A Deterministic Inventory Model with Biquadratic Demand, Static Rate of Deterioration and Linear Carrying Cost

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Abstract: In this paper, a Deterministic Model is evolving for the items decline by Demand as well as by Deterioration, where we take Demand as a Biquadratic Polynomial function of Time with Static rate of Deterioration and linear carrying cost. Here, Shortage is allowing and fully backlogging. An Organization can use this model where carrying cost is time dependent and demand increases with time biquadratically with the static rate of Deterioration.

Keywords: EOQ Model, demand, Deterioration, shortage

1. Introduction

Inventory is the priority to run a business. But it may be a blessing or crush for a Business Owner. It is a blessing because when an owner buys a large no of goods then he gets benefit due to the lower wholesale price, which increases the quality of customer service. But it becomes a curse for an owner due to its large cost of maintenance: like its Deteriorating cost (because many of the physical goods undergo damage or chemically change with time for example product like milk, fruits, bread, butter, etc gets spoil with time), cost arises due to out of fashion of the product, storage and handling cost of goods, ordering cost, etc. So, an owner has to decide two main things; How much to order called EOQ and When to order to control Inventory. So, Inventory Management is very important to use working Capital Effectively. Because an owner's main aim is to Maximize Profit and Minimize Cost. So, in previous years various mathematical models have been created by researchers to minimize cost. Some of them are discussed here. Datta & Pal (1998) [4], Lee & Wu (2002) [9], Sharma, Sharrma & Ramani (2012) [15] and Sharma and Preeti (2013) [14] considered Power Demand pattern for Deteriorating Items with time varying deterioration in their respective models. Wu (1999) [23], Wu (2002) [24] considered Weibull distributed Deterioration in their respective models. Giri & Chaudhuri (1998) [5] considered demand rate as a function of on hand inventory in their model. Bhowmic & Samanta(2007) [2] considered stock dependent time - varying demand rate, Mishra and Singh (2011)[11],Singh & Srivastava (2017) [22] considered Linear Demand ,Mishra and Singh (2013) [10] considered time dependent demand and deterioration, Bhowmi Samanta(2011) [3] considered constant demand rate and variable production cycle, Roy (2008) [13] considered time dependent deterioration rate and assumed Demand rate as a function of the selling price, Karmakar & Choudhury (2014) [6] assumed general ramp type demand rate, Kumar & Kumar (2015) [8] assumed time-dependent demand, Rasel (2017) [12] considered power distribution deterioration, Priya & Senbagam (2018) [7] assumed two 1parameter Weibull deterioration with quadratic time-dependent demand, Bansal, Kumar et al.(2021) [1] took stock-dependent demand rates ,in their respective model. In this paper, I have developed a model by assuming demand as a biquadratic polynomial function of time with constant rate of deterioration and linear carrying cost.

2. Assumptions and Notations

Notations

- h(t) Inventory Carrying Charge per object per unit time.
- C₂ Cost due to deficiency of one object per unit time
- C₃Cost of one Decayed Unit.
- T Length of every Production cycle.
- C(t) Average entire cost
- S Inventory at t = 0, where t is used for time.
- I(t) Inventory at any time t.
- D(t) Demand rate.
- $\theta(t)$ Deterioration rate function.

Assumptions

- 1) $D(t) = \alpha + \beta t + \gamma t^2 + \delta t^3 + \varepsilon t^4$, where $\alpha, \beta, \gamma, \delta, \varepsilon$ are > 0.
- 2) h(t) = h + mt, h > 0; m > 0
- 3) $\theta(t) = \theta_0$, where $0 < \theta_0 << 1$.
- 4) Lead time has been taken as 0.
- 5) Shortages are allowed and completely reserved.
- 6) Refill magnitude is static and the refill rate is unbounded.
- 7) During the time period T ,there is neither replacement nor repair of deteriorated units.

