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Bayesian Shift Point Estimation of Generalized Compound Rayleigh Distribution under General Entropy Loss Function

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Abstract: ***We consider the problem of estimating the location of a single change point in a sequence of independent lifetimes $x_1, x_2, \ldots, x_m, x_{m+1}, \ldots x_n (n \geq 3)$ were observed from Generalized Compound Rayleigh Distribution with parameter α, β, γ but it was found that there was a change in the system at some point of time m and it is reflected in the sequence after x_m by change in sequences as well as change in the parameter values. The Bayes estimates of γ and m are derived for asymmetric loss function known as General Entropy Loss Function under natural conjugate prior distribution. We propose Bayesian methods of estimating the change point, together with the model parameters, before and after its occurrence. Further, for Bayesian method under their respective identifiability and certain additional regularity conditions, we discussed the results by comparing it with real data with the help of 'R' Software.***

Keywords: Change Point Estimation, Generalized Compound Rayleigh Distribution, Bayesian Method, Natural Conjugate Inverted Gamma Prior, General Entropy Loss Function

1. Introduction

Bayesian inference is an approach to statistics in which all forms of uncertainty are expressed in terms of probability. A Bayesian approach to a problem starts with the formulation of a model that we hope is adequate to describe the situation of interest. We then formulated a *prior* distribution over the unknown parameters of the model, which is meant to capture our beliefs about the situation before seeing the data. After observing some data, we apply Bayes' Rule to obtain a *posterior* distribution for these unknowns, which takes account of both the prior and the data.

This theoretically simple process can be justified as the proper approach to uncertain inference by various arguments involving consistency with clear principles of rationality. Despite this, many people are uncomfortable with the Bayesian approach, often because they view the selection of a prior as being arbitrary and subjective. It is indeed subjective, but for this very reason it is not arbitrary. In theory there is just one correct prior, that captures our prior beliefs. In contrast, other statistical methods are truly arbitrary, in that there are usually many methods that are equally good according to non-Bayesian criteria of goodness, with no principled way of choosing between them.

In decision theory the loss criterion is specified in order to obtain best estimator. The simplest form of loss function is squared error loss function (SELF) which assigns equal magnitudes to both positive and negative errors. However this assumption may be inappropriate in most of the estimation problems. Some time overestimation leads to many serious consequences. In such situation many authors found the asymmetric loss functions, more appropriate. There are several loss functions which are used to deal such type of problem. In this research work we have considered some of the asymmetric loss function named general entropy loss functions (GELF) suggested by Calabria and Pulcini (1996). Such asymmetric loss functions are also studied by Parsian and Kirmani (2002) and Braess and Dette (2004).

1.1 Entropy Loss Function

In many practical situations, it appears to be more realistic to express the loss in terms of the ratio $\frac{\hat{\theta}}{\theta}$. In this case, Calabria and Pulcini (1994) points out that a useful asymmetric loss function is the Entropy loss

$$L(\delta) \propto [\delta^p - p\log_e(\delta) - 1], \quad (1.2.1)$$

Where $\delta = \frac{\hat{\theta}}{\theta}$

and whose minimum occurs at $\hat{\theta} = \theta$ when p › 0, a positive error $(\hat{\theta} > \theta)$ causes more serious consequences than a negative error and vice-versa. For small |p| value, the function is almost symmetric when both $\hat{\theta}$ and θ are measured in a logarithmic scale and approximately

$$L(\delta) \propto \frac{p^2}{2}\left[\log_e(\hat{\theta}) - \log_e(\theta)\right]^2$$

Also, the loss function $L(\delta)$ has been used in Dey et al (1987) and Dey and Lin (1992), in the original form having p = 1. Thus $L(\delta)$ can be written as

$$L(\delta) = b[\delta - \log_e(\delta) - 1]\,;\ b > 0, \quad \text{where } \delta = \frac{\hat{\theta}}{\theta} \quad (1.2.2)$$

In a Bayesian setup, the unknown parameter is viewed as random variable. The uncertainty about the true value of parameter is expressed by a prior distribution. The parametric inference is made using the posterior distribution which is obtained by incorporating the observed data in to the prior distribution using the Bayes theorem, the first theorem of inference. Hence we update the prior distribution in the light of observed data. Thus the uncertainty about the parameter prior to the experiment is represented by the prior distribution and the same, after the experiment, is represented by the posterior distribution. The various statistical models are considered as;

1.2 Generalized Compound Rayleigh Distribution

The Generalized Compound Rayleigh Distribution is a special case of the three- parameter Burr type XII distribution. Mostert, Roux, and Bekker (1999) considered a gamma mixture of Rayleigh distribution and obtained the compound Rayleigh model with unimodal hazard function. This unimodal hazard function is generalized and a flexible parametric model is thus constructed, which embeds the compound Rayleigh model, by adding shape parameter. Bain and Engelhardt (1991) studied this distribution (also known as the Compound Weibull distribution (Dubey 1968) from a Poisson perspective. (p.d.f.)

