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Wavelet Analytical Study of Rural and Urban Education Growth in India

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Abstract: ***Education is one of the important scales by which development level of a country is expressed. The geographical and social circumstances of rural and urban areas are different; therefore, the levels of education in rural and urban areas are also different. Wavelet transforms is a new and powerful analytical tool which is used to capture time frequency information of a signal/data. The education growth data of India from Jan. 2013 to Aug. 2021 imported from website data.gov.in are taken as raw data. Discrete wavelet transforms of these data are performed using Haar wavelet, decomposition level-5.In wavelet transforms, the data are decomposed into approximation and detail at each decomposition level. Approximation is the part of signal corresponding to the highest scale value and represents average behaviour or trend of the signal, while details represent differential behaviour of the signal corresponding to each decomposition level.Wavelet analysis reveals that overall growth in rural education is slightly greater than that of urban education of India in the recent years.The statistical results are strongly consistent with the wavelet analytical results.***

Keywords: Approximation, detail, education, rural, urban, wavelet

1. Introduction

Education is one of the most powerful tools which help in the development of any country [1]. It is an important part of the human development. From the very beginning, education is being provided to the students in the different mode and its syllabus is also being changed with time and demand of the society [2]. The education index is measured by combining the average adult years of schooling with expected years of schooling for students below age 25 years with 50% weighting of each. In the national education policy-2020, the main stress is on to enhance gross enrolment ratio (GER) and to provide vocational education along with traditional education. The national education policy-2020 has recommended for expenditure of 6% of the gross domestic products (GDP) on the education. Education learns a person to adjust and accommodate with the surroundings. It makes a person as a good civilian with full of knowledge and vocational skills, so that the person will be the true human resource and helps in the development of society as well as country [3]. Religious pomp and bigotry, population increasement, several social evils and many more are the effects of low education rate. Around 65% of the Indian population resides in the rural areas. The government of India has adopted special policy to increase the education level especially in rural areas, so that the rate of education is continuously increasing in rural and urban areas both. The geographical and social circumstances in rural and urban areas are different, so that, there is a difference in education rate in rural and urban areas.Due to lack of education, Indian society is involved in many social evils and these are the main causes of backwardness of India. The education which is being provided to our children and youths should help toprovide employment and jobs to them. With this education students will be trained technically and vocationally with the knowledge of literature and science. The education helps us to know about facts and laws of nature and matter. The new education helps us to develop scientific approach about everything happening in the surroundings, so that, scientific understanding among people is developed. By assimilating scientific and realistic approach, one can live a longer, healthy and happy life [4-5].

Fourier transforms has been a magical mathematical tool for long time to analyse several problems in science and engineering. Fourier transforms is suitable to analyse finite, single valued and stationary signals. In fourier transforms every finite, single valued and stationary signal or function is expressed in to sum of infinite functions having frequencies integral multiple to the frequency of the original function. But fourier transforms is not suitable for non-stationary and transient signals' analysis. To overcome problem of fourier transforms, window fourier transforms (WFT) or short time fourier transforms (STFT) is introduced, in which original function is multiplied by a window function and then its fourier transforms is performed. The time and frequency size of window function in WFT is restricted by Heisenberg uncertainty principle, so that its time and frequency analysis is also restricted. Wavelet transforms is suitable for analysis of non-stationary and transient signals. In wavelet transforms, the original function is multiplied by wavelet functions and then its fourier transforms is performed. The wavelet transforms capture time and frequency information of a signal [6-7]. The wavelet transform has an additionally time localization propertiesalong with frequency localization over fourier transforms which has frequency localization property only. The literal meaning of wavelet is a small wave which exists for very short time interval and then dies out. This wavelet is called mother wavelet and can be dilated and translated, so that a wavelet family can be generated. Wavelet is a zero average oscillating function which is well localized over a short period of time. For analysing and transforming discrete data, wavelets have been widely useful. Wavelet transform is a hierarchical set of wavelet functions. All wavelet transforms are related to harmonic analysis because we can assume, wavelet transforms as a form of time-frequency representation for analogue signals. Discrete-timefilter banks are used by all discrete wavelet transforms. In wavelet nomenclature, these filter banks are named as wavelet and scaling coefficients [8].In recent years, the wavelet transforms have attracted a lot of attention of physicists, mathematician and engineers. With help of multiresolution analysis, a signal can be represented in a better way as per requirement [9]. Wavelet transform is a powerful tool which is frequently being used in wide range of applications indata compression, signal and image processing, person identification, face and speech recognition, computer graphics, etc. [10-12].In this paper, we have studied and analysed education index of rural and urban areas of India using Haar discrete wavelet transforms.