3. Analysis of Model

Let the no of objects in stock at any time t be I(t).In time period $0 < t < t_1$,I(t) lessens gradually due to requirement and decaying of items and falls to zero at $t = t_1$.In the time period (t_1 , T), deficiency of items occurs which are wholly backlogged, where $t_1 < T$. The equations of this process are given by:

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -\{D(t)\}\ 0 \le t \le t_1(1)$$

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(2)

$$\frac{dI(t)}{dt} = -\{D(t)\} t_1 \le t \le T$$

Put $\theta = \theta_0$ and $D(t) = \alpha + \beta t + \gamma t^2 + \delta t^3 + \varepsilon t^4$ in (1) and (2), we get

$$\frac{dI(t)}{dt} + \theta_0 I(t) = -\{ \alpha + \beta t + \gamma t^2 + \delta t^3 + \varepsilon t^4 \}$$
(3)

$$\frac{dI(t)}{dt} = -\{\alpha + \beta t + \gamma t^2 + \delta t^3 + \varepsilon t^4\}$$
(4)

Solution of (3) is

$$I(t)e^{\theta_0 t} = -\int (\alpha + \beta t + \gamma t^2 + \delta t^3 + \varepsilon t^4) e^{\theta_0 t} dt + C$$

$$= -\int (\alpha + \beta t + \gamma t^2 + \delta t^3 + \varepsilon t^4) (1 + \theta_0 t) dt + C$$

$$= -\left[(\alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\varepsilon t^5}{5}) + \theta_0 (\frac{\alpha t^2}{2} + \frac{\beta t^3}{3} + \frac{\gamma t^4}{4} + \frac{\delta t^5}{5} + \frac{\varepsilon t^6}{6})\right] + C$$
(5)

Place t = 0 in (5), we get I(0) = C, but I(0) = S. So C = S.

Hence (5) implies

$$I(t) \ e^{\theta_0 t} = S - \left[\left(\alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\delta t^5}{5} \right) + \theta_0 \left(\frac{\alpha t^2}{2} + \frac{\beta t^3}{3} + \frac{\gamma t^4}{4} + \frac{\delta t^5}{5} + \frac{\delta t^6}{6} \right) \right]; \ 0 \le t \le t_1$$
(6)

Since $I(t_1) = 0$, So (6) implies

$$S = \left[\left(\alpha t_1 + \frac{\beta t_1^2}{2} + \frac{\gamma t_1^3}{3} + \frac{\delta t_1^4}{4} + \frac{\varepsilon t_1^5}{5} \right) + \theta_0 \left(\frac{\alpha t^2}{2} + \frac{\beta t^3}{3} + \frac{\gamma t^4}{4} + \frac{\delta t^5}{5} + \frac{\varepsilon t^6}{6} \right) \right]$$
(7)

$$I(t) = (1 - \theta_0 t) \left[\left\{ \alpha(t_1 - t) + \frac{\beta(t_1^2 - t^2)}{2} + \frac{\gamma(t_1^3 - t^3)}{3} + \frac{\delta(t_1^4 - t^4)}{4} + \frac{\varepsilon(t_1^5 - t^5)}{5} \right\} + \theta_0 \left\{ \frac{\alpha(t_1^2 - t^2)}{2} + \frac{\beta(t_1^3 - t^3)}{3} + \frac{\gamma(t_1^4 - t^4)}{4} + \frac{\delta(t_1^5 - t^5)}{5} + \frac{\varepsilon(t_1^6 - t^6)}{6} \right\} \right]$$

$$I(t) = \left[\left\{ \alpha(t_1 - t) + \frac{\beta(t_1^2 - t^2)}{2} + \frac{\gamma(t_1^3 - t^3)}{3} + \frac{\varepsilon(t_1^5 - t^5)}{5} \right\} + \theta_0 \left\{ \frac{\alpha(t_1^2 - t^2)}{2} + \frac{\beta(t_1^3 - t^3)}{3} + \frac{\gamma(t_1^4 - t^4)}{4} + \frac{\delta(t_1^5 - t^5)}{5} + \frac{\varepsilon(t_1^6 - t^6)}{6} \right\} \right] - \theta_0 \left\{ \alpha(t_1 t - t^2) + \frac{\beta(t_1^2 t - t^3)}{3} + \frac{\gamma(t_1^3 t - t^4)}{3} + \frac{\varepsilon(t_1^3 t - t^4)}{3} + \frac{\varepsilon(t_1^4 t - t^5)}{5} + \frac{\varepsilon(t_1^5 t - t^6)}{6} \right\} \right] - \theta_0 \left\{ \alpha(t_1 t - t^2) + \frac{\beta(t_1^2 t - t^3)}{3} + \frac{\gamma(t_1^3 t - t^4)}{3} + \frac{\varepsilon(t_1^3 t - t^4)}{3} + \frac{\varepsilon(t_1^5 t - t^6)}{5} \right\} \right\}$$
(8)

