Implementation of Coupled Fixed Point Theorems with β –Monotone Property in Soft b-Metric Space

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Abstract: ***In this paper we define β -Monotone property, Soft Metric Space, b-Soft Metric Space and then we demon start some results. In the first result we prove coupled soft fixed point theorem in ordered soft b- metric space with β -monotone property. In the second theorem demonstrate coupled soft co-incidence fixed point theorem for mapping satisfying generalized contractive conditions with β –monotone property in an ordered soft b-metric space.***

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1. Introduction and Preliminaries

Metric fixed point theory is an necessary part of mathematics because its applications use in different areas like variational and linear inequalities, improvement and approximation theory. Ali et al. [9], Agraval et al. ([14], [15], [16]), Pathak et al. [7] and many authors (see [1], [2], [3], [8], [13]) established fixed theorems in different spaces like partially ordered metric space, Metric space, Manger space, Banach space, generalized Banach space etc. A concept of soft theory as new mathematical tool for dealing with uncertainties is discussed in 1999 by Molodtsov [6]. A soft set is a collection of approximate descriptions of an object this theory has rich potential applications. On soft set theory many structures contributed by many researchers (see [5], [10], [12]). Shabir and Naz [11] were studied about soft topological spaces. In these studies, the concept of soft point is explained by different techniques. Later a notion of soft metric space and investigated some basic properties of these spaces. Recently Wadkar et.al [4] proved fixed point results in soft metric space. In this paper we first prove soft coupled fixed point theorem in ordered soft b-metric space using monotone property for contractive condition. In the next theorem we prove the coupled soft coincidence fixed point theorem for mapping satisfying generalized contractive conditions with α –monotone property in an ordered soft b-metric space.

Definition 1.1: Let Z and H are respectively an initial inverse set and a parameter set. A soft set over Z is pair denoted by (G, H) if and only if G is a mapping from E into the set of all subsets of the set X. That is G: H ⟶ P(z), where P(z) is the power set of Z.

Definition 1.2: The intersection of two soft sets (Z , C) and (Y , E) over X is a soft set denoted by (I , H) over X and is given by $(Z , C)\tilde{\cap} (Y , E) = (I ,H)$ where $H = C \cap E$ and $\forall \varepsilon \in H$, $I (\varepsilon) = Z (\varepsilon) \cap Y (\varepsilon)$.

Definition 1.3: A soft set (Z, B) over Y is said to be a null soft set denoted by ∅

if Z p = empty, for all p in A.

Definition 1.4: For all $p \in B$, if Z (p) = Y then (Z , B) is called an absolute soft set over Y.

Definition 1.5: The difference of two soft sets (H , P) and (L, P) over Y is a soft set (G , P) over X, denoted by (h , E) \ (L, E) , is defined as G(y) = F(y) \ G(y), $\forall y \in H$.

Definition 1.6: The complement of soft set (H , B) is denoted by $(H , B)^C$ and is defined as $(H, B)^C = (H^c , B)$,where $H^c : B \to P(Y)$ is a mapping given by $H^c (\beta) = Y - H (\beta)$, for all β .

Definition 1.7: Let R be the set of real numbers and B(R) be the collection of all nonempty bounded subsets of R and E taken as a set of parameters. Then a mapping Y:E→B(R) is called a soft real set, and it is denoted by (Y,E).

2. Main Results

Let (Y, ≤) be a partially ordered soft set and $\tilde{\rho}$ be a soft metric on Y such that $(Y, \tilde{\rho}, H)$ is a complete soft b-metric space. Consider a product $(Y, \tilde{\rho}, H)$ X $(Y, \tilde{\rho}, H)$ with the following partial order. For all $(x_\varphi, y_\delta) \in (Y, \tilde{\rho}, H)x (Y, \tilde{\rho}, H)$. We have $(u_\varphi, v_\delta) \geq (x_\varphi, y_\delta) \Leftrightarrow x_\varphi \geq u_\varphi,\ y_\delta \leq v_\delta$

Theorem 2.1: Let $((Y, \tilde{\rho}, H), \leq)$ be a partially ordered complete soft b- metric space and let S be a continuous mapping having the mixed monotone property such that For all $x_\varphi, y_{\delta,}\ u_\varphi, v_\delta \in (Y, \tilde{\rho}, H)$ and $\left(0, \frac{1}{s}\right)$ and $s \geq 1$,

We have

$$\tilde{\rho}\left(F(x_\varphi, y_\delta), (u_\varphi, v_\delta\right) \leq \beta \max\left[\begin{array}{c} \tilde{\rho}\ (x_\varphi , F(u_\varphi, v_\delta) + \tilde{\rho}\left(y_\delta, F(v_\delta, u_\varphi)\right) \\ +\tilde{\rho}\ (u_\varphi , F(x_\varphi, y_\delta) + \tilde{\rho}\left(v_\delta, F(y_\delta, x_\varphi)\right) \\ +\tilde{\rho}(y_\delta , \frac{F(u_\varphi, v_\delta)F(v_\delta, u_\varphi)F(x_\varphi, y_\delta)}{1+F(x_\varphi, y_\delta)F(u_\varphi, v_\delta)}) \end{array}\right] \quad (2.1.1)$$

Then F has a coupled soft fixed point in soft b-metric space $(Y, \tilde{\rho}, H)$.

