Double Diffusive Convection in a Layer of Saturated Rotating Couple-Stress Nanofluid in a Porous Medium

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Abstract: A linear stability analysis is performed on Double-diffusive instability problem in a layer of rotating Couple-stressed Nanofluid saturated with a porous medium. The fluid layer is perceived to be heated and courtesy from below. The normal mode technique and Galerkin-type weighted method are used to obtained are exact solution for the rotating Couple-stress nanofluid layer between two free boundaries. The effect of different non dimensional parameters such as modified Taylor number, couple-stress parameter, Rayleigh Number, thermo-nanofluid Lewis number, modified diffusivity ratio, thermo-solutal Lewis number etc. on the stationary convection have been investigated. The Rayleigh number is determined analytically and numerically at the onset of instability. In the case of stationary convection, as the Taylor number $T_a$ increase so does the Rayleigh number $R_a$ also increase which means that a nanofluid, in rotation is stable state. Graph also has been plotted between Rayleigh number $R_a$ and the wave number $\alpha$ for $T_a$.

Keywords: double-diffusive convection, rotation, couple-stress, nanofluid, porous medium

1. Introduction

The double-diffusive convection in the rotating couple stress nanofluid applications in the fields of chemical science, food processing, nuclear engineering and industries, geophysics, bioengineering and cancer treatment, biofluidic motion, oceanography [1-6]. Choi and Eastman first introduced the term nanofluid. Nanofluids are colloidal suspension in the range of 1-100 nm nanosheet particles in the base fluid such as water, oil, ethylene glycol etc [7]. Nanoparticles are of materials such as metallic oxides (Al2 O3, CuO), nitride ceramics (AlN, SiN), carbide ceramics (SiC, TiC), metals (Cu, Ag, Au) etc. have been used for the preparation of nanofluids. Due to the small dimensions, the suspended nanoparticles can behave like a base fluid molecule, which helps us to reduce problems like particle clogging, sedimentation etc. The combination of highly stable and highly conductivity of the suspended nanoparticles which make them highly preferable for making heat transfer fluids [8-10]. Stokes [11] proposed and postulated the theory of couple-stress fluid. The application of couple-stress fluid is to the study of the mechanism of lubrication of synovial joints, which has become the main objective of scientific research and found that the synovial fluid in human joints behaves like a couple-stress fluid [12]. Sharma and Thakur [13] studied the couple-stress fluid heated from below in hydro magnetics and found that couple stress parameter has stabilizing effect on the stationary convection. The onset of convection in a porous medium layer saturated by a couple-stress nanofluid discussed by Umavathi [14]. A convective transport in nanofluid was studied by [12-20]. In this paper, more realistic boundary conditions are used, see [21-22]. We assume that there is no flux at the plate and the nanoparticle flux value adjust accordingly. There is a need of changing the scale of dimensionless parameters. The basic solution of nanoparticle volume fraction is changed. Keeping in view of various applications of couple-stress nanofluid, see [23-29], our main aim in the present paper is to study the effect of cross-diffusion rotating couple stress parameter on the onset of convection in a horizontal layer of nanofluid saturating a porous medium. This problem is of triple-diffusion type as it contains nanoparticles, heat and the solute.

2. Mathematical Formulation

We consider an infinite horizontal porous medium layer of thickness $d$, bounded by the planes $z = 0$ and $z = d$ of a rotating couple stress nanofluid as shown in Fig. 1. The gravity force $g = (0, 0, -g)$ is acted aligned in the $z$ direction of the layer which is heated and soluted from below. Let $T_0$, $C_0$ and $\phi_0$ be the constant values of temperature, concentration and the volumetric fraction of nanoparticles at the lower boundary and $T_1$, $C_1$ and $\phi_1$ be the constant values of temperature, concentration and the volumetric fraction of nanoparticles at the upper boundary. We know that keeping a constant volume fraction of nanoparticles at the horizontal boundaries will be almost impossible in a realistic situation.

