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Double Diffusive Convection in a Layer of Saturated Rotating Couple-Stress Nanofluid in a Porous Medium

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Abstract: ***A linear stability analysis is performed on Double-diffusive instability problem in a layer of rotating Couple-stressed Nanofluid saturated with a porous medium. The fluid layer is perceived to be heated and courtesy from below. The normal mode technique and Galerkin-type weighted method are used to obtained are exact solution for the rotating Couple-stress nanofluid layer between two free boundaries. The effect of different non dimensional parameters such as modified Taylor number, couple-stress parameter, Rayleigh Number, thermo-nanofluid Lewis number, modified diffusivity ratio, thermo-solutal Lewis number etc. on the stationary convection have been investigated. The Rayleigh number is determined analytically and numerically at the onset of instability. In the case of stationary convection, as the Taylor number T_a increase so does the Rayleigh number R_a also increase which means that a nanofluid, in rotation is stable state. Graph also has been plotted between Rayleigh number R_a and the wave number a for T_a.***

Keywords: double-diffusive convection, rotation, couple-stress, nanofluid, porous medium

1.Introduction

The double-diffusive convection in the rotating couple stress nanofluid applications in the fields of chemical science, food processing, nuclear engineering and industries, geophysics, bioengineering and cancer treatment, biofluidic motion, oceanography [1-6]. Choi and Eastman first introduced the term nanofluid. Nanofluids are colloidal suspension in the range of 1-100 nm nanosheet particles in the base fluid such as water, oil, ethylene glycol etc [7]. Nanoparticles are of materials such as metallic oxides (Al2 O3, CuO), nitride ceramics (AlN, SiN), carbide ceramics (SiC, TiC), metals (Cu, Ag, Au) etc. have been used for the preparation of nanofluids. Due to the small dimensions, the suspended nanoparticles can behave like a base fluid molecule, which helps us to reduce problems like particle clogging, sedimentation etc. The combination of highly stable and highly conductivity of the suspended nanoparticles which make them highly preferable for making heat transfer fluids [8-10]. Stokes [11] proposed and postulated the theory of couple-stress fluid. The application of couple-stress fluid is to the study of the mechanism of lubrication of synovial joints, which has become the main objective of scientific research and found that the synovial fluid in human joints behaves like a couple-stress fluid [12]. Sharma and Thakur [13] studied the couple-stress fluid heated from below in hydro magnetics and found that couple stress parameter has stabilizing effect on the stationary convection. The onset of convection in a porous medium layer saturated by a couple-stress nanofluid discussed by Umavathi [14]. A convective transport in nanofluid was studied by [12-20]. In this paper, more realistic boundary conditions are used, see [21-22]. We assume that there is no flux at the plate and the nanoparticle flux value adjust accordingly. There is a need of changing the scale of dimensionless parameters. The basic solution of nanoparticle volume fraction is changed. Keeping in view of various applications of couple-stress nanofluid, see [23-29], our main aim in the present paper is to study the effect of cross-diffusion rotating couple stress parameter on the onset of convection in a horizontal layer of nanofluid saturating a porous medium. This problem is of triple-diffusion type as it contains nanoparticles, heat and the solute.

2.Mathematical Formulation

We consider an infinite horizontal porous medium layer of thickness d, bounded by the planes z = 0 and z = d of a rotating couple stress nanofluid as shown in Fig. 1. The gravity force g = (0, 0, -g) is acted aligned in the z direction of the layer which is heated and soluted from below. Let T_0, C_0 and ϕ_0 be the constant values of temperature, concentration and the volumetric fraction of nanoparticles at the lower boundary and T_1, C_1 and ϕ_1 be the constant values of temperature, concentration and the volumetric fraction of nanoparticles at the upper boundary. We know that keeping a constant volume fraction of nanoparticles at the horizontal boundaries will be almost impossible in a realistic situation.

