Casson Fluid Flow and Heat Transfer Past an Exponentially Stretching Surface

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Abstract: ***Numerical solutions for a class of nonlinear differential equations arising in heat and mass transfer of a non-Newtonian fluid towards an exponentially stretching surface are obtained. Casson fluid model is used to characterize the non-Newtonian fluid behaviour. Using the similarity transformations, the governing equations have been transformed into a system of ordinary differential equations. These differential equations are highly nonlinear which cannot be solved analytically. Therefore, bvp4c MATLAB solver has been used for solving it. The flow fields and heat and mass transfer are significantly affected by the physical parameters. The effect of increasing values of the Casson parameter and momentum slip parameter is to suppress the velocity field. But the temperature and concentration is enhanced with increasing Casson parameter and momentum slip parameter.***

Keywords: Casson Fluid, Heat Transfer, Slip Flow, Chemical Reaction

1. Introduction

The fluid flow behavior of non-Newtonian fluid has attracted special interest in recent years due to the wide application of these fluids in the chemical, pharmaceutical, petrochemical, food industries and electronic industries. A large number of fluids that are used extensively in industrial application are non-Newtonian fluids exhibiting a yield stress τ_y, stress that has to be exceeded before the fluid moves. As a result the fluid cannot sustain a velocity gradient unless the magnitude of the local shear stress is higher than this yield stress. Fluids that belong to this category include cement, drilling mud, sludge, granular suspensions, aqueores foams, slurries, paints, plastics, paper pulp and food products. Dharmaiah et al.[1] proposed perturbation analysis of thermophoresis, hall current and heat source on flow dissipative aligned convective flow about an inclined plate. Rani et al. [2] have analyzed hall and ion slip effects on ag - water based mhd nano fluid flow over a semi infinite vertical plate embedded in a porous medium. Vedavathi et al. [3] reported an incitement of hall and aligned mhd casson flow. Rani et al. [4] discussed hall and ion slip effects on mhd ag – $h_2 0$ nano fluid using biomedical engineering applications. Sudarshan Reddy et al. [5] formulated an analytical study on biomedical and its mechanisms using casson fluid over a vertical plate in presence of heat absorption. Dharmaiah et al. [6] have been reviewed hall and ion slip impact on magneto-titanium alloy nanoliquid with diffusion thermo and radiation absorption. Vedavathi et al. [7] have drafted entropy analysis of magnetohydrodynamic nanofluid transport past an inverted cone: buongiorno's model. Vedavathi et al.[8] composed entropy analysis of nanofluid magnetohydrodynamic convection flow past an inclined surface with a numerical review manner. Vedavathi et al. [9] have been expressed numerical study of radiative non-darcy nano fluid flow over a stretching sheet with a convective nield conditions and energy activation. Dharmaiah et al. [10] have been discussed non-newtonian nanofluid characteristics over a melting wedge: a numerical study. Dharmaiah et al. [11] have been carried out mhd radiative ohmic heating nanofluid flow of a stretching penetrable wedge:a numerical analysis. Dharmaiah et al. [12] have been investigated thermal diffusion effects on transient free convective magneto nanofluid in the presence of thermal radiation. Dharmaiah et al. [13] have been analyzed influence of ion-slip and hall current on magneto hydrodynamic free convective flow past an accelerated plate with dufour effect and ramped temperature. Sridhar et al. [14] have been scrutinized unsteady titanium alloy water-based nano fluid past an oscillatory moving vertical permeable semi-infinite flat plate with thermophoresis effect. Vedavathi et al. [15] have been scrutinized metallic nano particle effect on unsteady convective mhd flow of radiation with rotating frame of reference.

2. Mathematical Formulation

Let us consider the flow of an incompressible viscous fluid past a flat sheet coinciding with the plane y = 0. The fluid flow is confined to y > 0. Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed (see Fig.1). The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + p_y/\sqrt{2\pi}\right)e_{ij}, \pi > \pi_c \\ 2\left(\mu_B + p_y/\sqrt{2\pi_c}\right)e_{ij}, \pi < \pi_c \end{cases}$$

Here $\pi = e_{ij}e_{ij}$ and e_{ij} is the (i,j)-th component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is plastic dynamic viscosity of the non- Newtonian fluid, and p_y is the yield stress of the fluid. Let T_w, C_w be the temperature and concentration at the sheet respectively and temperature and concentration far away from the sheet is T_∞, C_∞. Also the reaction of species is the first order homogeneous chemical reaction with rate constant k.