(neglecting terms containing θ_0^2 , which is very small)

Also, solution of (4) is

$$I(t) = -\left(\alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\xi t^5}{5}\right) + A$$
(9)

Applying
$$I(t_1) = 0$$
 in (9), we get

$$I(t) = \alpha(t_1 - t) + \frac{\beta(t_1^2 - t^2)}{2} + \frac{\gamma(t_1^3 - t^3)}{3} + \frac{\delta(t_1^4 - t^4)}{4} + \frac{\varepsilon(t_1^5 - t^5)}{5};$$

$$t_1 \le t \le T$$
(10)

Therefore no. of Deteriorated Items = I(0) - Stock loss due to Demand

$$= S - \int_0^{t_1} (\alpha + \beta t + \gamma t^2 + \delta t^3 + \varepsilon t^4) dt$$
 (11)
[using (7)]

$$= \theta_0(\frac{\alpha t_1^2}{2} + \frac{\beta t_1^3}{3} + \frac{\gamma t_1^4}{4} + \frac{\delta t_1^5}{5} + \frac{\varepsilon t_1^6}{6})$$
(12)

Using (8), $I_1 = \int_0^{t_1} I(t) dt =$

$$\int_{0}^{t_{1}} \left\{ \alpha(t_{1}-t) + \frac{\beta(t_{1}^{2}-t^{2})}{2} + \frac{\gamma(t_{1}^{3}-t^{3})}{3} + \frac{\delta(t_{1}^{4}-t^{4})}{4} + \frac{\varepsilon(t_{1}^{5}-t^{5})}{5} \right\} dt \\ + \theta_{0} \int_{0}^{t_{1}} \left\{ \frac{\alpha(t_{1}^{2}-t^{2})}{2} + \frac{\beta(t_{1}^{3}-t^{3})}{3} + \frac{\gamma(t_{1}^{4}-t^{4})}{4} + \frac{\delta(t_{1}^{5}-t^{5})}{5} + \frac{\varepsilon(t_{1}^{6}-t^{6})}{6} \right\} dt \\ - \theta_{0} \int_{0}^{t_{1}} \left\{ \alpha(t_{1}t-t^{2}) + \frac{\beta(t_{1}^{2}t-t^{3})}{2} + \frac{\gamma(t_{1}^{3}t-t^{4})}{3} + \frac{\delta(t_{1}^{4}t-t^{5})}{4} + \frac{\varepsilon(t_{1}^{5}t-t^{6})}{4} + \frac{\varepsilon(t_{1}^{5}t-t^{6})}{5} \right\} dt (13) \\ = \left[\left(\frac{\alpha t_{1}^{2}}{2} + \frac{\beta t_{1}^{3}}{4} + \frac{\gamma t_{1}^{4}}{4} + \frac{\delta t_{1}^{5}}{5} + \frac{\varepsilon t_{1}^{6}}{6} \right) + \theta_{0} \left(\frac{\alpha t_{1}^{3}}{4} + \frac{\beta t_{1}^{4}}{4} + \frac{\gamma t_{1}^{5}}{5} + \frac{\varepsilon t_{1}^{7}}{5} \right) \right]$$

$$= \left[\left(\frac{\alpha t_1^2}{2} + \frac{\beta t_1^3}{3} + \frac{\gamma t_1^4}{4} + \frac{\delta t_1^5}{5} + \frac{\varepsilon t_1^6}{6} \right) + \theta_0 \left(\frac{\alpha t_1^3}{6} + \frac{\beta t_1^4}{8} + \frac{\gamma t_1^5}{10} + \frac{\delta t_1^6}{12} + \frac{\varepsilon t_1^7}{14} \right) \right]$$
(14)