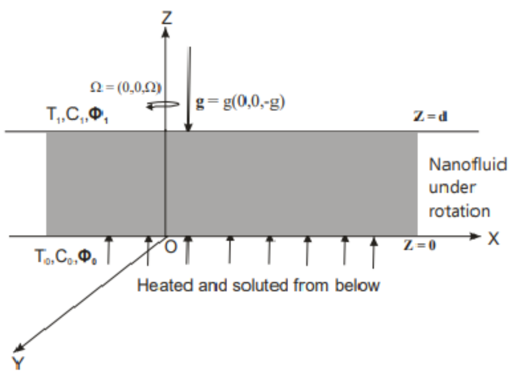

Figure1: Schematic sketch of physical situational

$$\nabla \cdot q = 0 \tag{1}$$

$$\frac{\rho}{\varepsilon}\left(\frac{\partial}{\partial t}+\frac{1}{\varepsilon}q\cdot\nabla\right)q = -\nabla p - \frac{1}{k_1}\left(\mu-\mu_c\nabla^2\right)q + \frac{2\rho}{\varepsilon}\left(q\times\Omega\right) + \left(\phi\rho_b+\left(1-\phi\right)\rho_f\left\{1-\alpha_T\left(T-T_0\right)-\alpha_C\left(C-C_0\right)\right\}\right)g \tag{2}$$

$$\frac{\partial\phi}{\partial t}+\frac{1}{\varepsilon}q\cdot\nabla\phi = D_B\nabla^2\phi+\frac{D_T}{T_1}\nabla^2 T \tag{3}$$

$$\left(\rho c\right)_m\frac{\partial}{\partial t}+\left(\rho c\right)_f q\cdot\nabla T = k_m\nabla^2 T+\varepsilon\left(\rho c\right)_p\left(D_B\nabla\phi\cdot\nabla T+\frac{D_T}{T_1}\nabla T\cdot\nabla T\right)+\left(\rho c\right)D_{TC}\nabla^2 C \tag{4}$$

$$\frac{\partial C}{\partial t}+\frac{1}{\varepsilon}q\cdot\nabla C = D_S\nabla^2 C+D_{CT}\nabla^2 T \tag{5}$$

We assume that the temperature to be constant and the thermophoretic nanoparticles flux to be zero at the boundaries [27].

The boundary conditions relevant to the problem [38, 39] are

$$w=0,\frac{\partial w}{\partial z}=0,\mathrm{T}=\mathrm{T}_0,\phi=\phi_0,C=C_0,D_B\frac{\partial\phi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{\partial z}=0 \quad at\ z=0 \tag{6}$$

$$\mathrm{w}=0,\frac{\partial w}{\partial z}=0,\mathrm{T}=\mathrm{T}_1,\phi=\phi_1,C=C_1,D_B\frac{\partial\phi}{\partial z}+\frac{D_T}{T_1}\frac{\partial T}{\partial z}=0 \quad at\ z=1 \tag{7}$$

Introducing non-dimensional variables as

$$\left(x',y',z'\right)=\left(\frac{x,y,z}{d}\right),\left(u',v'w'\right)=\left(\frac{u,v,w}{k_f}\right)d,t'=\frac{tk_f}{\sigma d^2},p'=\frac{pk_1}{\mu k_f}$$

$$\phi'=\frac{\phi-\phi_0}{\phi_0},T'=\frac{T-T_0}{T_0-T_1},C'=\frac{C-C_0}{C_0-C_1} \tag{8}$$

Where $k_f=\dfrac{k}{\left(\rho c\right)_f}$ is thermal diffusivity of the fluid and $\sigma=\dfrac{\left(\rho c_p\right)_m}{\left(\rho c_p\right)_f}$ is thermal capacity ratio and we dropping the dashes for convenience.

In non-dimensional form, Eq. (1)- (8) can be written as

$$\nabla\cdot q=0 \tag{9}$$

$$\gamma_a\frac{\partial q}{\partial t}=-\nabla p-\left(1-\eta\nabla^2\right)q+\sqrt{T_a}\left(q\times\hat{e}_z\right)-R_m\hat{e}_z-R_n\phi\hat{e}_z+R_aT\hat{e}_z+\frac{R_s}{L_e}C\hat{e}_z \tag{10}$$

$$\frac{1}{\sigma}\frac{\partial\phi}{\partial t}+\frac{1}{\varepsilon}q\cdot\nabla\phi=\frac{1}{L_n}\nabla^2\phi+\frac{N_A}{L_n}\nabla^2 T \tag{11}$$