Proof: Let two points $x_\varphi^0, y_\delta^0 \in (Y,\tilde{\rho},H) x (Y,\tilde{\rho},H)$ and $x_{\varphi_1}^1 = F(x_\varphi^0, y_\delta^0)$; $y_{\delta_1}^1 = F(y_\delta^0, x_\varphi^0)$.

In general $y_{\delta_{n+2}}^{n+2} = F(y_{\delta_{n+1}}^{n+1}, x_{\varphi_{n+1}}^{n+1}), y_{\delta_{n+1}}^{n+1} = F(y_{\delta_n}^n, x_{\varphi_n}^n)$

Similarly $x_{\varphi_{n+2}}^{n+2} = F(x_{\varphi_{n+1}}^{n+1}, y_{\delta_{n+1}}^{n+1}), x_{\varphi_{n+1}}^{n+1} = F(x_{\varphi_n}^n, y_{\delta_n}^n)$

with $x_\varphi^0 \le F(x_\varphi^0, y_\delta^0) = x_{\varphi_1}^1$ and $y_\delta^0 \ge F(y_\delta^0, x_\varphi^0) = y_{\delta_1}^1$
(2.1.2)

by iteartive process, we have

$$x_{\varphi_2}^2 = F\left(x_{\varphi_1}^1, y_{\delta_1}^1\right) = F(F(x_\varphi^0, y_\delta^0), F(y_\delta^0, x_\varphi^0)) = F^2(x_\varphi^0, y_\delta^0)$$

and $y_{\delta_2}^2 = F\left(y_{\delta_1}^1, x_{\varphi_1}^1\right) = F(F(y_\delta^0, x_\varphi^0), F(x_\varphi^0, y_\delta^0)) = F^2(y_\delta^0, x_\varphi^0)$

By the mixed monotone property of F we obtain

$$x_{\varphi_2}^2 = F^2(x_\varphi^0, y_\delta^0) = F\left(x_{\varphi_1}^1, y_{\delta_1}^1\right) \ge F(x_\varphi^0, y_\delta^0) = x_{\varphi_1}^1$$

And $y_{\delta_2}^2 = F^2(y_\delta^0, x_\varphi^0) = F\left(y_{\delta_1}^1, x_{\varphi_1}^1\right) \le F(y_\delta^0, x_\varphi^0) = y_{\delta_1}^1$

In general we have for $n \in N$

$$x_{\varphi_{n+1}}^{n+1} = F^{n+1}(x_\varphi^0, y_\delta^0) = F(F^n(x_\varphi^0, y_\delta^0), F^n(y_\delta^0, x_\varphi^0)) \quad (2.1.3)$$

and $y_{\delta_{n+1}}^{n+1} = F^{n+1}(y_\delta^0, x_\varphi^0) = F(F(y_\delta^0, x_\varphi^0), F(x_\varphi^0, y_\delta^0))$

It is obvious that

$$x_\varphi^0 \le F(x_\varphi^0, y_\delta^0) = x_{\varphi_1}^1 \le F^2(x_\varphi^0, y_\delta^0) = x_{\varphi_2}^2 \le \cdots \le F^n(x_\varphi^0, y_\delta^0) = x_{\varphi_n}^n \le$$

$$y_\delta^0 \ge F(y_\delta^0, x_\varphi^0) = y_{\delta_1}^1 \ge F\left(y_{\delta_1}^1, x_{\varphi_1}^1\right) = y_{\delta_2}^2 \ge \cdots \ge F^n(x_\varphi^0, y_\delta^0) = y_{\delta_n}^n \ge \ldots \quad (2.1.4)$$

Thus by mathematical induction principal we have for $n \in N$
$x_\varphi^0 \le x_{\varphi_1}^1 \le x_{\varphi_1}^2 \le \cdots \le x_{\varphi_n}^n \le x_{\varphi_{n+1}}^{n+1} \ldots$ and $y_\delta^0 \ge y_{\delta_1}^1 \ge y_{\delta_2}^2 \ge \cdots \ge y_{\delta_n}^n \ge y_{\delta_{n+1}}^{n+1}$