Figure1: Schematic sketch of physical situational
\[ \nabla \cdot \mathbf{q} = 0 \]  

\[ \frac{\rho}{\varepsilon} \left( \frac{\partial}{\partial t} + \frac{1}{\varepsilon} \mathbf{q} \cdot \nabla \right) \mathbf{q} = -\nabla p - \frac{1}{k_i} \left( \mu - \mu_i \nabla^2 \right) \mathbf{q} + \frac{2 \rho}{\varepsilon} (q \times \Omega) + \left( \phi \phi_b + (1 - \phi) \rho_f \left[ 1 - \alpha_r (T - T_0) - \alpha_c (C - C_0) \right] \right) g \]  

\[ \frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon} \mathbf{q} \cdot \nabla \phi = D_b \nabla^2 \phi + \frac{D_T}{T_i} \nabla^2 T \]  

\[ (\rho c)_m \frac{\partial}{\partial t} + (\rho c)_f \mathbf{q} \cdot \nabla T = k_m \nabla^2 T + \varepsilon (\rho c)_p \left( D_b \nabla \phi \cdot \nabla T + \frac{D_T}{T_i} \nabla T \cdot \nabla T \right) + (\rho c) D_{rc} \nabla^2 C \]  

\[ \frac{\partial C}{\partial t} + \frac{1}{\varepsilon} \mathbf{q} \cdot \nabla C = D_N \nabla^2 C + D_{CT} \nabla^2 T \]  

We assume that the temperature to be constant and the thermophoretic nanoparticles flux to be zero at the boundaries [27].

The boundary conditions relevant to the problem [38, 39] are

\[ w = 0, \frac{\partial w}{\partial z} = 0, T = T_0, \phi = \phi_0, C = C_0, D_b \frac{\partial \phi}{\partial z} + \frac{D_T}{T_i} \frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = 0 \]  

\[ w = 0, \frac{\partial w}{\partial z} = 0, T = T_1, \phi = \phi_1, C = C_1, D_b \frac{\partial \phi}{\partial z} + \frac{D_T}{T_i} \frac{\partial T}{\partial z} = 0 \quad \text{at} \quad z = 1 \]  

Introducing non-dimensional variables as

\[ (x', y', z') = \left( \frac{x, y, z}{d}, \frac{u', v', w'}{k_f} \right), (t', \phi', C') = \left( \frac{t, \phi, C}{k_f}, \frac{\rho c_f}{\rho c_m} \right), \]  

\[ k_f = \frac{k}{(\rho c)_f} \] is thermal diffusivity of the fluid and \( \sigma = \frac{(\rho c)_m}{(\rho c)_f} \) is thermal capacity ratio and we dropping the dashes for convenience.

In non-dimensional form, Eq. (1) - (8) can be written as

\[ \nabla \cdot \mathbf{q} = 0 \]  

\[ \gamma_\alpha \frac{\partial \mathbf{q}}{\partial t} = -\nabla p - \left( 1 - \eta \nabla^2 \right) \mathbf{q} + \sqrt{T_a} \left( q \cdot e_\alpha \right) - R_m \mathbf{e}_\alpha - R_\phi \phi \mathbf{e}_\alpha + R_T e_\alpha + \frac{R_c}{L_c} C e_\alpha \]  

\[ \frac{1}{\sigma} \frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon} \mathbf{q} \cdot \nabla \phi = \frac{1}{L_n} \nabla^2 \phi + \frac{N_b}{L_n} \nabla^2 T \]  

\[ \left( \frac{\partial T}{\partial t} + q \cdot \nabla T \right) = \nabla^2 T + \frac{N_b}{L_n} \nabla \phi \cdot \nabla T + \frac{N_a N_b}{L_n} \nabla T \cdot \nabla T + N_{TC} \nabla^2 C \]  

\[ \frac{1}{\sigma} \frac{\partial C}{\partial t} + \frac{1}{\varepsilon} \mathbf{q} \cdot \nabla C = \frac{1}{L_c} \nabla^2 C + N_{CT} \nabla^2 T \]
\[ w = 0, \frac{\partial w}{\partial z} = 0, T = 1, C = 1, \phi = 0 \quad \text{at} \quad z = 0 \quad \text{(14)} \]

\[ w = 0, \frac{\partial w}{\partial z} = 0, T = 0, C = 0, \phi = 1 \quad \text{at} \quad z = 1 \quad \text{(15)} \]