No of shortage units =

$$-\int_{t_{1}}^{T} I(t) dt =$$

$$\int_{T}^{t_{1}} \{ \alpha(t_{1} - t) + \frac{\beta(t_{1}^{2} - t^{2})}{2} + \frac{\gamma(t_{1}^{3} - t^{3})}{3} + \frac{\delta(t_{1}^{4} - t^{4})}{4} + \varepsilon(t15 - t5)5 \} dt (using (10))$$

$$= T \{ \alpha(\frac{T}{2} - t_{1}) + \frac{\beta}{2}(\frac{T^{2}}{3} - t_{1}^{2}) + \frac{\gamma}{3}(\frac{T^{3}}{4} - t_{1}^{3}) + \frac{\delta}{4}(\frac{T^{4}}{5} - t_{1}^{4}) + \frac{\varepsilon}{5}(\frac{T^{5}}{6} - t_{1}^{5}) \} + \{ \frac{\alpha t_{1}^{2}}{2} + \frac{\beta t_{1}^{3}}{3} + \frac{\gamma t_{1}^{4}}{4} + \frac{\delta t_{1}^{5}}{5} + \frac{\varepsilon t_{1}^{6}}{6} \}$$
(15)

 $= h I_1 + m \int_0^{t_1} tI(t) dt$ = $h I_1 + m \int_0^{t_1} t \left[\left\{ \alpha(t_1 - t) + \frac{\beta(t_1^2 - t^2)}{2} + \frac{\gamma(t_1^3 - t^3)}{3} + \delta t 14 - t 44 + \xi t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 13 - t 33 + \xi t 14 - t 44 + \xi t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 13 - t 33 + \xi t 14 - t 44 + \xi t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 13 - t 33 + \xi t 14 - t 44 + \xi t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 13 - t 33 + \xi t 14 - t 44 + \xi t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 13 - t 33 + \xi t 14 - t 44 + \xi t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 13 - t 33 + \xi t 14 - t 44 + \xi t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 13 - t 33 + \xi t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 13 - t 33 + \xi t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 13 - t 33 + \xi t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 13 - t 33 + \xi t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 13 - t 33 + \xi t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 13 - t 33 + \xi t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 13 - t 33 + \xi t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 13 - t 33 + \xi t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 22 + \beta t 15 - t 55 + \theta 0 \ \alpha t 12 - t 55 + \theta 0 \ \alpha t 12 - t 55 + \theta 0 \ \alpha t 12 + t 55 + \theta 0 \ \alpha t 12 + t 55 + \theta 0 \ \alpha t 12 + t 55 + \theta 0 \ \alpha t 12 + t 55 + \theta 0 \ \alpha t 12 + t 55 + \theta 0 \ \alpha t 12 + t 55 + \theta 0 \ \alpha t 12 + t 55 + \theta 0 \ \alpha t 12 + t 55 + \theta 0 \ \alpha t 12 + t 55 + \theta 0 \ \alpha t 12 + t 55 + \theta 0 \ \alpha t 12 + t 55 + \theta 0 \ \alpha t 12 + t 55 + \theta 0 \ \alpha t 12 + t 55 + \theta 0 \ \alpha t 12 + t 55 + \theta$ $\gamma t 14 - t 44 + \delta t 15 - t 55 + \varepsilon t 16 - t 66 - \theta 0 \alpha t 1t - t 2 +$ $\beta t 12t - t 32 + \gamma t 13t - t 43 + \delta t 14t - t 54 + \varepsilon t 15t - t 65$ dt using (8)

Using (14), we get

Inventory carrying cost = $= \int_0^{t_1} h(t)I(t)dt$ $= \int_0^{t_1} (h+mt)I(t)dt$ $= h \int_0^{t_1} I(t) dt + m \int_0^{t_1} t I(t) dt$

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$$= h \left(\frac{\alpha t_1^2}{2} + \frac{\beta t_1^3}{3} + \frac{\gamma t_1^4}{4} + \frac{\delta t_1^5}{5} + \frac{\varepsilon t_1^6}{6} \right) + \frac{\theta_0 h}{2} \left(\frac{\alpha t_1^3}{3} + \frac{\beta t_1^4}{4} + \frac{\gamma t_1^5}{5} + \frac{\delta t_1^6}{6} + \frac{\varepsilon t_1^7}{7} \right) + \frac{m}{2} \left(\frac{\alpha t_1^3}{3} + \frac{\beta t_1^4}{4} + \dots + \frac{\varepsilon t_1^7}{7} \right) + \frac{\theta_0 m}{6} \left(\frac{\alpha t_1^4}{4} + \frac{\beta t_1^5}{5} + \dots + \frac{\varepsilon t_1^8}{8} \right) (16)$$