Thus we have by condition (2.1.1) that

$$\tilde{\rho}\left(x_{\varphi_{n+1}}^{n+1}, x_{\varphi_n}^n\right) = \tilde{\rho}\left(F\left(x_{\varphi_n}^n, y_{\delta_n}^n\right), F(x_{\varphi_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n-1})\right)$$

$$\le \beta \max\left\{\begin{array}{c} \tilde{\rho}\left(x_{\varphi_n}^n, F\left(x_{\varphi_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n-1}\right)\right) + \tilde{\rho}\left(y_{\delta_n}^n, F\left(y_{\delta_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n-1}\right)\right), \\ \tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, F\left(x_{\varphi_n}^n, y_{\delta_n}^n\right)\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, F\left(y_{\delta_n}^n, x_{\varphi_n}^n\right)\right), \\ \tilde{\rho}\left(y_{\delta_n}^n, \frac{F\left(x_{\varphi_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n-1}\right).F\left(y_{\delta_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n-1}\right).F\left(x_{\varphi_n}^n, y_{\delta_n}^n\right)}{1+F\left(x_{\varphi_n}^n, y_{\delta_n}^n\right).\left(x_{\varphi_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n-1}\right)}\right) \end{array}\right\}$$

$$\le \beta \max\left\{\begin{array}{c} \tilde{\rho}\left(x_{\varphi_n}^n, x_{\varphi_n}^n\right) + \tilde{\rho}\left(y_{\delta_n}^n, y_{\delta_n}^n\right), \\ \tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n+1}\right), \\ \tilde{\rho}\left(y_{\delta_n}^n, \frac{F\left(x_{\varphi_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n-1}\right).F\left(y_{\delta_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n-1}\right).F\left(x_{\varphi_n}^n, y_{\delta_n}^n\right)}{F\left(x_{\varphi_n}^n, y_{\delta_n}^n\right).F\left(x_{\varphi_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n-1}\right)}\right) \end{array}\right\}$$

$$\le \beta \max\left\{\begin{array}{c} \tilde{\rho}\left(x_{\varphi_n}^n, x_{\varphi_n}^n\right) + \tilde{\rho}\left(y_{\delta_n}^n, y_{\delta_n}^n\right), \\ \tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n+1}\right), \\ \tilde{\rho}\left(y_{\delta_n}^n, y_{\delta_n}^n\right) \end{array}\right\}$$

$$\le \beta \max\left[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n+1}\right)\right]$$

$$\tilde{\rho}\left(x_{\varphi_{n-1}}^{n+1}, x_{\varphi_{n-1}}^n\right) \le \beta\left[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n+1}\right)\right] \quad (2.1.5)$$

Similarly since $y_{\delta_{n-1}}^{n-1} \ge y_{\delta_n}^n$ and $x_{\varphi_{n-1}}^{n-1} \le x_{\varphi_n}^n$ (2.1.6)

$$\tilde{\rho}\left(y_{\delta_{n-1}}^{n+1}, y_{\delta_n}^n\right) \le \beta\left[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n+1}\right)\right]$$

$$\begin{aligned} \tilde{\rho}\left(x_{\varphi_{n-1}}^{n+1}, x_{\varphi_{n-1}}^n\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n+1}, y_{\delta_n}^n\right) &\le \beta\left[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n+1}\right)\right] \\ &+ \beta\left[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_{n-1}}^{n+1}\right)\right] \end{aligned}$$

$$\begin{aligned} \le 2\beta\Big[s\left\{\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_n}^n\right) + \tilde{\rho}\left(x_{\varphi_n}^n, x_{\varphi_{n-1}}^{n+1}\right)\right\} \\ + s\left\{\tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_n}^n\right) + \tilde{\rho}\left(y_{\delta_n}^n, y_{\delta_{n-1}}^{n+1}\right)\right\}\Big] \end{aligned}$$

$$\begin{aligned} 2\beta s\left[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_n}^n\right) + \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_n}^n\right)\right] \\ + 2\beta s\left[\tilde{\rho}\left(x_{\varphi_n}^n, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_n}^n, y_{\delta_{n-1}}^{n+1}\right)\right] \end{aligned}$$

(1-
$2\beta s)\left[\tilde{\rho}\left(x_{\varphi_n}^n, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_n}^n, y_{\delta_{n-1}}^{n+1}\right)\right] \le 2\beta s\left[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_n}^n\right) +\right.$
ρyδn−1n−1,yδnn

$$\begin{aligned} \left[\tilde{\rho}\left(x_{\varphi_n}^n, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_n}^n, y_{\delta_{n-1}}^{n+1}\right)\right] &\le \frac{2\beta s}{(1-2\beta s)}\Big[\tilde{\rho}\left(x_{\varphi_{n-1}}^{n-1}, x_{\varphi_n}^n\right) \\ &+ \tilde{\rho}\left(y_{\delta_{n-1}}^{n-1}, y_{\delta_n}^n\right)\Big] \end{aligned}$$

Let suppose that $h = \frac{2\beta s}{(1-2\beta s)}$ and $d_m = \tilde{\rho}\left(x_{\varphi_n}^n, x_{\varphi_{n-1}}^{n+1}\right) + \tilde{\rho}\left(y_{\delta_n}^n, y_{\delta_{n-1}}^{n+1}\right)$ Then $d_m \le hd_{m-1}$

Similarly it can be proved that $d_{m-1} \le hd_{m-2}$.