Where, \( \gamma_{a} = \frac{\varepsilon}{\sigma V_{a}} \); (nondimensional acceleration coefficient), \( V_{a} = \frac{\varepsilon^{2} P_{r}}{D_{a}} \); (Vadasz number), \( P_{r} = \frac{\mu}{\rho k_{f}} \); (Prandtl number), \( D_{a} = \frac{k_{l}}{d^{2}} \); (Darcy number), \( \eta = \frac{\mu_{c}}{\mu d^{2}} \); (Couple stress parameter), \( T_{a} = \left( \frac{2 \eta k_{l} \Omega}{\varepsilon_{\mu}} \right)^{2} \); (Modified Tayler number), \( R_{m} = \left( \rho_{p} \phi_{0} + \rho (1 - \phi_{b}) \right) k_{l} g d \frac{\mu k_{f}}{\mu k_{f}} \); (Basic Density Rayleigh number), \( R_{n} = \frac{(1 - \phi_{b})(T_{0} - T_{b}) \rho \alpha_{b} k_{l} d}{\mu k_{f}} \); (Rayleigh Number), \( R_{s} = \frac{(1 - \phi_{b})(C_{0} - C_{b}) \rho \alpha_{c} d k_{l} g}{\mu D_{s}} \); (Concentration Rayleigh Number), \( R_{n} = \frac{(1 - \phi_{b})(T_{0} - T_{b}) \rho \alpha_{b} k_{l} d}{\mu k_{f}} \); (Rayleigh Number), \( R_{s} = \frac{(1 - \phi_{b})(C_{0} - C_{b}) \rho \alpha_{c} d k_{l} g}{\mu D_{s}} \); (Solute Rayleigh Number), \( L_{c} = \frac{k_{l}}{D_{s}} \); (Thermo-solute Lewis Number), \( L_{a} = \frac{k_{l}}{D_{a}} \); (Thermo-nanofluid Lewis Number), \( N_{a} = \frac{D_{f}(T_{0} - T_{b})}{D_{f} T_{b} \phi_{0}} \); (Modify diffusivity ratio), \( N_{b} = \frac{\varepsilon (\rho c)_{f} \phi_{0}}{(\rho c)_{f}} \); (Modify nano particle density ratio), \( N_{c} = \frac{D_{f}(C_{0} - C_{b})}{k_{f}(T_{0} - T_{b})} \); (Dufour parameter), \( N_{cr} = \frac{D_{cf}(T_{0} - T_{b})}{k_{f}(C_{0} - C_{b})} \); (Soret parameter)

### 3. Basic State and Perturbation Solutions

We assume a quiescent basic state by following Kuznetsov and Nield (2010a, b) and Chand and Rana (2012b) that verifies

\[ u = v = w = 0, p = p_{b}(z), C = C_{b}(z), T = T_{b}(z), \phi = \phi_{b}(z) \quad \text{(16)} \]

Substituting the basic state defined in (16) into Eq. (9) to (13), the basic solution is found to be

\[ T_{b} = 1 - z, C_{b} = 1 - z, \phi_{b} = z \quad \text{(17)} \]

These results are identical with the results obtained by Kuznetsov and Nield (2010a) and Nield and Kuznetsov (2011).

We superimposed infinite small small perturbations on the basic state

\[ q(u, v, w) = q^{+}(u, v, w), T = T_{b} + T^{+}, C = C_{b} + C^{+}, \phi = \phi_{b} + \phi^{+}, p = p_{b} + p^{+} \quad \text{(18)} \]

Using perturb solution given in eq. (18) in eq. (9) to (15) and linearizing the resulting equations by neglecting non liner terms that are product of prime quantities and dropping the double dashes for convenience, we obtained

\[ \nabla \cdot q = 0 \quad \text{(19)} \]

\[ \gamma_{a} \frac{\partial q}{\partial t} = -\nabla p - (1 - \eta \nabla^{2}) q + \sqrt{\sqrt{T_{a}} \left( q \times e_{\xi} \right)} + R_{a} T e_{\xi} + \frac{R_{c}}{L_{c}} C e_{\xi} - R_{a} \phi e_{\xi} \quad \text{(20)} \]