$f(x;\alpha;\beta;\gamma)=\alpha\gamma\beta^{\gamma}x^{\alpha-1}(\beta+x^{\alpha})^{-(\gamma+1)} \qquad x;\alpha,\beta,\gamma>0$ (1.3.1)

With Probability Distribution Function

$\mathrm{F(x)}=1-(1-\beta x^{\alpha})^{-\gamma} \qquad x;\alpha,\beta,\gamma>0$ (1.3.2)

Reliability function is

$\mathrm{R(t)}=\left(\frac{\beta}{\beta+t^{\alpha}}\right)^{\gamma}$ (1.3.3)

Hazard rate function

$\mathrm{H(t)}=\alpha\gamma\frac{t^{\alpha-1}}{\beta+t^{\alpha}}$ (1.3.4)

The Generalized compound Rayleigh model includes various well-known pdfs, namely

(i) Beta-Prime pdf (Patil, et al., 1984), if $\alpha=\beta=1$
(ii) $\alpha=1$
(iii) Burr XII pdf (Burr, 1942), if $\beta=1$

Compound Rayleigh pdf (Siddiqui &Weiss, 1963), if $\alpha=2$

1.3 Bayesian Estimation of Change Point in Generalized Compound Rayleigh Distribution under General Entropy Loss Function (GELF)

A sequence of independent lifetimes x_1, x_2, ..., x_m, , x_{m+1}, ...x_n($n\geq 3$) were observed from Generalized Compound Rayleigh Distribution with parameterα,β,γ but it was found that there was a change in the system at some point of time m and it is reflected in the sequence afterx_m by change in sequences as well as change in the parameter values. The Bayes estimates of γ and m are derived for symmetric and asymmetric loss functions under natural conjugate prior distribution.

1.3.1 Likelihood, Prior, Posterior and Marginal

Let $x_1,x_2,\dots\dots,x_n$, be a sequence of observed life times. First let observations $x_1,x_2,\dots\dots,x_n$ have come from Generalized Compound Rayleigh Distribution (G.C.R.D.) with probability density function as

$\mathrm{f}\,(\mathrm{x}|\alpha,\beta,\gamma)=\alpha\,\beta^{\gamma}\gamma\,x^{(\alpha-1)}(\beta+x^{\alpha})^{-(\gamma+1)}(x;\alpha,\beta,\gamma>0)$ (1.5.1.1)

Let 'm' is change point in the observation which breaks the distribution in two sequences as $(x_1,x_2,\dots\dots\dots..x_m)$ & $(x_{(m+1)},x_{(m+2)},\dots\dots.x_n)$.

The probability density functions of the above sequences are

$f_1(x)=\alpha_1\beta_1^{\gamma_1}\gamma_1x^{(\alpha_1-1)}(\beta_1+x^{\alpha_1})^{-(\gamma_1+1)}$ (1.5.1.2)

Where $x_1,\dots,x_m>0;\alpha_1,\beta_1,\gamma_1>0$

$f_2(x)=\alpha_2\beta_2^{\gamma_2}\gamma_2x^{(\alpha_2-1)}(\beta_2+x^{\alpha_2})^{-(\gamma_2+1)}$ (1.5.1.3)

Where $(x_{m+1},\dots,x_n;\ \alpha_2,\beta_2,\gamma_2>0$

The likelihood functions of probability density function of the sequence are

$$L_1(x|\alpha_1,\beta_1,\gamma_1)=\prod_{j=1}^{m}f(x_j|\alpha_1,\beta_1,\gamma_1)$$

$$L_1(x|\alpha_1,\beta_1,\gamma_1)=(\alpha_1\gamma_1)^mU_1e^{-\gamma_1T_{1m}} \qquad (1.5.1.4)$$