Therefore $d_m \le hd_{m-1} \le h^2 d_{m-2} \le \cdots h^n d_0$. (2.1.7)

This implies that $\lim_{n\to\infty} d_n = 0$. Thus$\lim_{n\to\infty} d\left(x_{\varphi_{n-1}}^{n+1}, x_{\varphi_n}^n\right) = \lim_{n\to\infty} d\left(y_{\delta_{n-1}}^{n+1}, y_{\delta_n}^n\right) = 0$

For each p≥ n, by (2.1.7) and repeat the application of triangle inequality that we optain that

$$\begin{aligned} \left(y_{\delta_n}^n, y_{\delta_p}^p\right) \le s\tilde{\rho}\left(y_{\varphi_n}^n, y_{\varphi_{n-1}}^{n+1}\right) + s^2\tilde{\rho}\left(y_{\varphi_{n-1}}^{n+1}, y_{\varphi_{n+2}}^{n+2}\right) \\ + s^3\tilde{\rho}\left(y_{\varphi_{n-2}}^{n+2}, y_{\varphi_{n-1}}^{n+1}\right) + \cdots s^m\tilde{\rho}\left(y_{\varphi_n}^n, y_{\varphi_{n-1}}^{n+1}\right) \end{aligned}$$

And

$$\left(x^n_{\delta_n}, x^p_{\delta_p}\right) \le s\tilde{\rho}\left(x^n_{\varphi_n}, x^{n+1}_{\varphi_{n-1}}\right) + s^2\tilde{\rho}\left(x^{n+1}_{\varphi_{n-1}}, x^{n+2}_{\varphi_{n+2}}\right) + s^3\tilde{\rho}\left(x^{n+2}_{\varphi_{n-2}}, x^{n+1}_{\varphi_{n-1}}\right) + \cdots s^m\tilde{\rho}\left(x^n_{\varphi_n}, x^{n+1}_{\varphi_{n-1}}\right)$$

Adding these we get

$$\begin{aligned}\left(y^n_{\delta_n}, y^p_{\delta_p}\right) + \left(x^n_{\delta_n}, x^p_{\delta_p}\right) &\le s\left[\tilde{\rho}\left(y^n_{\varphi_n}, y^{n+1}_{\varphi_{n-1}}\right) + \tilde{\rho}\left(x^n_{\varphi_n}, x^{n+1}_{\varphi_{n-1}}\right)\right] \\ &+ s^2[\tilde{\rho}\left(x^{n+1}_{\varphi_{n-1}}, x^{n+2}_{\varphi_{n+2}}\right) + \tilde{\rho}\left(x^{n+1}_{\varphi_{n-1}}, x^{n+2}_{\varphi_{n+2}}\right) \\ &+ s^3\left[\tilde{\rho}\left(x^{n+2}_{\varphi_{n-2}}, x^{n+1}_{\varphi_{n-1}}\right)\right. \\ &+ \left.\tilde{\rho}\left(y^{n+2}_{\varphi_{n-2}}, y^{n+1}_{\varphi_{n-1}}\right)\right] \ldots s^m[\tilde{\rho}\left(x^n_{\varphi_n}, x^{n+1}_{\varphi_{n-1}}\right) \\ &+ \tilde{\rho}\left(y^{n+2}_{\varphi_{n-2}}, y^{n+1}_{\varphi_{n-1}}\right)]\end{aligned}$$

$\le s[h^n+h^{n+1}+h^{n+2}+h^{n+3}+h^{n+4} \ldots\ldots\ldots h^{m-1}]d_0 \le s\frac{h^n}{1-h^n}$

$d_0 \to 0$ as $n \to \infty$

There $\left\{x^n_{\varphi_n}\right\}$ and $\left\{y^n_{\mu_n}\right\}$ are chouchy sequences.Since(X, $\tilde{\rho}$, E)is complete b- soft metric space. There exist y_μ, x_φ in (X, $\tilde{\rho}$, E) such that $\lim_{n\to\infty} x^n_{\varphi_n} = x_\varphi$

And $\lim_{n\to\infty} y^n_{\mu_n} = y_\mu$. Thus by taking limit $n \to \infty$, in equation we get