2. Basics of wavelet transforms

The continuous wavelet transforms(CWT) of a function $f(t)$ can be expressed as follows [13]: -

$$W_{(a,b)}f(t) = \int_t f(t)\frac{1}{\sqrt{|a|}}\,\psi\left(\frac{t-b}{a}\right)dt$$
$$= \int f(t)\psi_{a,b}(t)dt$$

Where a and b are two real numbers. It is obtained via $a = 2^{-j}$, and $\frac{b}{a} = k$ where j and k are integers. By introducing this substitution, the discrete wavelet transform is expressed as follows [14]: -

$$W_{j,k}f(t) = \int f(t)2^{\frac{j}{2}}\psi(2^j t - k)\ dt$$

In discrete wavelet transforms, the discrete wavelet is expressed as follows: -

$$\psi_{j,k}(t) = 2^{\frac{j}{2}}\psi(2^j t - k)$$

These wavelet coefficients for all j and k develop an orthonormal basis $\psi_{0,0}(t)$ called as a mother wavelet. Translation and dilation of the mother wavelet produces other wavelets. There is an infinite number of such types of functions and we can select one of them according to our application.

Multiresolution analysis (MRA) consists of a sequence $V_j : j\in\mathbb{Z}$ corresponding to a closed subspaces of a square integrable functions $L^2(\mathbb{R})$, satisfying the following properties [15-16]: -

1)$V_{j+1}\subset V_j : j \in \mathbb{Z}$

2)$\cap_{j\in\mathbb{Z}} V_j = \{0\}$, $\cup_{j\in\mathbb{Z}} = L^2(\mathbb{R})$

3)For every$L^2(\mathbb{R})$, $f(t) \in V_j \Rightarrow f(2t) \in V_{j+1}$, $\forall\ j \in \mathbb{Z}$

4) There is a function $\phi(t) \in V_0$, such that $\{\phi(t-k) : k \in \mathbb{Z}\}$ is an orthonormal basis of V_0.

These properties lead to a dilation equation as follows: -

$$\phi(t) = \sum_{k\in\mathbb{Z}} \alpha_k\ \phi(2t-k)$$

where α_k is called low pass filter and defined as follows: -

$\alpha_k = \left(\frac{1}{\sqrt{2}}\right)\int_{-\infty}^{\infty}\phi(t)\phi(2t-k)dt$

The W_1 be the orthogonal complement of V_1 in V_0 i.e.

$V_0 = V_1 \oplus W_1$

If $\psi \in W_0$be any function then dilation equation can be written as follows: -

$$\psi(t) = \sum_{k\in Z}\beta_k\ \phi(2t-k)$$

Where$\beta_k = (-1)^{k+1}\alpha_{1-k}$ is called high pass filter.

In terms of bases of space V_1 and W_1, the space V_0can be expressed as follows:

$$V_0 = V_1 \oplus W_1$$

In general,

$V_j = V_{j+1}\oplus W_{j+1}$

But, $V_{j+1} = V_{j+2}\oplus W_{j+2}$

Therefore, we can write,

$V_j = W_{j+1}\oplus W_{j+2}\oplus V_{j+2}$

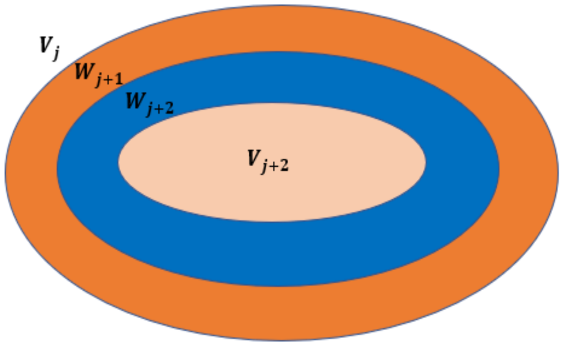

Figure 1: Vector space decomposition

Proceeding in the same manner and by $j_0$iteration, we can write,

$V_j = W_{j+1}\oplus W_{j+2}\oplus W_{j+3}\oplus \ldots\ldots\ldots\ldots W_{j_0}\oplus V_{j_0}$