Where

$$U_1=\prod_{j=1}^{m}\frac{x_j^{(\alpha_1-1)}}{\beta_1+x_j^{\alpha_1}}$$

$$T_{1m}=\sum_{j=1}^{m}\log\left(1+\frac{x_j^{\alpha_1}}{\beta_1}\right)$$

$$L_2(x|\alpha_2,\beta_2,\gamma_2)=\prod_{j=(m+1)}^{n}f(x_j|\alpha_2,\beta_2,\gamma_2)$$

$$L_2(x|\alpha_2,\beta_2,\gamma_2)=(\alpha_2\gamma_2)^{(n-m)}U_2e^{-\gamma_2\,(T_{1n}-T_{1m}\,)} \qquad (1.5.1.5)$$

where

$$U_2=\prod_{j=m+1}^{n}\frac{x_j^{(\alpha_2-1)}}{(\beta_2+x_j^{\alpha_2})}$$

and $\quad T_{1n}-T_{1m}=\sum_{j=(m+1)}^{n}\log\left(1+\frac{x_j^{\alpha_2}}{\beta_2}\right)$

The joint likelihood function is given by

$$L(\gamma_1,\gamma_2|\underline{x})\propto(\alpha_1\gamma_1)^mU_1\,e^{-\gamma_1T_{1m}}\,(\alpha_2\gamma_2)^{n-m}U_2\,e^{-\gamma_2\,(T_{1n}-T_{1m}\,)} \qquad (1.5.1.6)$$

Suppose the marginal prior distribution of γ_1 and γ_2 are natural conjugate prior

$$\pi_1(\gamma_1,\underline{x})=\frac{b_1^{a_1}}{\Gamma a_1}\gamma_1^{(a_1-1)}e^{-\gamma_1b_1};a_1,b_1>0,\gamma_1>0 \qquad (1.5.1.7)$$

$$\pi_2(\gamma_2,\underline{x})=\frac{b_2^{a_2}}{\Gamma a_2}\gamma_2^{(a_2-1)}e^{-\gamma_2b_2};\ a_2,b_2>0,\gamma_2>0 \qquad (1.5.1.8)$$

The joint prior distribution of γ_1,γ_2 and change point 'm' is

$$\pi(\gamma_1,\gamma_2,m)\propto\frac{b_1^{a_1}}{\Gamma a_1}\frac{b_2^{a_2}}{\Gamma a_2}\gamma_1^{(a_1-1)}e^{-\gamma_1b_1}\gamma_2^{(a_2-1)}e^{-\gamma_2b_2} \qquad (1.5.1.9)$$

where $\gamma_1,\gamma_2>0\&m=1,2,\dots\dots\dots..(n-1)$

The joint posterior density of$\gamma_1,\gamma_2$and m say $\rho(\gamma_1,\gamma_2,m/\underline{x})$ is obtained by using equations (1.5.1.6) & (1.5.1.9)

$$\rho(\gamma_1,\gamma_2,m|\underline{x})=\frac{\mathrm{L}(\gamma_1,\gamma_2/\underline{x})\pi(\gamma_1,\gamma_2,m)}{\sum_m\iint_{\gamma_1,\gamma_2}\mathrm{L}(\gamma_1,\gamma_2/\underline{x})\pi(\gamma_1,\gamma_2,m)d\gamma_1d\gamma_2} \qquad (1.5.1.10)$$

$$\rho(\gamma_1,\gamma_2,m|\underline{x})=\frac{\gamma_1{}^{(m+a_1-1)}e^{-\gamma_1(T_{1m}+b_1)}\gamma_2{}^{(n-m+a_2-1)}e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}}{\sum_m\int_0^{\infty}\gamma_1{}^{(m+a_1-1)}e^{-\gamma_1(T_{1m}+b_1)}\,d\gamma_1\int_0^{\infty}\gamma_2{}^{(n-m+a_2-1)}e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}\,d\gamma_2}$$

