α- Continuity of Fuzzy Multifunctions

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The aim of this paper is to initiate the study and to find different characterizing properties of lower and upper α-continuous fuzzy multifunction's ,where the domain of these functions is a topological space with these values as arbitrary fuzzy sets in fuzzy topological spaces. The study of different classes of non-continuous functions between topological spaces and there genralizations to multi valued cases has long been of interest to topologist. Some of these functions have been extended to fuzzy topological space by many authors. A further step ahead in this process of generalization is the introduction of fuzzy multifunction's.

According to Papageorgious(1985) ,If (X ,τ) is a topological space in the sense Chang (1968), then a fuzzy multifunction F: X → Y is a function which maps a point of X to a set in Y. We shell adhere to this terminology and simply by X and Y, we shell denote the topological space(X ,τ) and the fuzzy topological space (Y,σ) respectively.

Fuzzy α-continuous Mappings

A mapping f from am fuzzy topological space (X ,τ) to a fuzzy topological space (Y,σ) is fuzzy α-continuous if the inverse of every fuzzy open set of Y is fuzzy α open in X.

Remark: The concept of fuzzy α-continuous and fuzzy continuous mappings are independent.

Example: Let X = { a,b,c } and Y = { x,y,z } Define λ and μ as

λ (a) = 0 , λ (b) = 0.2 , λ (c) = 0.2 ;
μ (x) = 0 , μ (y) = 0.2 , μ (z) = 0.7 ;

Let τ = { 0 , λ , 1 } and σ = { 0, μ , 1 }. Then the mapping f : (X ,τ) → (Y,σ) defined by f(a) = x , f (b) = y , f (c) = z is fuzzy continuous but not fuzzy α-continuous.

Example: Ler X = { a ,b ,c } and Y = {x ,y} Define λ and μ as :

λ (a) = 0 , λ (b) = 0 ;
μ (x) = 0, μ (y) = 0.8 ;

Let τ = { 0 ,λ, 1 } and σ = { 0 , μ, 1 }. Then the mapping f : (X , τ) → (Y, σ) defined by f(a) = x , f (b) = y is fuzzy α-continuous but not fuzzy continuous.

Remark: Every fuzzy α-continuous mappings is fuzzy continuous but the converse may be False for the mapping f: (X, τ) → (Y , σ) is fuzzy α-continuous but not fuzzy continuous.

Space (Y, σ) is fuzzy pre-continuous if the inverse image of every fuzzy open set of Y is fuzzy preopen in X.

Remark: Every fuzzy α-continuous mapping is fuzzy precontinuous, But the converse may not be true.

Remark: Every fuzzy α-continuous mapping is fuzzy precontinuous but the converse may not be true.

Theorem: Let f : (x ,τ) → (Y , σ) is fuzzy α- continuous and g : (Y , σ) → (Z , γ) be Fuzzy continuous , then g of : (X ,τ) → (Z, γ) is fuzzy α- continuous.

Theorem: A mapping f: (X , τ) → (Y , σ) is fuzzy α-continuous if and only if f: X → f(x) is fuzzy continuous.

Theorem: Let f: (X , τ) → (Y , σ) be fuzzy α- continuous and μbe fuzzy open set of Y.

Then Cl f^{-1}(μ) ≤ f^{1} (cl (μ)).

Theorem: The following are equivalent for a fuzzy multifunction F: X → Y.

(a) F is fuzzy upper α- continuous.
(b) F^{+}(λ) Ɛ α (x) for any fuzzy open set λ of Y.
(c) F^{-}(λ) Ɛ α -closed in X for any fuzzy closed set λ of Y.
(d) Cl (int(cl(F^{-} (μ)))) ⊂ F^{-} (cl(μ)) for each fuzzy set μ in Y.
(e) αcl (F^{-} (μ)) ⊂ F^{-} (cl (μ)) for each fuzzy set μ of Y.