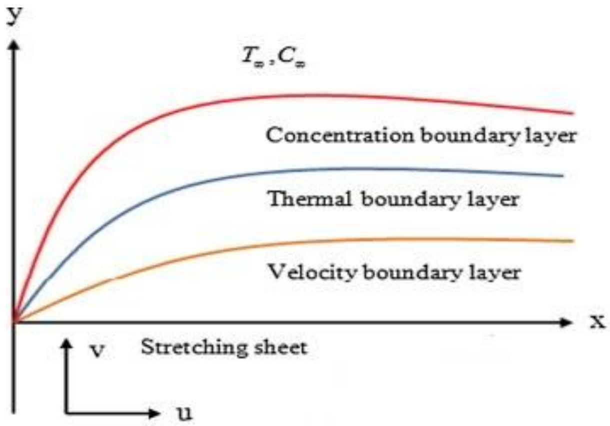

Figure 1: Physical flow of the problem

Under these assumptions along with the Boussinesq and boundary layer approximations, the system of equations, which models the flow is given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (2.1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} \quad (2.2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \quad (2.3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k\left(C-C_\infty\right) \quad (2.4)$$

The boundary conditions are

$$u=u_w+u_{slip}=U_0e^{\frac{x}{L}}+N\upsilon\frac{\partial u}{\partial y}, v=v_w\left(x\right), T=T_w, C=C_w$$

at $y=0$

$$u\to 0, T\to T_\infty, C\to C_\infty \quad \text{as} \quad y\to\infty \quad (2.5)$$

We introduce also a stream functions ψ is defined by

$$u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x} \quad (2.6)$$

Introducing the similarity variables as

$$\eta=\sqrt{\frac{U_0}{2\upsilon L}}e^{\frac{x}{2L}}y, \psi=\sqrt{2\upsilon U_0 L}e^{\frac{x}{2L}}f\left(\eta\right), u=U_0e^{\frac{x}{L}}f'\left(\eta\right)$$

$$v=-\sqrt{\frac{\upsilon U_0}{2L}}\left(f\left(\eta\right)+\eta f'\left(\eta\right)\right), T=T_\infty+T_0e^{\frac{x}{2L}}\theta\left(\eta\right)$$

$$C=C_\infty+C_0e^{\frac{x}{2L}}\phi\left(\eta\right)$$

(2.7)

where $f\left(\eta\right)$ is the dimensionless stream function, $\theta\left(\eta\right)$ is the dimensionless temperature, $\phi\left(\eta\right)$ is the dimensionless concentration, η is the similarity variable, T_0, C_0 are the reference temperature and concentration respectively.

Substituting equations (2.6) and (2.7) in (2.2) – (2.5), we have

$$\left(1+\frac{1}{\beta}\right)f'''+ff''-2f'^2=0 \quad (2.8)$$

$$\theta''+\Pr\left(f\theta'-f'\theta\right)=0 \quad (2.9)$$

$$\phi''+Sc\left(f\phi'-f'\phi-2kr\phi\right)=0 \quad (2.10)$$

The corresponding boundary conditions are

$$f'\left(0\right)=1+\gamma f''(0), f\left(0\right)=fw, \theta\left(0\right)=1, \phi\left(0\right)=1$$

$$f'\left(\infty\right)=\theta\left(\infty\right)=\phi\left(\infty\right)=0 \quad (2.11)$$

3. Solution of the Problem

The set of equations (2.8) to (2.11) were reduced to a system of first-order differential equations and solved using a MATLAB boundary value problem solver called **bvp4c**. This program solves boundary value problems for ordinary differential equations of the form $y'=f\left(x,y,p\right), a\le x\le b$, by implementing a collocation method subject to general nonlinear, two-point boundary conditions $g\left(y(a), y(b), p\right)$. Here $\boldsymbol{p}$ is a vector of unknown parameters. Boundary value problems (BVPs) arise in most diverse forms. Just about any BVP can be formulated for solution with **bvp4c**. The first step is to write the *ODEs* as a system of first order ordinary differential equations.

4. Results and Discussion

Numerical solutions for a class of nonlinear differential equations arising in heat and mass transfer of a non-Newtonian fluid towards an exponentially stretching surface are obtained. Casson fluid model is used to characterize the non-Newtonian fluid behaviour. Using the similarity transformations, the governing equations have been transformed into a system of ordinary differential equations. These differential equations are highly nonlinear which cannot be solved analytically. Therefore, bvp4c MATLAB solver has been used for solving it. The flow fields and heat and mass transfer are significantly affected by the physical parameters. The effect of increasing values of the Casson parameter and momentum slip parameter is to suppress the velocity field. But the temperature and concentration is enhanced with increasing Casson parameter and momentum slip parameter.