Assuming

$\gamma_1(T_{1m}+b_1)=x \qquad \& \gamma_2(T_{1n}-T_{1m}+b_2)=y$

$$\gamma_1=\frac{x}{(T_{1m}+b_1)} \& \gamma_2=\frac{y}{(T_{1n}-T_{1m}+b_2)}$$

$$d\gamma_1=\frac{dx}{(T_{1m}+b_1)} \& d\gamma_2=\frac{dy}{(T_{1n}-T_{1m}+b_2)}$$

$$\rho(\gamma_1,\gamma_2,m/\underline{x})=\frac{\gamma_1{}^{(m+a_1-1)}e^{-\gamma_1(T_{1m}+b_1)}\gamma_2{}^{(n-m+a_2-1)}e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}}{\xi(a_1,a_2,b_1,b_2,m,n)} \quad (1.5.1.11)$$

where

$$\xi(a_1,a_2,b_1,b_2,m,n)=\sum_{m=1}^{(n-1)}\frac{\Gamma\,(m+a_1)}{(T_{1m}+b_1)^{(m+a_1)}}\frac{\Gamma\,(n-m+a_2)}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2)}} \quad (1.5.1.12)$$

The Marginal posterior distribution of change point 'm' using the equations (1.5.1.6), (1.5.1.7) & (1.5.1.8)

$$\rho(m|\underline{x})=\frac{L(\gamma_1,\gamma_2/\underline{x})\,\pi(\gamma_1)\,\pi(\gamma_2)}{\sum_m L(\gamma_1,\gamma_2/\underline{x})\,\pi(\gamma_1)\,\pi(\gamma_2)} \quad (1.5.1.13)$$

$$\rho(m|\underline{x})=\frac{\int_0^\infty \gamma_1{}^{(m+a_1-1)}e^{-\gamma_1(T_{1m}+b_1)}\,d\gamma_1\int_0^\infty \gamma_2{}^{(n-m+a_2-1)}e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}\,d\gamma_2}{\sum_m\int_0^\infty \gamma_1{}^{(m+a_1-1)}e^{-\gamma_1(T_{1m}+b_1)}\,d\gamma_1\int_0^\infty \gamma_2{}^{(n-m+a_2-1)}e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}\,d\gamma_2}$$

Assuming

$\gamma_1(T_{1m}+b_1)=y \qquad \& \gamma_2(T_{1n}-T_{1m}+b_2)=z$

$$\gamma_1=\frac{y}{(T_{1m}+b_1)} \& \gamma_2=\frac{z}{(T_{1n}-T_{1m}+b_2)}$$

$$d\gamma_1=\frac{dy}{(T_{1m}+b_1)} \& \qquad d\gamma_2=\frac{dz}{(T_{1n}-T_{1m}+b_2)}$$

then

$$\rho(m|\underline{x})=\frac{\frac{\Gamma\,(m+a_1)}{(T_{1m}+b_1)^{(m+a_1)}}\frac{\Gamma\,(n-m+a_2)}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)} \quad (1.5.1.14)$$

The marginal posterior distribution of γ_1 using equation (1.5.1.6) & (1.5.1.7) is given by

$$\rho(\gamma_1|\underline{x})=\frac{L(\gamma_1,\gamma_2/\underline{x})\,\pi(\gamma_1)}{\int_0^\infty L(\gamma_1,\gamma_2/\underline{x})\pi(\gamma_1)d\gamma_1} \quad (1.5.1.15)$$

$$\rho(\gamma_1|\underline{x})=\frac{\sum_m \gamma_1{}^{(m+a_1-1)}e^{-\gamma_1(T_{1m}+b_1)}\int_0^\infty \gamma_2{}^{(n-m+a_2-1)}e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}\,d\gamma_2}{\sum_m\int_0^\infty \gamma_1{}^{(m+a_1-1)}e^{-\gamma_1(T_{1m}+b_1)}\,d\gamma_1\int_0^\infty \gamma_2{}^{(n-m+a_2-1)}e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}\,d\gamma_2}$$

Assuming $\quad \gamma_2(T_{1n}-T_{1m}+b_2)=y,\ \&\gamma_2=\frac{y}{(T_{1n}-t_{1m}+b_2)}$

then

$$\rho(\gamma_1|\underline{x})=\frac{\sum_m e^{-\gamma_1(T_{1m}+b_1)}\gamma_1{}^{(m+a_1-1)}\frac{\Gamma\,(n-m+a_2)}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2)}}}{\xi(\alpha_1,\alpha_2,\beta_1,\beta_2,m,n)} \quad (1.5.1.16)$$