$x_\varphi = \lim_{n\to\infty} x^n_{\varphi_n} = \lim_{n\to\infty} S[x^{n-1}_{\varphi_{n-1}}, y^{n-1}_{\delta_{n-1}}] = S\lim_{n\to\infty}[x^{n-1}_{\varphi_{n-1}}, y^{n-1}_{\delta_{n-1}}] = S(x_\varphi, y_\delta)$ and

$y_\delta = \lim_{n\to\infty} y^n_{\delta_n} = \lim_{n\to\infty} S[y^{n-1}_{\delta_{n-1}}, x^{n-1}_{\delta_{n-1}}] = \lim_{n\to\infty}[x^{n-1}_{\delta_{n-1}}, y^{n-1}_{\delta_{n-1}}] = S(x_\varphi, y_\delta)$

Therefore $x_\varphi = S(x_\varphi, y_\delta)$and $y_\delta = S(y_\delta, x_\varphi)$.Thus S has coupled soft fixed point in (X, $\tilde{\rho}$,E).

In next theorem we prove coupled soft coincidence fixed point theorem for mapping satisfying generalized contractive conditions with –monotone in an ordered soft b-metric space.

Theorem2.2: Let ((X,$\tilde{\rho}$, E),≤) be a partially ordered set and $\tilde{\rho}$: (X, $\tilde{\rho}$,E) x (X, $\tilde{\rho}$,E)→ R be a soft b-mtric defined on X with coefficient $s \ge 1$. Let β: (X, $\tilde{\rho}$,E)→ (X, $\tilde{\rho}$, E) and S: (X, $\tilde{\rho}$,E)→ (X, $\tilde{\rho}$, E)be two mappings such that

$$\tilde{\rho}\left(S\left(x_\varphi, y_\delta\right). S(u_\varphi, v_\delta)\right) + \tilde{\rho}\left(S\left(y_\delta, x_\varphi\right). S(v_\delta, u_\varphi)\right) \le k\left\{\tilde{\rho}\left(\beta x_\varphi, \beta u_\varphi\right) + \tilde{\rho}\left(\beta y_\delta, \beta v_\delta\right)\right\}; \quad (2.2.1)$$

for some $k \in \left[0, \frac{1}{s}\right)$ and for all x_φ, y_δ $u_\varphi, v_\delta \in$(X, $\tilde{\rho}$, E) with $\beta x_\varphi \ge \beta u_\varphi$ and $\beta y_\delta \ge \beta v_\delta$.We further assume the following hypothesis:

1) S(X x X)⊆ β(X)
2) β(X) is complete
3) β is continuous and commute with S.
4) S has the mixed β-monotone property on X and eighter S is continuous or X has the following property
   a) If a non decreasing sequence and $\left\{x^n_{\varphi_n}\right\} \to x_\varphi$ then $\left\{x^n_{\varphi_n}\right\} \le x_\varphi$.
   b) If a non increasing sequence and $\left\{y^n_{\delta_n}\right\} \to y_\delta$ then $\left\{y^n_{\delta_n}\right\} \le y_\delta$.

If there exist two elements $x^0_{\varphi_0}, y^0_{\delta_0}$ in (X, $\tilde{\rho}$,E) with $\beta x^0_{\varphi_0} \le S(x^0_{\varphi_0}, y^0_{\delta_0})$

$\beta y^0_{\delta_0} \ge S(y^0_{\delta_0}, x^0_{\varphi_0})$,

Then S and β have unique a coupled soft fixed point that is there exist a unique $x_\varphi \in (X, \tilde{\rho}, E)$ such that $x_\varphi = S(x_\varphi, x_\varphi) = \beta\, x_\varphi$.

Proof: Let$x_{\varphi_0}, y_{\delta_0} \in (X, \tilde{\rho}, E)$ be such that $\beta x_{\varphi_0} \le S\left(x_{\varphi_0}, y_{\delta_0}\right)$ and $\beta y_{\delta_0} \ge S\left(y_{\delta_0}, x_{\varphi_0}\right)$.Since S(X x X) ≥ β(X), We can choose $x^1_{\varphi_0}, y^1_{\delta_0} \in$ (X, $\tilde{\rho}$,E) such that $\beta x^1_{\varphi_0} = S(x_{\varphi_0}, y_{\delta_0})$ and $\beta y^1_{\delta_0} = S(y_{\delta_0}, x_{\varphi_0})$.Again since S(X x X) ≥ β(X),we can choose $x^2_{\varphi_0}, y^2_{\delta_0} \in$ (X, $\tilde{\rho}$,E) such that $\beta x^2_{\delta_0} = S(x^1_{\varphi_0}, y^1_{\delta_0})$ and $\beta y^2_{\delta_0} = S(y^1_{\delta_0}, x^1_{\varphi_0})$.

