into play to teach a certain topic.

However, to solve the problem of the segmentation and lack of dynamics of the Ball, Thames and Phelps (2008), Ponte (2012)states that the teacher's professional knowledge includes various aspects, being interested mainly in that which refers to educational practice, and that it assumes as Didactic Knowledge, differentiating four dimensions: "1) knowledge of mathematics, 2) knowledge of the curriculum, 3) knowledge of students and their

learning processes, and 4) knowledge of work processes in the classroom "(p. 5).

Regarding the first dimension, Ponte (2012) states that: "beyond the fundamental concepts and procedures within the discipline, the forms of representation of those concepts and procedures emerge that give a general perspective on the character of school mathematics" (p. 5). The second dimension is related to the knowledge of the curriculum; It implies the recognition of the purposes, objectives of the teaching of mathematics and the organization of the contents; in addition, it is decisive in decision-making and ways of guiding teaching.

The third dimension is determined by the knowledge of the students and their learning processes; where, understanding their interests, tastes, behaviors, ways of thinking, among others, are decisive in professional practice. Finally, the dimension corresponding to educational practice constitutes a fundamental nucleus that includes "long or medium term planning, such as the plan thought for each class session, the elaboration of the tasks to be carried out, and all those questions related to conducting the activity in the mathematics classroom "(p. 6).

In addition, it can be inferred that professional knowledge can be produced from teaching practices. In this sense, the idea of professional knowledge presented by Ponte (2012) takes value, who considers that, this knowledge:

Its fundamental basis is experience and reflection on experience, not only individual, but of the entire professional body. Its quality is not determined by abstract criteria of conceptual or logical coherence (as occurs in the case of academic knowledge), but by criteria of effectiveness in solving practical problems and criteria of adequacy of solutions to existing resources. The value of this knowledge is based on the experience discussed, systematized and validated by a specific professional group, which has in turn been recognized by society. (p. 4)

On the other hand, the same author states that "The professional knowledge of teachers is, above all, oriented to a practical activity (teaching mathematics to groups of students)" (p. 3) and that such knowledge is fundamentally based on experience and reflection on it; in the same way, it states that, although it is important to characterize the professional knowledge of the teaching staff; It is even more important to study its development process.

Apparently, the advantage of the perspective presented by Ponte (2012), occurs because it adopts an integrating vision of the different aspects of the teacher's knowledge, instead of presenting it in categories. This author does not conceive of the possibility of separating some dimensions from others, and therefore it is perceived that they are closely linked, but clearly differentiated, therefore, he emphasizes that all of them are always in some way present in the activity of teachers when teaching math. It is noted then that the school mathematics, the curricular objectives and priorities, the vision of the students and the learning modes are present, as well as the knowledge of

the work dynamics, of the resources and forms of practical action of the teachers.

However, the Didactic-Mathematical Knowledge model not only contemplates various components of knowledge that a teacher who teaches mathematics must have, but also offers analysis tools to determine both the components of knowledge and their relevance to the context in which teachers develops his work. In addition, those same types of knowledge that Ponte (2012) raises, but much more explicit and differentiated, together with analysis tools are considered in the CDM model.

3.3. Professional Development of the Mathematics Teacher

Regarding this issue (Imbernon and Canto, 2013) they state that training is an important element for professional development, but not the only one and, perhaps not the decisive one. The authors also present a possible approach to the concept of professional development of teachers, indicating that it may be a systematic attempt to improve work practice, professional knowledge and beliefs. Professional knowledge can be seen as a support for teacher development and as a product of said development (Climent et. Al, 2014). According to these authors, reflection on practice, in a broad sense of practice, not only that which takes place in the classroom, is nourished, among others, by the teacher's knowledge.

In professional development, five main lines or axes of action stand out in the light of Imbernon (2011): theoretical practical reflection on one's own practice through the analysis of reality; the exchange of experiences among equals to enable updating in all fields of educational intervention; linking training to a work project; training as a critical analysis of labor practices, such as hierarchy, sexism, proletarianization, individualism, low status, among others; and professional development occurs predominantly in school. For this author, some aspects that contribute to DPD are: the work climate in educational institutions, dealing with colleagues, promotion within the profession, training throughout life, teaching career and experience; and that part of this professionalization occurs individually and another is collective as a professional group.