The marginal posterior distribution of γ_2, using the equation (1.5.1.6) & (1.5.1.8) is given by

$$\rho(\gamma_2|\underline{x})=\frac{L(\gamma_1,\gamma_2/\underline{x})\,\pi(\gamma_2)}{\int_0^\infty L(\gamma_1,\gamma_2/\underline{x})\,\pi(\gamma_2)\,d\gamma_2} \quad (1.5.1.17)$$

$$\rho(\gamma_2|\underline{x})=\frac{\sum_m\int_0^\infty e^{-\gamma_1(T_{1m}+b_1)}\gamma_1{}^{(m+a_1-1)}\left[\gamma_2{}^{(n-m+a_2-1)}e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}\right]d\gamma_1}{\sum_m\int_0^\infty \gamma_1{}^{(m+a_1-1)}e^{-\gamma_1(T_{1m}+b_1)}\,d\gamma_1\int_0^\infty \gamma_2{}^{(n-m+a_2-1)}e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}\,d\gamma_2}$$

Assuming $\quad \gamma_1(T_{1m}+b_1)=y\,,\ \gamma_1=\frac{y}{(T_{1m}+b_1)}$

$$\rho(\gamma_2\,/\,\underline{x})=\frac{\sum_m\left|\frac{\Gamma\,(m+a_1)}{(T_{1m}+b_1)^{(m+a_1)}}\right|\gamma_2{}^{(n-m+a_2-1)}e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}}{\xi(a_1,a_2,b_1,b_2,m,n)} \quad (1.5.1.18)$$

1.3.2Bayes Estimators under General Entropy Loss Function (GELF)

Occasionally, the use of symmetric loss function, namely SELF, was found to inappropriate, since for example, overestimations of the reliability function usually much more serious than an underestimation. Here was consider asymmetric loss function namely GELF proposed by Calabria and Pulcini (1994), is given by

$$L_5(\theta,d)=\left(\frac{d}{\theta}\right)^{\alpha_2}-\alpha_2\ln\left(\frac{d}{\theta}\right)-1\ ;\quad(\alpha_2\neq 0) \quad (1.5.2.1)$$

Where as for the Change or Shift point m, the loss function is defined as

$$L_5(m,\widehat{m}_{BE})=\left(\frac{\widehat{m}_{BE}}{m}\right)^{\alpha_2}-\alpha_2\ln\left(\frac{\widehat{m}_{BE}}{m}\right)-1\ ;\quad(\alpha_2\neq 0) \quad (1.5.2.2)$$

Where, $\alpha_2\neq 0$, m = 1, 2, …. (n-1), and $\widehat{m}_G=1,2,\ldots\ldots(n-1)$. Here, $\widehat{m}_{BE}$ is the smallest integer greater than the analytical solution. The sign of the shape parameter $\alpha_2>0$, if overestimation is more serious than underestimation, and vice versa, and the magnitude of α_2 reflects the degree of asymmtery. The Bayes estimator of θ under the GELF is given by

$$\hat{\theta}_{BE} = \left[E_\rho(\theta^{-k_2})\right]^{1/k_2} \quad (1.5.2.3)$$

The Bayes estimate $\hat{m}_{BE}$ of m under GELF using marginal posterior distribution equation (1.5.1.14), we get as

$$\hat{m}_{BE} = \left[\sum_m m^{-k_2}\, \rho(m|\underline{x})\right]^{-1/k_2}$$

$$\hat{m}_{BE} = \left[\frac{\sum_m m^{-k_2} \frac{\Gamma(m+a_1)}{(T_{1m}+b_1)^{(m+a_1)}} \frac{\Gamma(n-m+a_2)}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{-1/k_2} \quad (1.5.2.5)$$

The Bayes Estimate $\hat{\gamma}_{1BE}$ of γ_1 under GELF using marginal posterior distribution equation (1.5.1.16), we get

$$\hat{\gamma}_{1BE} = \left[E_\rho(\gamma_1{}^{-k_2})\right]^{-1/k_2} (1.5.2.6)\hat{\gamma}_{1BE}$$

$$= \left[\sum_m \gamma_1{}^{-k_2}\, \rho(\gamma_1|\underline{x})\right]^{-1/k_2}$$

$$\hat{\gamma}_{1BE} = \left[\frac{\sum_m \gamma_1{}^{-k_2} e^{-\gamma_1(T_{1m}+b_1)} \gamma_1{}^{(m+a_1-1)} \frac{\Gamma(n-m+a_2)}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{-1/k_2}$$

$$\hat{\gamma}_{1BE} = \left[\frac{\sum_m \frac{\Gamma(n-m+a_2)}{(T_{1n}-T_{1m}+b_2)^{n-m+a_2}} \int_0^\infty e^{-\gamma_1(T_{1m}+b_1)} \gamma_1{}^{m+a_1-k_2-1}\, d\gamma_1}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{-1/k_2}$$