Continuing this process we can constract two sequences $\left\{x^n_{\varphi_n}\right\}$and $\left\{y^n_{\delta_n}\right\}$ in X.

Such that $\beta x^{n+1}_{\varphi_{n+1}} = S(x^n_{\varphi_0}, y^n_{\delta_0})$ (2.2.2)

And $\beta y^{n+1}_{\delta_{n+1}} = S(y^n_{\delta_0}, x^n_{\varphi_0})$ for all n (2.2.3)

Now we will prove that for all $n \ge 0$.

$\beta x^n_{\varphi_n} \le \beta x^{n+1}_{\varphi_{n+1}}$, (2.2.4)

$\beta y^n_{\delta_n} \ge \beta y^{n+1}_{\delta_{n+1}}$ (2.2.5)

We shall use the mathematical law of induction. Let n = 0.

Since $\beta x_{\varphi_0} \le S\left(x_{\varphi_0}, y_{\delta_0}\right)$ and $\beta y_{\delta_0} \ge S\left(y_{\delta_0}, x_{\varphi_0}\right)$ and $\beta x^1_{\varphi_0} = S(x_{\varphi_0}, y_{\delta_0})$ and $\beta y^1_{\delta_0} = S(y_{\delta_0}, x_{\varphi_0})$ we have $\beta x_{\varphi_0} \le x^1_{\varphi_0}$ and and $\beta y_{\delta_0} \ge \beta y^1_{\delta_0}$

That is (2.2.4) and (2.2.5) holds for all n = 0. We assume that (2.2.4) and (2.2.5) holds for n> *0*

As S has the mixed monotone property and $\beta x^n_{\varphi_n} \le \beta x^{n+1}_{\varphi_{n+1}}$ and

$\beta y^n_{\delta_n} \ge \beta y^{n+1}_{\delta_{n+1}}$,we get

$\beta x^{n+1}_{\varphi_{n+1}} = S\left(x^n_{\varphi_n}, y^n_{\delta_n}\right) \le S(x^{n+1}_{\varphi_{n+1}}, y^n_{\delta_n})$ and

$S(y^{n+1}_{\delta_{n+1}}, x^n_{\varphi_n}) \le S\left(y^n_{\delta_n}, x^n_{\varphi_n}\right) = \beta y^{n+1}_{\delta_{n+1}}$

Also for the same reason we have

$\beta x^{n+2}_{\varphi_{n+1}} = S\left(x^{n+1}_{\varphi_{n+1}}, y^{n+1}_{\delta_{n+1}}\right) \ge S(x^{n+1}_{\varphi_{n+1}}, y^n_{\delta_n})$ and $(y^{n+1}_{\delta_{n+1}}, x^n_{\varphi_n}) \le S\left(y^n_{\delta_n}, x^{n+1}_{\varphi_n}\right) = \beta y^{n+1}_{\delta_{n+1}}$

From (2.2.2) and (2.2.3) we obtain $\beta x^{n+1}_{\varphi_{n+1}} \le \beta x^{n+2}_{\varphi_{n+1}}$ and $\beta y^{n+1}_{\delta_{n+1}} \ge \beta y^{n+2}_{\delta_{n+2}}$. Thus by mathematical induction we conclude that (2.2.4) and (2.2.5) holds for all $n \ge 0$. Continuing this process one can easily verify that

$\beta x_{\varphi_0} \le \beta x^1_{\varphi_1} \le \beta x^2_{\varphi_2} \le \beta x^3_{\varphi_3} \le \cdots . \beta x^n_{\varphi_n} \le \beta x^{n+1}_{\varphi_{n+1}} \le \cdots \ldots$ (2.2.6)

$\beta y_{\delta_0} \geq \beta y^1_{\delta_1} \geq \beta y^2_{\delta_2} \geq \beta y^3_{\delta_3} \ldots\ldots \geq \beta y^n_{\delta_n} \geq \beta y^{n+1}_{\delta_{n+1}} \geq \cdots \ldots$
(2.2.7)

Now if $(x^{n+1}_{\varphi_{n+1}}, y^{n+1}_{\delta_{n+1}}) = (x^n_{\varphi_n}, y^n_{\delta_n})$,then S and β have coupled soft coincidence point.
So assume $(x^{n+1}_{\varphi_{n+1}}, y^{n+1}_{\delta_{n+1}}) \neq (x^n_{\varphi_n}, y^n_{\delta_n})$ for all n ≥ 0.i.e. we assume that either
$\beta x^{n+1}_{\varphi_{n+1}} = (x^n_{\varphi_n}, y^n_{\delta_n}) \neq \beta x^n_{\varphi_n}$ or $\beta y^{n+1}_{\delta_{n+1}} = S\left(y^n_{\delta_n}, x^n_{\varphi_n}\right) \neq \beta y^n_{\delta_n}$.