Located in the field of professional development of primary school teachers regarding the teaching of mathematics, Climent (2010) expanded his initial conceptualization of development, linked above all to the change of conceptions and the expansion of professional knowledge, to an understanding of the practice (including in this the understanding of their performance, their conceptions, their knowledge) and their reflection on it. Professional knowledge can be seen as a support for teacher development and as a product of said development (Climent et. Al, 2014). According to these authors, reflection on practice, in a broad sense of practice, not only that which takes place in the classroom, is nourished, among others, by the teacher's knowledge.

The expression professional development could well be associated according to Vaillant (2016), with permanent training, continuous training, in-service training, human resource development, lifelong learning or training; it can also be understood in other perspectives according to the author:

[...] The notion of "professional development" is the one that best adapts to the conception of the teacher as a teaching professional. Likewise, the concept of "development" has a connotation of evolution and continuity, which goes beyond the traditional juxtaposition between initial training and teacher improvement. (p. 8)

In another perspective, Bautista and Ortega (2015) describe the approaches to Teacher Professional Development (DPD) in five nations highly committed to research and practice in this field (United States, Australia, Hong Kong, Finland and Singapore). According to the authors, professional development is about teachers' learning, how they learn to learn and transform their professional knowledge into practice to benefit the development of their students. It implies the capacity and availability to examine where each one is in terms of convictions and beliefs, as well as the analysis and implementation of appropriate alternatives for improvement or change.

The reflections on educational practices, motivated by the decision to change the possible unfavorable circumstances for learning, would be showing an opportunity for professional development for mathematics teachers (Ponte, 2012), understood as "the progressive development of potentialities and the construction of new knowledge is marked by social and collective dynamics, and depends on the ways of articulating interests, needs and resources of teachers" (p. 9).

When contrasting training with Professional Development, Ponte (2012) considers that:

Training tends to be seen as a movement "from the outside in", where teachers are expected to assimilate the knowledge and information transmitted to them, while professional development represents a movement "from the inside out", where expects the teaching staff to decide on the issues to be considered, the projects to be undertaken and the way to carry them out. On the one hand, the training focuses mainly on what teachers do not have and which, however, "should have". On the other hand, professional development pays special attention to the achievements of teachers. The most important thing is that the teaching staff ceases to be an object to become a subject of training. (pp. 7-8)

That is why it is important to guide teacher training processes towards Professional Development. In the light of Ponte, it is possible for Professional Development to benefit from training opportunities that meet the needs and achievement goals of teachers.

In this order of ideas, teacher training spaces then become an input to consolidate processes associated with Teacher

Professional Development, which, according to Ponte (2012), is the mathematics teachers who learn from their activity and from the reflection around it. Only in this way, so far, it could be said that it is possible to strengthen teacher training spaces, to support, transform or consolidate Professional Development.

In this sense, Ponte (2012) shows that it is possible to find training contexts that lead teachers to learn mathematics and didactics, and to function more properly. The author also considers the fact that professional knowledge and therefore didactic knowledge have a personal character, associated with action and reflection on experience, implies that their development requires imaginative and diversified ways of working in situations that are as close as possible. Possible to practice situations. Therefore, the creation of teacher training and professional development contexts constitutes an inexhaustible field of work for teachers who teach mathematics.

In this sense, the creation of training and professional development contexts for teachers constitutes an inexhaustible field of work for mathematics educators (Ponte, 2012). However, when reviewing the research on Teacher Professional Development, (Vaillant and Marcelo, 2015) consider that teacher preparation is rarely concerned with medium or long-term results.

Otherwise, Niemi (2015) states that the teacher's work is increasingly complex and cooperating with other professionals is necessary and important, especially when students have different needs. In addition, it points out the need to focus not only on continuing training but also on initial training, since this is one of the weakest points in all countries.

Thus, the professional development of the mathematics teacher can be perceived through the changes in their performance in relation to various elements that their practice involves, in the words of (Ponte et al., 2005) implies a natural process of professional growth in which You gradually acquire confidence, gain new perspectives, increase in knowledge, discover new methods and roles.