Assuming $\gamma_1(T_{1m}+\beta_1) = y \quad \& \gamma_1 = \frac{y}{T_{1m}+\beta_1}$

$$\hat{\gamma}_{1BE} = \left[\frac{\sum_m \frac{\Gamma(n-m+a_2)}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2)}} \int_0^\infty e^{-y} \frac{y^{(m+a_1-k_2-1)}}{(T_{1m}+b_1)^{(m+a_1-k_2-1)}} \frac{dy}{(T_{1m}+b_1)}}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{-1/k_2}$$

$$\hat{\gamma}_{1BE} = \left[\frac{\sum_m \frac{\Gamma(n-m+a_2)}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2)}} \frac{\Gamma(m+a_1-k_2)}{(T_{1m}+b_1)^{(m+a_1-k_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{-1/k_2}$$

$$\hat{\gamma}_{1BE} = \left[\frac{\xi[(a_1-k_2),a_2,b_1,b_2,m,n]}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{-1/k_2} \quad (1.5.2.7)$$

The Bayes Estimate $\hat{\gamma}_{2BE}$ of γ_2 under GELF using marginal posterior distribution equation (1.5.1.18), we get

$$\hat{\gamma}_{2BE} = \left[E_\rho(\gamma_2{}^{-k_2})\right]^{-1/k_2} (1.5.2.8)\hat{\gamma}_{2BE} = \left[\sum_m \gamma_2{}^{-k_2}\, \rho(\gamma_2|\underline{x})\right]^{-1/k_2}$$

$$\hat{\gamma}_{2BE} = \left[\frac{\sum_m \gamma_2{}^{-k_2} \left[\frac{\Gamma(m+a_1)}{(T_{1m}+b_1)^{(m+a_1)}}\right] \gamma_2{}^{(n-m+a_2-1)} e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{-1/k_2}$$

$$\hat{\gamma}_{2BE} = \left[\frac{\sum_m \left[\frac{\Gamma(m+a_1)}{(T_{1m}+b_1)^{(m+a_1)}}\right] \int_0^\infty \gamma_2{}^{(n-m+a_2-k_2-1)} e^{-\gamma_2(T_{1n}-T_{1m}+b_2)}\, d\gamma_2}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{-1/k_2}$$

Assuming $\gamma_2(T_{1n}-T_{1m}+b_2) = y \ \& \gamma_2 = \frac{y}{T_{1n}-T_{1m}+b_2}$

$$\hat{\gamma}_{2BE} = \left[\frac{\sum_m \left[\frac{\Gamma(m+a_1)}{(T_{1m}+b_1)^{(m+a_1)}}\right] \int_0^\infty e^{-y} \frac{y^{(n-m+a_2-k_2-1)}}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2-k_2-1)}} \frac{dy}{(T_{1n}-T_{1m}+b_2)}}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{-1/k_2}$$

$$\hat{\gamma}_{2BE} = \left[\frac{\sum_m \left[\frac{\Gamma(m+a_1)}{(T_{1m}+b_1)^{(m+a_1)}}\right] \frac{\Gamma(n-m+a_2-k_2)}{(T_{1n}-T_{1m}+b_2)^{(n-m+a_2-k_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{-1/k_2}$$

$$\hat{\gamma}_{2BE} = \left[\frac{\xi[a_1,(a_2-k_2),b_1,b_2,m,n]}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{-1/k_2} \quad (1.5.2.9)$$

Numerical Comparison for Generalized Compound Rayleigh Sequences

We have generated 20 random observations from Generalized Compound Rayleigh distribution with parameter $\alpha = 2, \beta = 0.5$ and $\gamma = 2$. The observed data mean is 0.9639 and variance is 2.3071. Let the change in sequence is at 11th observation, so the means of both sequences $(x_1, x_2, \ldots, x_m)$ and $(x_{(m+1)}, x_{(m+2)}, \ldots, x_n)$ are γ_1= 1.2682, γ_2= 0.5920. If the target value of γ_1 is unknown, its estimating $(\hat{\gamma}_1)$ is given by the mean of first m sample observation given m = 11, $\gamma = 1.268$.