3. Research methodology

Any square integrable function f, ca be expressed in the series expansion form as follows: -

$f(t) =$

$\sum_{k\in\mathbb{Z}}\langle f, \tilde{\phi}_{j_0,k}\rangle\phi_{j_0,k}(t) + \sum_{p=j+1}^{j_0}\sum_{k\in\mathbb{Z}}\langle f, \tilde{\psi}_{p,k}\rangle\,\psi_{p,k}(t)$

And similarly with the roles of the basis and the dual basis are interchanged [18]. Here, just like for the Haar wavelet $\sum_{k\in\mathbb{Z}}\langle f, \tilde{\phi}_{j_0,k}\rangle\phi_{j_0,k}(t)$ is a coarse scale V_{j_0} – approximation of f and for every p, the sum $\sum_{k\in\mathbb{Z}}\langle f, \tilde{\psi}_{p,k}\rangle\,\psi_{p,k}(t)$, adds the detail in W_p, that is exactly the detail needed to get from the V_j – approximation to the next finer approximation with a function in V_{j+1}.It follows that any function$f$ in $V_j = V_{j+1} \oplus W_{j+1}$ can be expressed in the equivalent form as follows: -

$$f(t) = \sum_{k\in\mathbb{Z}} a_{j,k}\phi_{j,k}(t)$$
$$= \sum_{k\in\mathbb{Z}} a_{j_0,k}\phi_{j_0,k}(t) + \sum_{p=j+1}^{j_0}\sum_{k\in\mathbb{Z}} d_{p,k}\psi_{p,k}(t)$$

The decomposition part of the pyramid algorithm computes the coefficients $a_{j+1,k}$, and $d_{j+1,k}$ from the coefficients $a_{j,k}$. Insertion in the standard formula for the coefficients $a_{j,k}$ is expressed as follows: -

$a_{j,k} = \langle f,\ \tilde{\phi}_{j,k}\rangle = \sum_{n\in\mathbb{Z}}\overline{\tilde{\alpha}[n]}\,\langle f, \tilde{\phi}_{j+1,2k+n}\rangle$ =

$\sum_{n\in\mathbb{Z}}\overline{\tilde{\alpha}[-n]}\,\langle f, \tilde{\phi}_{j+1,2k-n}\rangle$

$= \sum_{n \in \mathbb{Z}} \overline{\tilde{\alpha}[-n]}\, a_{j+1,2k-n}$

Hence, if we set $u_j[k] \stackrel{\text{def}}{=} a_{j,k}$ and $\tilde{\alpha}^*[n] \stackrel{\text{def}}{=} \overline{\tilde{\alpha}[-n]}$ (the involution of $\tilde{\alpha}$) then the last equation can be reformulated as the convolution $v_{j+1}[k] = (v_j * \alpha^*)[2k] = (\downarrow$ $vj* \alpha*[k]$ with $\downarrow$ being the down sampling operator that throws away the odd numbered samples [19]. Similarly, we can write,

$$\psi_{j,k}(t) = \sum_{n \in \mathbb{Z}} \beta[n]\tilde{\phi}_{j+1,2k+n}(t)$$

And

$$\tilde{\psi}_{j,k}(t) = \sum_{n \in \mathbb{Z}} \tilde{\beta}[n]\tilde{\phi}_{j+1,2k+n}(t)$$

So that, it follows for the coefficients $d_{j,k}$,

$d_{j,k} = \langle f, \tilde{\psi}_{j,k} \rangle = \sum_{n \in \mathbb{Z}} \tilde{\beta}[-n] a_{j,2k-n}$

Let S= $\{S_n : n \in \mathbb{Z}\}$ is a signal, by N iteration signaland with help ofapproximation and detail coefficients at each decomposition level, S can be expressed as $S = a_N + d_1 + d_2 + d_3 + d_4 + d_5 + \ldots\ldots\ldots\ldots + d_N$ through low and high pass filters, shown in figure 2.