According to what is mentioned in this section, this study considers it important to develop strategies to strengthen the professional development of teachers who teach mathematics in primary school, understood as the understanding of practice (of their performance, their conceptions and their knowledge) and his reflection on it (Climent, 2010). Which implies understanding it as the learning of mathematics teachers, about how they learn to learn and transform their knowledge into practice to benefit the development of their students. Thus, the professional learning of teachers according to (Ávalos, 2011) is a complex process that requires the cognitive and emotional involvement of teachers, both individually and collectively.

4. Discussion

Considering the scenario of ideas that emerged in the process of inquiry and analysis of each of the studies, it is

inferred that the reference documents that guide the curriculum are fundamental and cannot be ignored, however, in the construction of the mathematics curriculum. Various factors contribute, including the teacher, who performs it at different levels, moments and contexts. Thus, to take advantage of the curricular role of the mathematics teacher, it would be important to analyze the teacher's didactic knowledge in the process of (re) construction of the curriculum; which has to do with the way in which the teacher interprets and manages the mathematics curriculum, implies the planning of educational practice, its implementation in the classroom and the evaluation of their curricular options. Therefore, In that order of ideas, it would be valid to think that the analysis of the teacher's performance in the face of his learning and didactic knowledge of mathematics could be a decisive element in the management of the mathematics curriculum and, in this sense, it becomes important and convenient think about the need to generate spaces for reflection for the primary school teacher, in which interactions that transform their teaching and enhance their knowledge and professional development predominate. The foregoing could benefit teachers to the extent that they manage to reflect on the planning of educational practice, its implementation in the classroom and the evaluation of their curricular options. In this regard, Vaillant (2016) states that:

The situation of teachers in Latin America has been widely portrayed by research (Ávalos, 2012, Vaillant and Marcelo García, 2015) and today there is enough information in relation to training processes, working conditions and performance evaluation. Most of the research works conclude that teachers are the key to improving educational results and that school directors are of vital importance so that teachers and professors can work better (p.8).

This is how, in terms of what the teacher knows and how he teaches what he knows, Ponte (2012), considers that the articulation between mathematical content and pedagogical knowledge contributes to the development of professional knowledge and that it probably contributes theoretically and methodologically to teaching practice.

Thus, the difficulties that teachers present with respect to school mathematics in primary education and its incidence in the curriculum have been analyzed through studies carried out by some researchers in different countries and contexts. Investigations such as that of Zapata, Santa and Jaramillo (2018) consider it convenient to generate reflection spaces for the primary school teacher, in which interactions that transform their knowledge of teaching and enhance their professional development predominate, where the teacher's reflection, determination and performance to improve their Practices could be decisive elements in the transformation of your professional knowledge and an essential part for a (re) construction of the mathematics curriculum.

It can then be inferred that it would be necessary to make contributions to the production of didactic knowledge of primary school teachers, in order to recognize ways of

thinking and promote improvements in teaching and, therefore, the teacher can achieve a (re) construction of the curriculum. In addition, in the light of Santa and Jaramillo (2015), consolidating spaces for professional reflection, forming collectives, understood as groups composed of teachers in interaction with themselves, with peers, with students and with the media, can promote transformations in the knowledge that teachers have about teaching.

The Ponte (2012) model refers to four types of knowledge "1) knowledge of mathematics, 2) knowledge of the curriculum, 3) knowledge of students and their learning processes, and 4) knowledge of work processes in the classroom", this model is found, based on the proposal of Ball, Thames and Phelps (2008). These same types of knowledge, much more explicit and differentiated, together with analysis tools are considered by the authors of the CDM and presented in the works already referred to above.

Therefore, according to the analysis and review of the literature that has been carried out so far, it is possible to ask the following research question: How the didactic knowledge of primary school teachers enables the (re) construction of the mathematics curriculum in the framework of a professional development program? Whose goal could it be, to Finalize the didactic knowledge of primary school teachers, in the (re) construction of the mathematics curriculum within the framework of a professional development program. (Re) constructing the curriculum has to do with the way in which the teacher interprets and manages the curriculum, it implies the planning of educational practice, its implementation in the classroom and the evaluation of their curricular options.