Again let p and n be two positive integer such that p> n then we can write

$$\tilde\rho\left(\beta x^n_{\varphi_n}, \beta x^p_{\varphi_p}\right) \leq$$
$$s\{\tilde\rho(\beta x^n_{\varphi_n}, \beta x^{n+1}_{\varphi_{n+1}})\} + s\{\tilde\rho(\beta x^{n+1}_{\varphi_{n+1}}, \beta x^p_{\varphi_p})\} \leq$$
$$s\{\tilde\rho(\beta x^n_{\varphi_n}, \beta x^{n+1}_{\varphi_{n+1}})\} + s^2\{\tilde\rho(\beta x^{n+1}_{\varphi_{n+1}}, \beta x^{n+2}_{\varphi_{n+2}})\} +$$
$$s^3\{\tilde\rho(\beta x^{n+2}_{\varphi_{n+2}}, \beta x^{n+3}_{\varphi_{n+3}})\} + \cdots . s^{p-n-1}\{\tilde\rho(\beta x^{p+1}_{\varphi_{p+1}}, \beta x^n_{\varphi_n})\}.$$

Similarly

$$\tilde\rho\left(\beta y^n_{\delta_n}, \beta y^p_{\delta_p}\right) \leq$$
$$\tilde\rho\{s(\beta y^n_{\delta_n}, \beta y^{n+1}_{\delta_{n+1}})\} + s^2\{(\tilde\rho(\beta y^{n+1}_{\delta_{n+1}}, \beta y^{n+2}_{\delta_{n+2}})\} +$$
$$\cdots s^{p-n-1}\{(\tilde\rho(\beta y^{p-1}_{\delta_{p-1}}, \beta y^p_{\delta_p})\}.$$

Therefore

$$\tilde\rho\left(\beta x^n_{\varphi_n}, \beta x^p_{\varphi_p}\right) + \tilde\rho\left(\beta y^n_{\delta_n}, \beta y^p_{\delta_p}\right)$$
$$\leq sd_n + s^2 d_{n+1} + \cdots . + s^{p-n-1} d_{p-1}$$
$$\leq sk^n d_0 + s^2 k^{n+1} d_0 + \cdots . + s^{p-n-1} k^n d_0$$
$$\leq sk^n d_0 \frac{1}{1-sk} \to 0 \text{ ; as } n \to \infty$$

Hence $\{\beta x^n_{\varphi_n}\}$ and $\{\beta y^n_{\delta_n}\}$ are two Cauchy sequences in βX and βX is complete.

Thus there exist two soft point say x_φ, y_δ in X such that $\{\beta x^n_{\varphi_n}\} = \beta x_\varphi = \Omega$
and
that $\{\beta x^n_{\varphi_n}\} \to \beta x_\varphi = \xi$ as $n \to \infty$. Hence S is complete and so
$\beta\{\beta x^{n+1}_{\varphi_{n+1}}\} = \beta\left(S(\beta x^n_{\varphi_n}, \beta y^n_{\delta_n})\right) = S(\beta x^n_{\varphi_n}, \beta y^n_{\delta_n})$ (Since S andβ are commutative)
$\beta(\Omega) = S(\Omega, \xi)$ (Since S andβ are continuous)

Similarly we can show that $\beta(\Omega) = S(\Omega, \xi)$ .Thus $S(\Omega, \xi)$ is point of coincidence for S and β.Again le we getthat holds,by (2.2.6) $\{\beta x^n_{\varphi_n}\}$ is a non decreasing sequence and $\beta x^n_{\varphi_n} \to \Omega$, therefore $\beta x^n_{\varphi_n} \leq \Omega$ for all n.Similarly by (2.2.7) we get that $\{\beta y^n_{\delta_n}\}$ is non increasing sequence and
$\beta y^n_{\delta_n} \to \xi$, so $\beta y^n_{\delta_n} \geq \xi$ for all n,
then