$$\left(\frac{\partial T}{\partial t}+q\cdot\nabla T\right)=\nabla^2 T+\frac{N_B}{L_n}\nabla\phi\cdot\nabla T+\frac{N_AN_B}{L_n}\nabla T\cdot\nabla T+N_{TC}\nabla^2 C \tag{12}$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t}+\frac{1}{\varepsilon}q\cdot\nabla C=\frac{1}{L_e}\nabla^2 C+N_{CT}\nabla^2 T \tag{13}$$

$$w=0,\frac{\partial w}{\partial z}=0,T=1,C=1,\phi=0 \quad at\ z=0 \tag{14}$$

$$w=0,\frac{\partial w}{\partial z}=0,T=0,C=0,\phi=1 \quad at\ z=1 \tag{15}$$

Where, $\gamma_a=\frac{\varepsilon}{\sigma V_a}$; (nondimensional acceleration coefficient), $V_a=\frac{\varepsilon^2 P_r}{D_a}$; (Vadasz number), $P_r=\frac{\mu}{\rho k_f}$; (Prandtl number), $D_a=\frac{k_1}{d^2}$; (Darcy number), $\eta=\frac{\mu_c}{\mu d^2}$; (Couple stress parameter), $T_a=\left(\frac{2\rho k_f k_1\Omega}{\varepsilon\mu}\right)^2$; (Modified Tayler number), $R_m=\frac{\left(\rho_p\phi_0+\rho\left(1-\phi_0\right)\right)k_1 gd}{\mu k_f}$; (Basic Density Rayleigh number), $R_n=\frac{\left(\rho_p-\rho\right)k_1\phi_0 gd}{\mu k_f}$; (Concentration Rayleigh Number), $R_a=\frac{\left(1-\phi_0\right)\left(T_0-T_1\right)\rho\alpha_T k_1 d}{\mu k_f}$; (Rayleigh Number), $R_S=\frac{\left(1-\phi_0\right)\left(C_0-C_1\right)\rho\alpha_C dk_1 g}{\mu D_s}$; (Solute Rayleigh Number), $L_e=\frac{k_f}{D_S}$; (Thermo-solute Lewis Number), $L_n=\frac{k_f}{D_B}$; (Thermo-nanofluid Lewis Number), $N_A=\frac{D_T\left(T_0-T_1\right)}{D_B T_1\phi_0}$; (Modify diffusivity ratio), $N_B=\frac{\varepsilon\left(\rho c\right)_p\phi_0}{\left(\rho c\right)_f}$; (Modify nano particle density ratio), $N_{TC}=\frac{D_{TC}\left(C_0-C_1\right)}{k_f\left(T_0-T_1\right)}$; (Dufour parameter), $N_{CT}=\frac{D_{CT}\left(T_0-T_1\right)}{k_f\left(C_0-C_1\right)}$; (Soret parameter)

3.Basic State and Perturbation Solutions

We assume a quiescent basic state by following Kuznetsov and Nield (2010a, b) and Chand and Rana (2012b) that verifies

$$u=v=w=0,\ p=p_b\left(z\right),C=C_b\left(z\right),T=T_b\left(z\right),\phi=\phi_b\left(z\right) \tag{16}$$

Substituting the basic state defined in (16) into Eq. (9) to (13), the basic solution is found to be

$$T_b=1-z,\ C_b=1-z,\ \phi_b=z \tag{17}$$

These results are identical with the results obtained by Kuznetsov and Nield (2010a) and Nield and Kuznetsov (2011).
We superimposed infinite simal small perturbations on the basic state

$$q\left(u,v,w\right)=q''\left(u,v,w\right),T=T_b+T'',C=C_b+C'',\phi=\phi_b+\phi'',\ p=p_b+p'' \tag{18}$$