2. Sensitivity Analysis of Bayes Estimates

In this section we have studied the sensitivity of the Bayes estimates with respect to changes in the parameters of prior distribution a_1, b_1, a_2 and b_2. The means and variances of the prior distribution are used as prior information in computing these parameters. Then with these parameter values we have computed the Bayes estimates of m, γ_1 and γ_2 under general entropy loss function (GELF) considering different set of values of $(a_1, b_1)$and (a_2, b_2).We have also considered the different sample sizes n=10 (10)30. The Bayes estimates of the change point 'm' and the parameters γ_1 and γ_2 are given in table- (4.1) under GELF. Their respective mean squared errors (M.S.E's) are calculated by repeating this process 1000 times and presented in same table in small parenthesis under the estimated values of parameters. All these values appear to be robust with respect to correct choice of prior parameter values and appropriate sample size. From the below table we conclude that –

The Bayes estimates of the parameters γ_1 and γ_2 of GCRD obtained with GELF are seems to be efficient as the numerical values of their mse's are very small for $\hat{\gamma}_{1BE}$ and $\hat{\gamma}_{2BE}$ in comparison with $\hat{m}_{BE}$.The Bayes estimates of the parameters are robust with correct choice of prior parameters and sample size. This consistency is similar to the conclusions drawn by Calabria and Pulcini (1996). The Bayes estimates of the parameters are robust with a_1 = (1.5-2.5), a2= (1.70-2.50), b_1= (1.75-2.75) and b_2= (1.80-2.60) and all sample size.

Table 1.1: Bayes Estimates of m, γ_1&$\gamma_2$for GCRD sequences and their respective M.S.E.'s Under GELF

(a_1, b_1)	(a_2, b_2)	n	$\hat{m}_{BE}$	$\hat{\gamma}_{1BE}$	$\hat{\gamma}_{2BE}$
(1.25, 1.50)	(1.50, 1.60)	10	2.2375 **(0.2409)**	0.1839 **(3.5046)**	0.4150 **(2.2776)**
		20	3.2684 **(1.1614)**	0.1113 **(3.7381)**	0.2796 **(3.1288)**
		30	4.4147 **(0.0353)**	0.0644 **(3.7310)**	0.3043 **(3.0313)**
(1.50, 1.75)	(1.70, 1.80)	10	2.4056 **(0.4941)**	0.1139 **(2.8918)**	0.2982 **(2.4116)**
		20	3.1798 **(1.6132)**	0.0933 **(3.7161)**	0.2690 **(2.9710)**
		30	4.7601 **(7.9354)**	0.0806 **(3.7176)**	0.1949 **(2.8390)**
(1.75, 2.0)	(1.90, 2.0)	10	2.7368 **(0.2062)**	0.1504 **(3.5153)**	0.2799 **(2.800)**
		20	3.9204 **(3.1352)**	0.1049 **(3.5509)**	0.2485 **(3.0285)**
		30	8.3818 **(24.2671)**	0.1477 **(3.6236)**	0.2336 **(3.2257)**
(2.0, 2.25)	(2.10, 2.20)	10	2.3326 **(0.1569)**	0.1274 **(3.3872)**	0.2366 **(2.6159)**
		20	3.1266 **(2.1347)**	0.0834 **(3.6105)**	0.2399 **(2.7302)**
		30	4.1201 **(4.3081)**	0.0728 **(3.8154)**	0.1905 **(3.0615)**
(2.25, 2.50)	(2.30, 2.40)	10	2.9935 **(0.3429)**	0.1277 **(3.4797)**	0.5369 **(2.7068)**
		20	3.8048 **(2.2571)**	0.0871 **(3.7271)**	0.1959 **(3.0668)**
		30	4.4457 **(4.0216)**	.0723 **(3.7722)**	.1959 **(3.1464)**
(2.50, 2.75)	(2.50, 2.60)	10	2.4144 **(0.6308)**	0.1692 **(3.4158)**	0.3704 **(2.9467)**
		20	3.7902 **(8.5221)**	0.1007 **(3.5954)**	0.2604 **(3.2319)**
		30	4.3244 **(7.2033)**	0.0575 **(3.7195)**	0.2322 **(3.1164)**

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