\[ \frac{1}{\sigma} \frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon} w = \frac{1}{L_{n}} \nabla^{2} \phi + \frac{N_{b}}{L_{n}} \nabla^{2} T \quad \text{(21)} \]
\[
\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{N_B}{L_n} \left( \frac{\partial T}{\partial z} + \frac{\partial \phi}{\partial z} \right) - \frac{2N_A N_B}{L_n} \frac{\partial T}{\partial z} + N_{TC} \nabla^2 C
\]  
(22)

\[
\frac{1}{\sigma} \frac{\partial C}{\partial t} - \frac{1}{\varepsilon} w = \frac{1}{L_e} \nabla^2 C + N_{CT} \nabla^2 T
\]  
(23)

\[ w = 0, T = 0, C = 0, \phi = 0, \frac{\partial \phi}{\partial z} + N_A \frac{\partial T}{\partial z} = 0 \text{ at } z = 0 \text{ and } z = 1 \]  
(24)

The parameter \( R_m \) is not involved in Eq. (19) - (23), it is just a measure of the basic static pressure gradient. The seven unknown’s \( u, v, w, T, C \) and \( \phi \) can be reduced to four by operating Eq. (20) with \( \mathbf{e} \cdot \text{curl} \), which yields

\[
\gamma_a \frac{\partial}{\partial t} \nabla^2 w + \left(1 - \eta \nabla^2 \right) \nabla^2 w = R_a \nabla_u^2 T - \frac{R_s}{L_e} \nabla_u^2 C + R_a \nabla_u^2 \phi + \sqrt{T_a} \frac{\partial \xi}{\partial z}
\]  
(25)

Where \( \nabla_u^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \) is the 2-D Laplace operator on the horizontal plane and \( \xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \) is the \( z \)-component of vorticity.

Operating Eq. (20) by \( \mathbf{e} \cdot \text{curl} \), we get

\[
\left[ \gamma_a \frac{\partial}{\partial t} \left(1 - \eta \nabla^2 \right) \right] \xi = \sqrt{T_a} \frac{\partial w}{\partial z}
\]  
(26)

Eliminating \( \xi \) from Eq. (25) by using Eq. (26), we get

\[
\gamma_a \frac{\partial}{\partial t} \nabla^2 w + \left(1 - \eta \nabla^2 \right) \nabla^2 w = R_a \nabla_u^2 T + \frac{R_s}{L_e} \nabla_u^2 C - R_a \nabla_u^2 \phi - T_a \left\{ \gamma_a \frac{\partial}{\partial t} \left(1 - \eta \nabla^2 \right) \right\}^{-1} \frac{\partial^2 w}{\partial z^2}
\]  
(27)

4. Normal Mode Analysis

Express the disturbances into normal modes of the form

\[
[w, \phi, T, C] = [W(z), \phi(z), \Theta(z), \Gamma(z)] \exp(i lx + imy + n t)
\]  
(28)

Where \( l, m, \) are the wave numbers in the \( x \) and \( y \) direction resp. and \( n \) is the growth rate of the disturbances.

Substituting equation (28) into equation (27) and (19)- (23), we find the following eigenvalue problems

\[
\left( D^2 - a^2 \right) \left\{ \gamma_a n + \left(1 - \eta \left(D^2 - a^2\right) \right) \right\}^2 + T_a D^2 \left[ W(z) + a^2 R_a \left\{ \gamma_a n + \left(1 - \eta \left(D^2 - a^2\right) \right) \right\} \right. \\
+ \frac{R_s}{L_e} a^2 \left\{ \gamma_a n + \left(1 - \eta \left(D^2 - a^2\right) \right) \right\} \right] \Theta(z) - a^2 R_a \left\{ \gamma_a n + \left(1 - \eta \left(D^2 - a^2\right) \right) \right\} \phi(z) = 0
\]  
(29)

\[
\frac{1}{\varepsilon} W(z) + N_{CT} \left( D^2 - a^2 \right) \Theta(z) + \left\{ \frac{1}{L_e} \left(D^2 - a^2\right) - \frac{n}{\sigma} \right\} \Theta(z) = 0
\]  
(30)