Thus, there is abundant literature on aspects associated with the knowledge and professional development of the teacher, where the importance of didactic knowledge can be deduced before the (re) construction of the mathematics curriculum; an aspect that invites the reflection of teachers in training, in practice and teacher trainers; furthermore, the Research on the didactic knowledge of primary school teachers is very scarce Vásquez and Alsina (2015).

Given this problem, it is considered that this study could generate a space with teachers who teach math in elementary school, to analyze, according to their didactic knowledge, their needs and interests, the interpretation and management of the mathematics curriculum, the planning of educational practice and its implementation in the classroom, so that through an accompaniment strategy, it is possible to think about the production of knowledge, and contribute to the respective teacher professional development.

5. Conclusions

The themes underlying this review manage to account for the purpose that motivated the preparation of this article, since they allow us to identify a series of aspects that have been associated to the didactic knowledge of primary school teachers in the process of (re) construction of the mathematics curriculum, as well as the posing of questions

that could suggest future research. On the one hand, the teacher uses the curricular documents based on his interpretation of them, and this interpretation depends on how well he understands their content, which is reflected in his educational practice. In addition, Colombian mathematics teachers have diverse backgrounds in the discipline and in its didactics, which could affect the knowledge and preparation of the primary school teacher in mathematics.

Thus, there is abundant literature on aspects associated with the knowledge and professional development of the teacher, where the importance of didactic knowledge can be deduced before the (re) construction of the mathematics curriculum; an aspect that invites the reflection of teachers in training, in practice and teacher trainers; furthermore, the Research on the didactic knowledge of primary school teachers is scarce. On the other hand, it is possible to deduce the role of the teacher so that the management of the curriculum can be favored in his class; aspects that invite the reflection of teachers in training, in practice and teacher trainers.

Now, although the (re) construction of the mathematics curriculum in the context of didactic knowledge is not necessarily explicit in the studies reviewed, it is possible to infer that this may be due to the fact that this issue is not trivial for the primary school teacher. Therefore, it may be valid to think, in terms of the analyzes achieved within the framework of professional knowledge and development, that the teacher's reflection, determination and action to improve their practices could be decisive elements in the transformation of their professional knowledge, and In this sense, it is convenient to think about the urgency of generating spaces for reflection for the primary school teacher, in which interactions that transform their knowledge of teaching and enhance their professional development predominate.

The studies mentioned above report the importance of reflecting on the way in which the teacher interprets and manages the curriculum, which implies the planning of educational practice, its implementation in the classroom and the evaluation of their curricular options. In this sense, although professional knowledge is not a new issue in the field of mathematics education research, it could be somewhat novel in the (re) construction of the mathematics curriculum in the context of the didactic knowledge of the primary school teacher.

In relation to the above, another consensus among researchers shows that teachers not only need to know how to interpret and manage the mathematics curriculum in primary school, they should also know mathematics, but this disciplinary knowledge is not enough, and, in this sense, other analyzes should be elicited in the current doctoral study. Thus, the knowledge and professional development of teachers allow us to glimpse the role of the teacher in the planning of educational practice and its implementation in the classroom. Now, as researchers, it is worth reflecting on the conditions and possibilities for practicing primary school teachers, with or without

specific training in mathematics, to respond to the manifest need to evaluate their curricular options.

6.Future Scope

Faced with the Didactic-Mathematical Knowledge (CDM) model, this not only contemplates various components of knowledge that a teacher who teaches mathematics must have, but also offers analysis tools to determine both the components of knowledge and their relevance to the context in the which teachers develop their work. In addition, those same types of knowledge that other author proposes, but much more explicit and differentiated, together with analysis tools are considered in the CDM model.

And in relation to aspects related to what the teacher should know, progress is recognized, but it is a line that deserves further research reflection, for example, could models of teacher knowledge be linked to the teacher's curriculum? Do the teacher's conceptions and beliefs affect the staging of the mathematics curriculum? What competencies and abilities should be characteristic of mathematics teachers to develop the curriculum?

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