Proof:

(a) → (b) : Let λ be any fuzzy open set and let x ϵ F^{+} (λ) , then F (x) ≤ λ.Since F is fuzzy upper α- continuous , there exists U ϵ α (x) containing x such that F (U) ≤ λ for all u ϵ U.Hence , x ϵ U ⊂ F^{+} (λ) ϵ α (x).
(b) → (c) This follows from the fact that F^{+}(1- λ) = 1 –F^{-}(λ) for any fuzzy set λ of Y.
(c) → (d) Let μ be any fuzzy set of Y then F^{-}(cl(μ)) is α -closed in λ .Therefore cl(int(cl(F^{-}(μ)))) ⊂ cl(int(cl(F^{-}(cl(μ)))) ⊂ F^{-}(cl (μ)).
(d) →(e) Let λ be any fuzzy closed set of Y then by (d),

Cl(int(cl(F^{-}(μ)))) ⊂ F^{-}cl (λ)) ⊂ F^{-} (λ) ,Hence F^{-}(λ) is α -closed in X.
(c) → (e) : Let μ be any fuzzy set of Y then F^{-}(cl(μ)) is α-closed in X.Hence
α cl (F^{-} (μ)) ⊂ α cl F^{-}(cl(μ)) = F^{-} (cl(μ)).
(e) → (c) : Let λ be any fuzzy closed set of Y.Then we have α cl (F^{-}(λ)) ⊂ F^{-} (cl (λ) = F^{-}(λ).This shows that F^{-} (λ) is α-closed in X.
(b) → (f): Let x ϵ X and λ be a neighbourhood of F(x) .Then there exists an fuzzy open set μ of Y such that F(x) ≤ μ ≤ λ.Therefore ,x ϵ F^{+} (μ) ⊂ F^{+} (λ).Hence F^{+} (λ) is an α-neighbourhood of X .
(f) → (g) : Let x ϵ X and λ be a neighbourhood of F(X).Put U = F^{+} (λ) then U is an α-neighbourhood of X,and F(U) ≤ λ.
(g) →(a) : Let x ϵ X and λ be any fuzzy open set of Y such that F(x) ≤ λ then λ is a neighbourhood of F(x),Therefore there exist an α-neighbourhood A of X ,such that F(A) ≤ λ. Therefore there exsist U ϵ α (X) such that x ϵ U⊂ A..

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Hence ,U ϵ α (X) containing x such that f(u) ≤ λ., for all u ϵ U.

Theorem: The following are equivalent for multiplication F: X→ Y:

(a) F is lower α- continuous.
(b) F(λ) ϵ α(X) for any fuzzy open set λ of Y.
(c) F^{+}(λ) is α-closed in X for any closed set λ of Y.
(d) Cl (int(cl(F^{+}(μ)))) ⊂ F^{+}(cl(μ)) for each fuzzy set μ of Y.
(e) A cl (F^{+}(μ)) ⊂ F^{+}(cl (μ)) for any subset μ of Y.

Proof

(a) →(b) Let λ be any fuzzy open set and let x ϵ F^{-}(λ) , then F (x)qλ.Since F is fuzzy lower α-continuous ,there exist U ϵ α (X) containing X such that F(U)qλ.for all u ϵ U Therefore x ϵ U ⊂ F (λ) and 1 – F(λ) ϵ α (X).
(b) →(c): This follows from the fact that F^{-}(1-λ) = 1- F^{+}(λ) for any fuzzy set λ of Y.
(c) →(d): Let μ be any fuzzy set of Y then F^{+}(cl (μ)) is α -closed in X. Therefore
Cl (int(cl(F^{+}(μ)))) ⊂ cl (int (cl (F^{+}(cl (μ)))) ⊂ F^{+}(cl (μ)).

(d) →(c): Let λ be any fuzzy closed set of Y, then by (d), cl(int(cl(F^{+}(λ)))) ⊂ F^{+}(cl(λ)) = F^{+}(λ).Hence F^{+} (λ) is α-closed in X.
(c) → (e): Let μ be any fuzzy set of Y then F^{+}(cl(μ)) is α-closed in X.
Hence α cl(F^{+}(μ)) ⊂ α cl (F^{+}(cl(μ))) = F^{+}(cl(μ)).
(e) →(c) :Let λ be any fuzzy closed set of Y then we have α cl ($F^{-(}$(λ) ⊂ F^{-}(cl(λ)) = F^{-}(λ)
(e) → (a): Let x ϵ X and λ be any fuzzy open set such that F (x)qλ. Then x ϵ F^{-}(λ) .We shell show that F^{-} (λ) ϵ α (x).By the hypothesis ,we have F (cl(int(cl(F^{+}(1-λ)))) and hence F^{-}(λ) ϵ α (X).Put U = F^{-}(λ) .We have x ϵU ϵ α (X) and F(u)qλ for every uϵ U.

Thus F is lower α – continuous..

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