\[
W(z) + \left( D^2 - a^2 - n + \frac{N_B}{L_n} D - \frac{2N_A N_B}{L_n} D \right) \Theta(z) - \frac{N_B}{L_n} D \phi(z) + N_{TC} \left( D^2 - a^2 \right) \Gamma(z) = 0
\]  
(31)
\[
\frac{1}{\varepsilon} W(z) - \frac{N_A}{L_n} \left( D^2 - a^2 \right) \Theta(z) - \left( \frac{1}{L_n} \left( D^2 - a^2 \right) - \frac{n}{\sigma} \right) \varphi(z) = 0
\]  
(32)

\[
W = 0, D^2 W = 0, \Gamma = 0, \Theta = 0, \varphi = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1
\]  
(33)

where \( D = \frac{d}{dz} \) and \( a^2 = l^2 + m^2 \) is the dimensionless horizontal wave number.

Consider the solution of the form

\[
W = W_0 \sin(\pi z), \quad \Theta = \Theta_0 \sin(\pi z), \quad \Gamma = \Gamma_0 \sin(\pi z), \quad \phi = \phi \sin(\pi z)
\]  
(34)

by putting the solution in equations (29)-(32) and integrate each equation, we obtained the following matrix equations

\[
\begin{bmatrix}
J^2 \left( \gamma_a \frac{n}{(1 + \eta J^2)^2} \right) + \pi^2 T_u - a^2 R_n \left( \gamma_a \frac{n}{(1 + \eta J^2)^2} \right) - R_s \left( \gamma_a \frac{n}{(1 + \eta J^2)^2} \right) \frac{a^2 R_n}{L_n} & W_0 \\
\frac{1}{\varepsilon} - N_C \gamma_a J^2 & - \left( \frac{J^2}{L_n} + \frac{n}{\sigma} \right) & 0 & \Theta_0 \\
1 & -J^2 N_C & 0 & \Gamma_0 \\
\frac{1}{\varepsilon} \frac{N_A}{L_n} & 0 & \left( \frac{J^2}{L_n} + \frac{n}{\sigma} \right) & \varphi_0
\end{bmatrix}
\]  
(35)

where \( J^2 = \pi^2 + a^2 \) is the total wave number.

5. The Stationary Convection

For stationary convection, we put \( n=0 \) in (35) and take determinant, we get

\[
R_n = \frac{1}{(\varepsilon - N_C L_n)} \left[ \frac{\varepsilon J^2}{a^2} \left\{ J^2 \left( \frac{n}{1 + \eta J^2} \right) + \frac{\pi^2 T_u}{1 + \eta J^2} \right\} \left\{ 1 - L_n N_C N_C \right\} + R_s \left( N_C \varepsilon - 1 \right) \right]
\]  
(36)

Eq. (36) represent the thermal Rayleigh number as a function of the dimensionless wave number \( a \) and the parameters \( N_C, N_C, L_n, R_n, N_n, T_u \) and the couple stress parameter \( \eta \). Also in Eq. (36) the modify nanoparticle ratio \( N_B \) does not appear and the diffusivity ratio parameter \( N_A \) appears only with the nanoparticle Rayleigh number \( R_n \). The result given in equation (36) is a good agreement with the result finned by Kuznetsov and Nield (2010) and Chand and Rana (2016) in the absence of the Dufour, Soret and stable solute gradient parameters \( N_C, N_C \) and \( R_s \) respectively. Eq. (36) is analyzed numerically to depict the stability characteristics with respect to rotation.
The above graph that as $T_u$ increases then Rayleigh number $R_u$ also increases. This means that a couple stress nanofluid, in presence of rotation, is stable with respect to the onset of stationary convection when both the boundaries are free. This result is in good agreement with the result obtained by Chand and Rana (2016).

6. Conclusion

We investigated the effect of rotation on the onset of double-diffusive convection in a layer of couple-stress Nanofluid and using a Galerkin type weighted method for solving the problem. The Rayleigh Number, thermo-nanofluid Lewis number and Soret parameter have stabilizing effects on the stationary convection which are in good agreement with the results as discussed by Kuznetsov and Nield (2010) and Rana et al. (2016). The thermo-solutal Lewis number and Dufour parameter has destabilizing effects on the stationary convection of the system which are identical with the results as derived by Kuznetsov and Nield and Chand and Rana (2016). Rotation has stabilizing effect on the stationary convection as shown in figure 2 which is in good agreement with the result derived by and Rana (2014).

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