Using perturb solution given in eq. (18) in eq. (9) to (15) and linearizing the resulting equations by neglecting non liner terms that are product of prime quantities and dropping the double dashes for convenience, we obtained

$$\nabla\cdot q=0 \tag{19}$$

$$\gamma_a\frac{\partial q}{\partial t}=-\nabla p-\left(1-\eta\nabla^2\right)q+\sqrt{T_a}\left(q\times\hat{e}_z\right)+R_a T\hat{e}_z+\frac{R_S}{L_e}C\hat{e}_z-R_n\phi\hat{e}_z \tag{20}$$

$$\frac{1}{\sigma}\frac{\partial\phi}{\partial t}+\frac{1}{\varepsilon}w=\frac{1}{L_n}\nabla^2\phi+\frac{N_A}{L_n}\nabla^2 T \tag{21}$$

$$\frac{\partial T}{\partial t}-w=\nabla^2 T+\frac{N_B}{L_n}\left(\frac{\partial T}{\partial z}-\frac{\partial \phi}{\partial z}\right)-\frac{2N_A N_B}{L_n}\frac{\partial T}{\partial z}+N_{TC}\nabla^2 C \tag{22}$$

$$\frac{1}{\sigma}\frac{\partial C}{\partial t}-\frac{1}{\varepsilon}w=\frac{1}{L_e}\nabla^2 C+N_{CT}\nabla^2 T \tag{23}$$

$$w=0, T=0, C=0, \phi=0, \frac{\partial \phi}{\partial z}+N_A\frac{\partial T}{\partial z}=0 \ \ at \ \ z=0 \ \ and \ \ z=1 \tag{24}$$

The parameter *Rm* is not involved in Eq. (19) - (23), it is just a measure of the basic static pressure gradient. The seven unknown's u, v, w, p, T, C and φ can be reduced to four by operating Eq. (20) with $\vec{e}_z\cdot$ curl curl, which yields

$$\gamma_a\frac{\partial}{\partial t}\nabla^2 w+\left(1-\eta\nabla^2\right)\nabla^2 w=R_a\nabla_H{}^2 T-\frac{R_S}{L_e}\nabla_H{}^2 C+R_n\nabla_H{}^2\phi+\sqrt{T_a}\frac{\partial \xi}{\partial z} \tag{25}$$

Where $\nabla_H{}^2=\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}$ is the 2-D Laplace operator on the horizontal plane and $\xi=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ is the z-component of vorticity.

Operating Eq. (20) by $\vec{e}_z\cdot curl$, we get

$$\left[\gamma_a\frac{\partial}{\partial t}+\left(1-\eta\nabla^2\right)\right]\xi=\sqrt{T_a}\frac{\partial w}{\partial z} \tag{26}$$

Eliminating ξ from Eq. (25) by using Eq. (26), we get

$$\gamma_a\frac{\partial}{\partial t}\nabla^2 w+\left(1-\eta\nabla^2\right)\nabla^2 w=R_a\nabla_H{}^2 T+\frac{R_S}{L_e}\nabla_H{}^2 C-R_n\nabla_H{}^2\phi-T_a\left\{\gamma_a\frac{\partial}{\partial t}+\left(1-\eta\nabla^2\right)\right\}^{-1}\frac{\partial^2 w}{\partial z^2} \tag{27}$$

4.Normal Mode Analysis

Express the disturbances into normal modes of the form

$$\left[w,\phi,T,C\right]=\left[W(z),\varphi(z),\Theta(z),\Gamma(z)\right]\exp\left(ilx+imy+nt\right) \tag{28}$$

Where *l, m,* are the wave numbers in the x and y direction resp. and n is the growth rate of the disturbances.

Substituting equation (28) into equation (27) and (19)- (23), we find the following eigenvalue problems

$$\begin{aligned}&\left[\left(D^2-a^2\right)\left\{\gamma_a n+\left(1-\eta\left(D^2-a^2\right)\right)\right\}^2+T_a D^2\right]W(z)+a^2 R_a\left\{\gamma_a n+\left(1-\eta\left(D^2-a^2\right)\right)\right\}\Theta(z)\\&+\frac{R_S}{L_e}a^2\left\{\gamma_a n+\left(1-\eta\left(D^2-a^2\right)\right)\right\}\Gamma(z)-a^2 R_n\left\{\gamma_a n+\left(1-\eta\left(D^2-a^2\right)\right)\right\}\varphi(z)=0\end{aligned} \tag{29}$$

$$\frac{1}{\varepsilon}W(z)+N_{CT}\left(D^2-a^2\right)\Theta(z)+\left\{\frac{1}{L_e}\left(D^2-a^2\right)-\frac{n}{\sigma}\right\}\Gamma(z)=0 \tag{30}$$