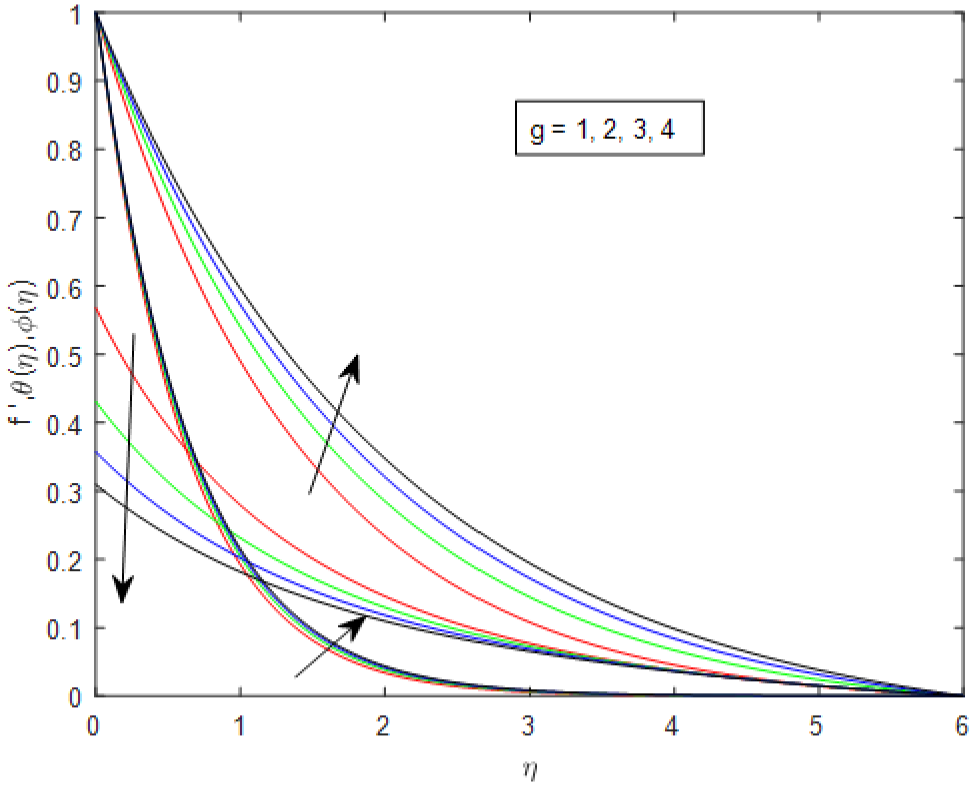

Figure 2: variations for parameter g

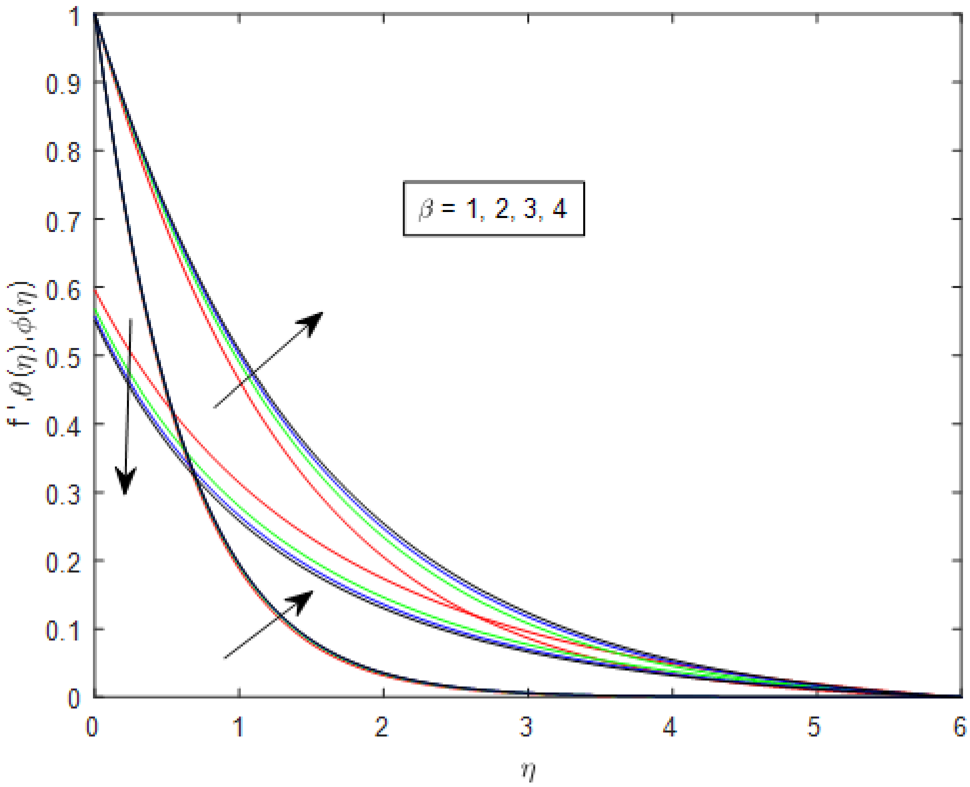

Figure 3: variations for parameter β

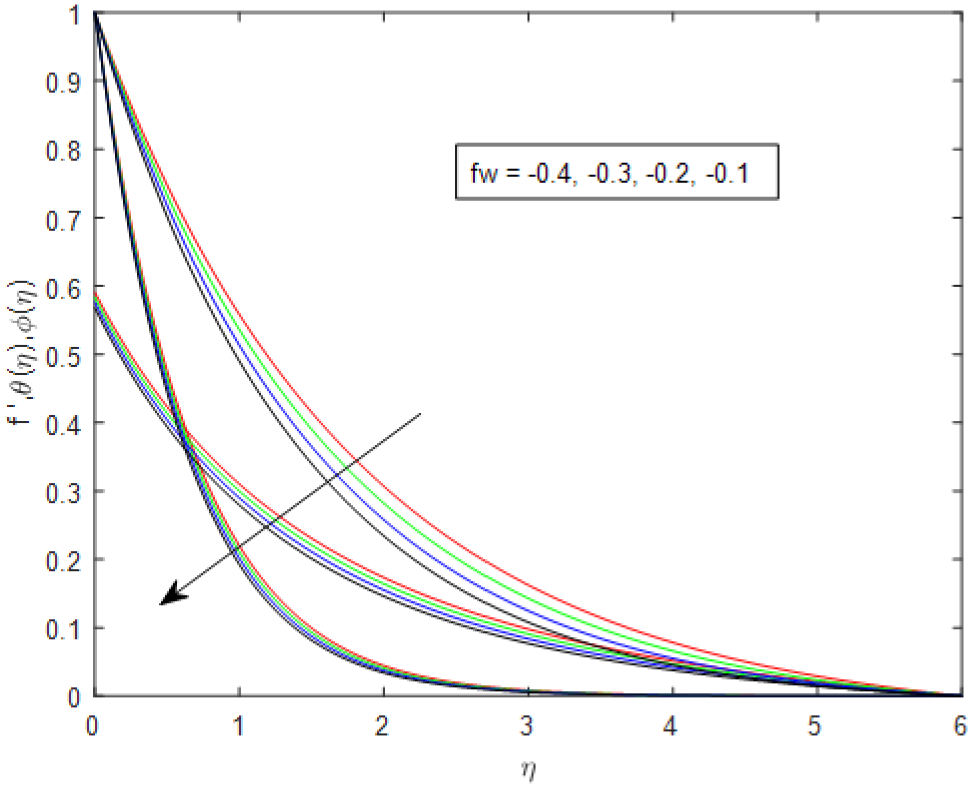

Figure 4: Variations for parameter fw

Figures 2-4 document the graphical computations for velocity, temperature and concentration for the effects of the dictating thermophysical parameters. Figs. 1-3 show that the effects of Casson parameter (β), suction/injection parameter (fw) and slip parameter on velocity, temperature and concentration profiles. From figures 2 and 3, velocity and concentration are increasing and temperature is decreasing. From figure 4, three profiles are decreasing.

5. Conclusions

In the present prater, Numerical solutions for a class of nonlinear differential equations arising in heat and mass transfer of a non-Newtonian fluid towards an exponentially stretching surface are obtained. Casson fluid model is used to characterize the non-Newtonian fluid behaviour. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. It has been found that

1) Momentum boundary layer thickness decreases with increasing Casson parameter and momentum slip parameter but the thermal and solute boundary layer thickness increases in this case.
2) Momentum and thermal as well as solute boundary layer thickness reduces with the influence of mass flux parameter.

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