$$\tilde\rho(\beta(\Omega), S(\Omega, \xi)) \leq s\tilde\rho$$
$$s\left(\beta(\Omega), \beta\beta x^{n+1}_{\varphi_{n+1}}\right) + s\,\tilde\rho\left(\beta\beta x^{n+1}_{\varphi_{n+1}}, S(\Omega, \xi)\right)$$
$$s\left(\beta(\Omega), \beta\beta x^{n+1}_{\varphi_{n+1}}\right) + s\tilde\rho\left(\beta(S(x^n_{\varphi_n}, y^n_{\delta_n})), S(\Omega, \xi)\right)$$
$$s\left(\beta(\Omega), \beta\beta x^{n+1}_{\varphi_{n+1}}\right) + s\,\tilde\rho\left(\beta(S(\beta x^n_{\varphi_n}, \beta y^n_{\delta_n})), S(\Omega, \xi)\right)$$
$$\leq \left(\beta(\Omega), \beta\beta x^{n+1}_{\varphi_{n+1}}\right) + s\tilde\rho\left(\beta(S(\beta x^n_{\varphi_n}, \beta y^n_{\delta_n})), S(\Omega, \xi)\right)$$
$$+ s\tilde\rho\left((S(\beta y^n_{\delta_n}, \beta x^n_{\varphi_n})), S(\Omega, \xi)\right)$$
$$\leq \left(\beta(\Omega), \beta\beta x^{n+1}_{\varphi_{n+1}}\right) + sk\left\{\beta\left(\beta\beta x^n_{\varphi_n}, \beta\Omega\right) + s\tilde\rho\left(\beta\beta y^n_{\delta_n}, \beta\xi\right)\right\}$$
(2.2.8)

Since β is continuous ,$\beta\, \beta x^n_{\varphi_n} \to \beta\Omega$ and $\beta\beta y^n_{\delta_n} \to \beta\,\xi$ and hence the right hand side of equation (2.2.8) becomes zero as $n \to \infty$.

Thus $\beta(\Omega) = S(\Omega, \xi)$, similarly we can show that $\beta(\xi) = S(\xi, \Omega)$.

$$\tilde\rho\,(\beta(\Omega), S(\Omega, \xi)$$
$$\tilde\rho(\beta(\Omega), S(\Omega, \xi)) + \tilde\rho(\beta\xi, \Omega)$$
$$= \tilde\rho(S(\Omega, \xi), S(\xi, \Omega))$$
$$+ \tilde\rho(S(\xi, \Omega), S(\Omega, \xi)$$
$$\leq k\{\tilde\rho\{(\beta\Omega, \beta\xi) + \tilde\rho\{(\beta\xi, \beta\Omega)\}$$
$$2\tilde\rho\{(\beta\Omega, \beta\xi) \leq 2k\tilde\rho\{(\beta\Omega, \beta\xi) \to \tilde\rho\{(\beta\Omega, \beta\xi) \leq k\tilde\rho\{(\beta\Omega, \beta\xi)$$

Since $k < \frac{1}{s}$;
$\tilde\rho\{(\beta\Omega, \beta\xi) = 0$. Thus $\beta\Omega = \beta\xi$, Hence $S(\Omega, \xi) = S(\Omega) = \beta(\xi) = S(\xi, \Omega)$

Finally

$$\tilde\rho(\Omega, \beta\Omega) \leq s\tilde\rho\left(\Omega, \beta x^{n+1}_{\varphi_{n+1}}\right) + s\tilde\rho\left(\beta x^{n+1}_{\varphi_{n+1}}, \beta\Omega\right) +$$
$$s\tilde\rho\left(\Omega, \beta x^{n+1}_{\varphi_{n+1}}\right) + s\tilde\rho\left(S\left(x^n_{\varphi_n}, y^n_{\delta_n}\right), S(\Omega, \xi)\right)$$

In the same way

$$\tilde\rho(\xi, \beta\xi) \leq s\tilde\rho\left(\xi, \beta y^{n+1}_{\varphi_{n+1}}\right) + s\tilde\rho\left(\beta y^{n+1}_{\varphi_{n+1}}, \beta\xi\right)$$
$$\leq s\tilde\rho\left(\xi, \beta y^{n+1}_{\varphi_{n+1}}\right)$$
$$+ s\tilde\rho\left(S\left(y^n_{\delta_n}, x^n_{\varphi_n}\right), S(\xi, \Omega)\right)$$

Therefore
$(1-ks^2)[\tilde\rho(\Omega, \beta\Omega) + \tilde\rho(\xi, \beta\xi)] \leq s\left\{\tilde\rho\left(\Omega, \beta x^{n+1}_{\varphi_{n+1}}\right) +\right.$
ρξ,βyφn+1n+1+

$$s^2\left\{\left(\beta y^n_{\varphi_n}, \Omega\right) + \left(\beta x^n_{\varphi_n}, \xi\right)\right\} \to 0 \ as \to \infty$$

Thus $\tilde\rho(\Omega, \beta\Omega) = 0 = \tilde\rho(\xi, \beta\xi)$
$\xi = \beta\xi$ and $\Omega = \beta\Omega$
$\beta y_\delta = S(y_\delta, y_\delta) = y_\delta$
This means that S and β have a common soft fixed point.

3. Conclusion

In this paper the investigations concerning the existence of coupled soft fixed point theorem for a contractive condition with monotone property and $\beta -$ monotony property in an ordered soft b- metric space are established.

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