$$W(z)+\left(D^2-a^2-n+\frac{N_B}{L_n}D-\frac{2N_A N_B}{L_n}D\right)\Theta(z)-\frac{N_B}{L_n}D\varphi(z)+N_{TC}\left(D^2-a^2\right)\Gamma(z)=0 \tag{31}$$

$$\frac{1}{\varepsilon}W(z)-\frac{N_A}{L_n}\left(D^2-a^2\right)\Theta(z)-\left(\frac{1}{L_n}\left(D^2-a^2\right)-\frac{n}{\sigma}\right)\varphi(z)=0 \tag{32}$$

$$W=0, D^2W=0, \Gamma=0, \Theta=0, \varphi=0 \quad at \quad z=0 \quad and \quad z=1 \tag{33}$$

where $D=\dfrac{d}{dz}$ and $a^2=l^2+m^2$ is the dimensionless horizontal wave number.

Consider the solution of the form

$$W=W_0\sin(\pi z),\ \Theta=\Theta_0\sin(\pi z),\ \Gamma=\Gamma_0\sin(\pi z),\ \varphi=\varphi\sin(\pi z) \tag{34}$$

by putting the solution in equations (29)- (32) and integrate each equation, we obtained the following matrix equations

$$\begin{pmatrix} J^2\left(\dfrac{\gamma_a n+}{\left(1+\eta J^2\right)}\right)^2+\pi^2T_a & -a^2R_a\left(\dfrac{\gamma_a n+}{\left(1+\eta J^2\right)}\right) & -\dfrac{R_S}{L_e}a^2\left(\dfrac{\gamma_a n+}{\left(1+\eta J^2\right)}\right) & a^2R_m\left(\dfrac{\gamma_a n+}{\left(1+\eta J^2\right)}\right) \\ \dfrac{1}{\varepsilon} & -N_{CT}J^2 & -\left(\dfrac{J^2}{L_e}+\dfrac{n}{\sigma}\right) & 0 \\ 1 & -\left(J^2+n\right) & -J^2N_{TC} & 0 \\ \dfrac{1}{\varepsilon} & \dfrac{N_A}{L_n} & 0 & \left(\dfrac{J^2}{L_n}+\dfrac{n}{\sigma}\right) \end{pmatrix}\begin{pmatrix} W_0 \\ \Theta_0 \\ \Gamma_0 \\ \varphi_0 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{35}$$

where $J^2=\pi^2+a^2$ is the total wave number.

5.The Stationary Convection

For stationary convection, we put n=0 in (35) and take determinant, we get

$$R_a=\frac{1}{\left(\varepsilon-N_{TC}L_e\right)}\left[\begin{matrix}\dfrac{\varepsilon}{a^2}J^2\left\{J^2\left(1+\eta J^2\right)+\dfrac{\pi^2T_a}{\left(1+\eta J^2\right)}\right\}\left\{1-L_eN_{TC}N_{CT}\right\}+R_S\left(N_{CT}\varepsilon-1\right) \\ -R_n\left\{L_n+\varepsilon N_A-N_{TC}L_e\left(N_A+L_nN_{CT}\right)\right\}\end{matrix}\right] \tag{36}$$

Eq. (36) represent the thermal Rayleigh number as a function of the dimensionless wave number a and the parameters $N_{TC}, N_{CT}, L_e, R_S, R_N, L_n, T_a$ and the couple stress parameter η.Also in Eq. (36) the modify nano particle ratio N_B does not appear and the diffusivity ratio parameter N_A appears only with the nanoparticle Rayleigh number R_n. The result given in equation (36) is a good agreement with the result finned by Kuznetsov and Nield (2010) and Chand and Rana (2016) in the absence of the Dufour, Soret and stable solute gradient parameters N_{CT}, N_{TC} and R_S respectively. Eq. (36) is analyzed numerically to depict the stability characteristics with respect to rotation.