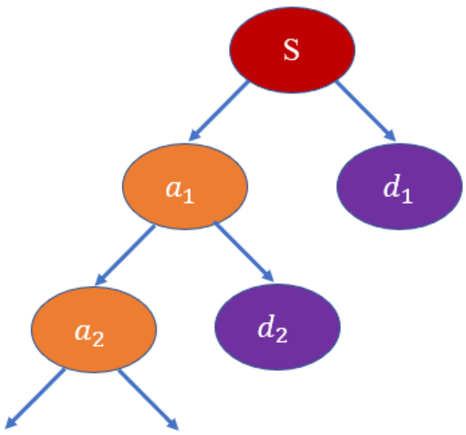

Figure 2: Signal decomposition

At decomposition level-1, any signal S can be expressed as follows: -

$$S(1) = a_1 + d_1$$

Similarly, at decomposition level-2, 3, 4 and 5, the signal is expressed as,

$$S(2) = a_2 + d_2 + d_1$$
$$S(3) = a_3 + d_3 + d_2 + d_1$$
$$S(4) = a_4 + d_4 + d_3 + d_2 + d_1$$
$$S(5) = a_5 + d_5 + d_4 + d_3 + d_2 + d_1$$

and so on [17]. We perform discrete Haar wavelet transforms of the rural and urban education growth data up to decomposition level-5. The approximation corresponding to maximum scale $a_5$represents average behaviour or trend of the data, while d_1, d_2, d_3, d_4 and d_5 represent differential behaviour of the data at each decomposition level.The statistical parameters like skewness, kurtosis, standard deviation and correlation are also determined and discussed [20-21].

4. Results and discussion

The rural and urban education index data of India from Jan. 2013 to Aug. 2021 are taken as raw data and plotted in figure 3.

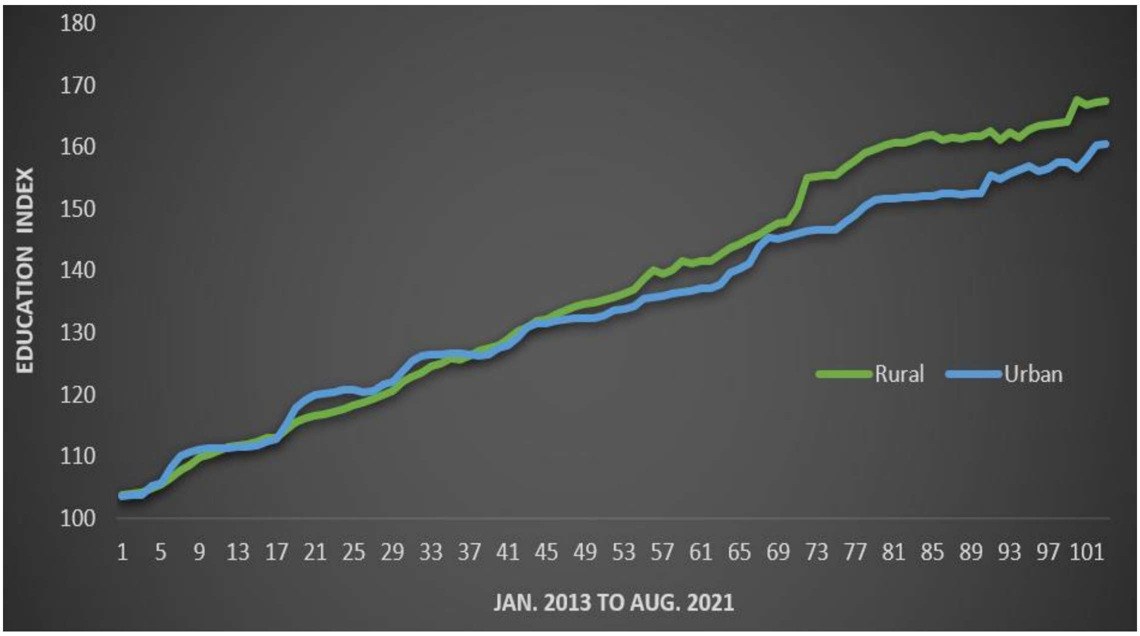

Figure 3: Rural and urban education growth from Jan. 2013 to Aug. 2021

It is clear from the graphs in figure 2 that in the recent years the education index of ruralareasis slightly greater than that of urban areas. This is very interesting result and consequence of the education policies followed by government in the recent years. The wavelet transforms of these data are performed using Haar wavelet, decomposition level-5 in figure 4 and 5.