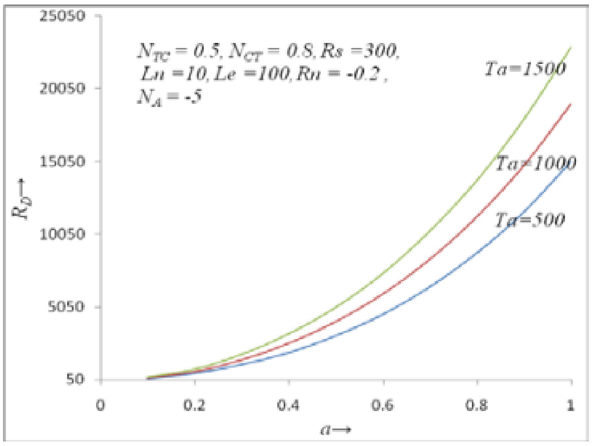

Figure 2: Variation of R_D with a for $T_a = 500$, $T_a = 1000$, $T_a = 1500$

The above graph that as T_a increases then Rayleigh number R_a also increases. This means that a couple stress nanofluid, in presence of rotation, is stable with respect to the onset of stationary convection when both the boundaries are free. This result is in good agreement with the result obtained by Chand and Rana (2016).

6.Conclusion

We investigated the effect of rotation on the onset of double-diffusive convection in a layer of couple-stress Nanofluid and using a Galerkin type weighted method for solving the problem. The Rayleigh Number, thermo-nanofluid Lewis number and Soret parameter have stabilizing effects on the stationary convection which are in good agreement with the results as discussed by Kuznetsov and Nield (2010) and Rana et al. (2016). The thermo-solutal Lewis number and Dufour parameter has destabilizing effects on the stationary convection of the system which are identical with the results as derived by Kuznetsov and Nield and Chand and Rana (2016). Rotation has stabilizing effect on the stationary convection as shown in figure 2 which is in good agreement with the result derived by and Rana (2014).

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References

[1] Eastman, J. A., Phillpot, S. R., Choi, S. U. S., Keblinski, P. "Thermal Transport in Nanofluids." Annual Review of Materials Research. 34, pp. 219-246. 2004.
[2] Das, S. K., Choi, S. U. S. "A Review of Heat Transfer in Nanofluids." Advances in Heat Transfer. 41, pp. 81-197. 2009.
[3] Kim, S. Y., Koo, J.-M., Kuznetsov, A. V. "Effect of Anisotropy in Permeability and Effective Thermal Conductivity on Thermal Performance of an Aluminum Foam Heat Sink." Numerical Heat Transfer, Part A: Applications. 40 (1), pp. 21-36. 2001.
[4] Kim, S. Y., Kuznetsov, A. V. "Optimization of Pin-fin Heat Sinks Using Anisotropic Local Thermal Nonequilibrium Porous Model in a Jet Impinging Channel." Numerical Heat Transfer, Part A: Applications. 44 (8), pp. 771-787. 2003.
[5] Chandrasekhar, S.“Hydrodynamic and hydromagnetic stability. Dover Publications”, New York. 1981.
[6] Boungiorno, J., Hu, L.-W. "Nanofluid coolants for advanced nuclear power plants." In: Proceedings of the American Nuclear Society - International Congress on Advances in Nuclear Power Plants 2005 (ICAPP'05), Seoul, South Korea, May 15-19, 2005. pp. 15-19.
[7] Choi, S. U. S., Eastman, J. A."Enhancing thermal conductivity of fluids with nanoparticles." In: International Mechanical Engineering Congress and Exhibition, San Francisco, USA, Nov. 12-17, 1995.
[8] Kuznetsov, A. V., Nield, D. A. "The Onset of Double-Diffusive Nanofluid Convection in a Layer of a Saturated Porous Medium." Transport in Porous Media. 85 (3), pp. 941-951. 2010.
[9] Nield, D. A., Kuznetsov, A. V. "The onset of double-diffusive convection in a nanofluid layer." International Journal of Heat and Fluid Flow. 32 (4), pp. 771-776. 2011.
[10] Yadav, D., Lee, D., Cho, H.-H., Lee, J. "The Onset of Double-Diffusive Nanofluid Convection in a Rotating Porous Medium Layer with Thermal Conductivity and Viscosity Variation: A Revised Model." Journal of Porous Media. 19 (1), pp. 31-46. 2016.
[11] Stokes, V. K. "Couple Stresses in Fluids." Physics of Fluids. 9, pp. 1709- 1715. 1966.
[12] Walicki, E., Walicka, A. "Interia effect in the squeeze film of couple-stress fluids in biological bearings." Applied Mechanics and Engineering. 4 (2), pp. 363-373. 1999.
[13] Sharma, R. C., Thakur, K. D. "On couple-stress fluid heated from below in hydromagnetics." Czechoslovak Journal of Physics. 50 (6), pp. 753- 758. 2000.
[14] Umavathi, J.C., Beg, O.A. “Simulation of the onset of convection in a porous medium layer saturated by a couple-stress nanofluid”. Microfluids and nanofluids. 3, pp 1171-1184, 2021.
[15] Tzou, D. Y. “Thermal instability of nanofluids in natural convection”.International Journal of Heat and MassTransfer. 51 (11-12), pp. 2967- 2979. 2008.

[16] Alloui, Z., Vasseur, P., Reggio, M. "Natural convection of nanofluids in a shallow cavity heated from below." International Journal of Thermal Sciences. 50 (3), pp. 385-393. 2011.
[17] Nield, D. A., Kuznetsov, A. V. "Thermal instability in a porous medium layer saturated by a nanofluid." International Journal of Heat Mass Transfer. 52 (25-26), pp. 5796–5801. 2009.
[18] Yadav, D., Agrawal, G. S., Bhargava, R. "Thermal instability of rotating nanofluid layer." International Journal of Engineering Science. 49 (11), pp. 1171-1184. 2011.
[19] Yadav, D., Bhargava, R., Agrawal, G. S. "Numerical solution of a thermal instability problem in a rotating nanofluid layer." International Journal of Heat and Mass Transfer. 63, pp. 313-322. 2013.
[20] Wang, S., Tan, W. "Stability analysis of soret-driven double-diffusive convection of Maxwell fluid in a porous medium." International Journal of Heat and Fluid Flow. 32 (1), pp. 88-94. 2011.
[21] Chand, R., Rana, G. C. "Dufour and Soret Effects on the Thermosolutal Instability of Rivlin-Ericksen Elastico-Viscous Fluid in Porous Medium." Zeitschrift für Naturforschung A. 67 (12), pp. 685–691. 2012.
[22] Singh, R., Bishnoi, J. and Tyagi, V.K. “Onset of Soret instability in a DarcyMaxwell nanofluid”. SN Applied Sci, 1, pp. 1273-1301. 2019.
[23] Rana, G. C., Thakur R. C., Kango, S. K. "On the Onset of Double-Diffusive Convection in a Layer of Nanofluid under Rotation Saturating a Porous Medium." Journal of Porous Media. 17 (8), pp. 657–667. 2014.
[24] Yadav, D., Lee, J. Cho, H. H. "Brinkman convection induced by purely internal heating in a rotating porous medium layer saturated by a nanofluid." Powder Technology. 286, pp. 592-601. 2015.
[25] Nield, D. A., Kuznetsov, A. V. "Thermal instability in a porous medium layer saturated by a nanofluid: A revised model." International Journal of Heat Mass Transfer. 68, pp. 211-214. 2014.
[26] Yadav, D., Agrawal, G. S., Lee, J. "Thermal instability in a rotating nanofluid layer: A revised model." Ain Shams Engineering Journal. 7 (1), pp. 431-440. 2016.
[27] Kapil, D., Pundir, S.K., Pundir, R. “Magnetic field effect on thermal impermanence in a nanofluid layer with low Prandtl number through porous medium”. Int. J. of Advanced Sci and Tech., 29 (4s), pp. 3115-3128. 2020.
[28] Yadav, D., Mahabaleshwar, U.S., Wakif, A., Chand, R. “Significance of the inconstant viscosity and internal heat generation on the occurrence of Darcy–Brinkman convective motion in a couple-stress fluid-saturated porous medium: An analytical solution,” Int. Commun. Heat Mass Transfer **122**, pp. 105165. 2021.
[29] Rana, G.C., Thakur, R.C. “Theonset of double-diffusive convection in a layer of nanofluid under rotation”. Thermal Engineering, 15 (1), pp. 88-95. 2016.