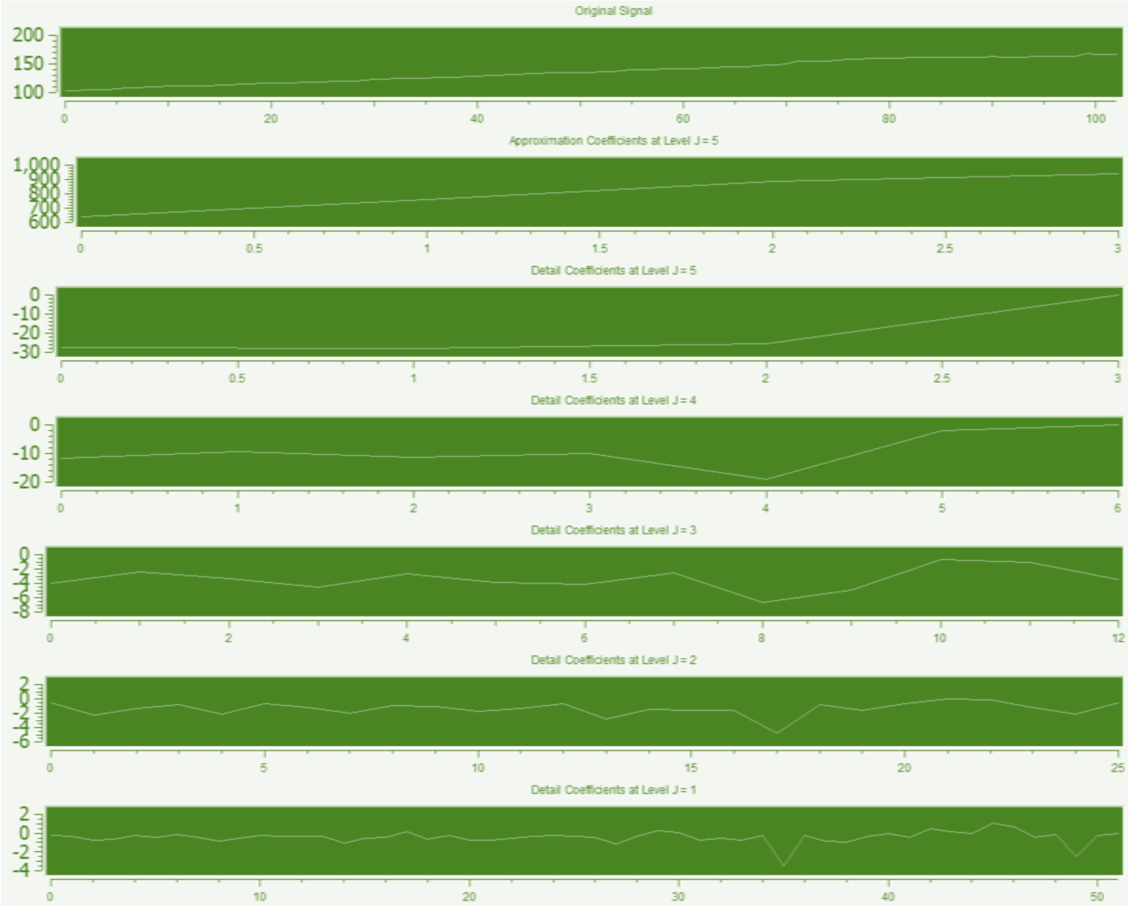

Figure 4: Discrete wavelet transforms of rural education

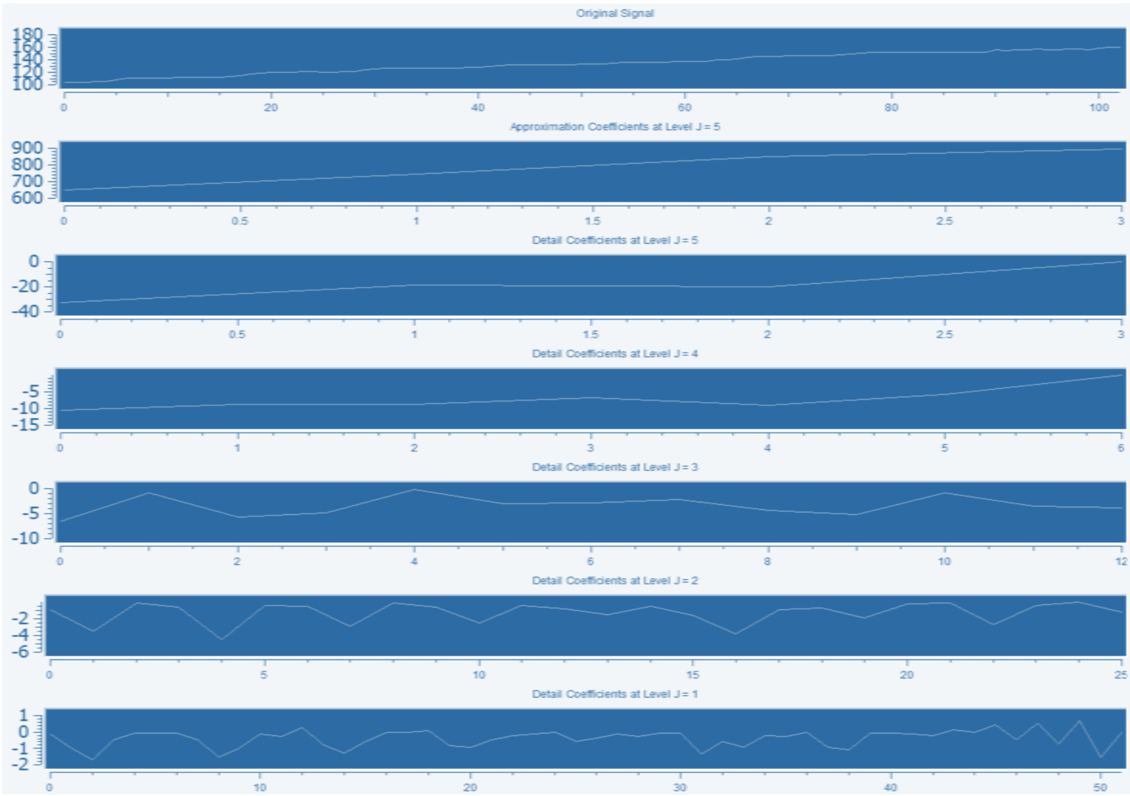

Figure 5: Discrete wavelet transforms of urban education

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Approximation represents average behaviour or trend of the signal, which represents the slowest part of a signal. In wavelet analysis term, it corresponds to the greatest scale value. As the scale increases, resolution decreases, producing a better estimate of unknown data. The detail represents the differential behaviour of the signal. The approximation of rural and urban education data reveals the continuous growth of education in India from Jan. 2013 to Aug. 2021. The approximation reveals that overall growth in rural education is slightly greater than that of urban education in the recent years. The differential behaviour of rural and urban education for the same tenure reveals the fluctuations in time to time.

The statistical parameters like skewness, kurtosis, standard deviation and correlation are also determined and enlisted in table 1.

Table 1: Statistical parameters

S. No.	Statistical Parameter	Rural	Urban
1	Skewness	-0.00976	-0.13603
2	Kurtosis	-1.33723	-1.11436
3	Standard Deviation	19.76787	16.16245
4	Correlation	0.99429	

Skewness is a measure of lack of symmetry of data, while Kurtosis is a measure of whether the data are flatness relative to a normal distribution. The negative and low value of skewness represents that the data are skewed to the left. Negative value of kurtosis indicates that the outlier character of given data is less extreme that expected from a normal distribution. High value of standard deviation indicates that the data points are widely spread over mean value. The positive and high value of correlation coefficient represents that both data are linearly and positively correlated.

5. Conclusion

Non-linear data of rural and urban education growth in India are well analysed by the laws and concepts ofdiscrete wavelet transforms.It is observed that the approximation of rural and urban education of India from Jan. 2013 to Nov. 2021 shows continuous increasing trend, while growth in rural education is slightly greater than that of urban education in recent years. The details of rural and urban education for the same tenure reveals the fluctuations in time to time. Data of rural and urban education both show weak intermittency. The rural and urban educationgrowth are positively and linearly related. The wavelet analytical results provide strong consistency with the statistical analytical results. By virtue of these results, we can say that discrete wavelet transforms of rural and urban education growth provides a simple and accurate framework to investigate the education development of India.

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