Shortage $cost = C_2 *$ quantity of shortage units

$$= C_{2} T \left\{ a \left(\frac{T}{2} - t_{1} \right) + \frac{\beta}{2} \left(\frac{T^{2}}{3} - t_{1}^{2} \right) + \frac{\gamma}{3} \left(\frac{T^{3}}{4} - t_{1}^{3} \right) + \frac{\delta}{4} \left(\frac{T^{4}}{5} - t_{1}^{4} \right) + \frac{\varepsilon}{5} \left(\frac{T^{5}}{6} - t_{1}^{5} \right) \right\} + C_{2} \left\{ \frac{\alpha t_{1}^{2}}{2} + \frac{\beta t_{1}^{3}}{3} + \frac{\gamma t_{1}^{4}}{4} + \frac{\delta t_{1}^{5}}{5} + \frac{\varepsilon t_{1}^{6}}{6} \right\}$$
(17)

Cost due to Deterioration = C₃ * quantity of Decayed units = $C_3 \theta_0 \left(\frac{\alpha t_1^2}{2} + \frac{\beta t_1^3}{3} + \frac{\gamma t_1^4}{4} + \frac{\delta t_1^5}{5} + \frac{\varepsilon t_1^6}{6}\right)$ (18)

Entire cost per unit time = Inventory carrying cost + shortage cost + cost due to Decaying objects

$$\begin{split} \mathcal{C}(t) &= h \left(\frac{\alpha t_1^2}{2} + \frac{\beta t_1^3}{3} + \frac{\gamma t_1^4}{4} + \frac{\delta t_1^5}{5} + \frac{\varepsilon t_1^6}{6} \right) + \frac{\theta_0 h}{2} \left(\frac{\alpha t_1^3}{3} + \frac{\beta t_1^4}{4} + \frac{\gamma t_1^5}{5} + \frac{\delta t_1^6}{6} + \frac{\varepsilon t_1^7}{7} \right) \\ &+ \frac{m}{2} \left(\frac{\alpha t_1^3}{3} + \frac{\beta t_1^4}{4} + \dots + \frac{\varepsilon t_1^7}{7} \right) + \frac{\theta_0 m}{6} \left(\frac{\alpha t_1^4}{4} + \frac{\beta t_1^5}{5} + \dots + \frac{\varepsilon t_1^8}{8} \right) \\ &+ C_2 T \left[\alpha \left(\frac{T}{2} - t_1 \right) + \frac{\beta}{2} \left(\frac{T^2}{3} - t_1^2 \right) + \frac{\gamma}{3} \left(\frac{T^3}{4} - t_1^3 \right) + \frac{\delta}{4} \left(\frac{T^4}{5} - t_1^4 \right) + \frac{\varepsilon}{5} \left(\frac{T^5}{6} - t_1^5 \right) \right] + C_2 \left\{ \frac{\alpha t_1^2}{2} + \frac{\beta t_1^3}{3} + \frac{\gamma t_1^4}{4} + \frac{\delta t_1^5}{5} + \frac{\varepsilon t_1^6}{6} \right\} + \\ C_3 \theta_0 \left(\frac{\alpha t_1^2}{2} + \frac{\beta t_1^3}{3} + \frac{\gamma t_1^4}{4} + \frac{\delta t_1^5}{5} + \frac{\varepsilon t_1^6}{6} \right) (19) \end{split}$$

Average entire cost per unit time

$$\begin{split} \mathcal{C}(t_{1}) &= \frac{1}{T} \left[\text{Complete cost per unit time} \right] \\ &= \frac{h}{T} \left(\frac{\alpha t_{1}^{2}}{2} + \frac{\beta t_{1}^{3}}{3} + \frac{\gamma t_{1}^{4}}{4} + \frac{\delta t_{1}^{5}}{5} + \frac{\delta t_{1}^{6}}{6} \right) \\ &\quad + \frac{\theta_{0}h}{2T} \left(\frac{\alpha t_{1}^{3}}{3} + \frac{\beta t_{1}^{4}}{4} + \frac{\gamma t_{1}^{5}}{5} + \frac{\delta t_{1}^{6}}{6} \right) \\ &\quad + \frac{\theta_{0}h}{2T} \left(\frac{\alpha t_{1}^{3}}{3} + \frac{\beta t_{1}^{4}}{4} + \cdots + \frac{\delta t_{1}^{7}}{7} \right) \\ &\quad + \frac{\theta_{0}m}{6T} \left(\frac{\alpha t_{1}^{4}}{4} + \frac{\beta t_{1}^{5}}{5} + \cdots + \frac{\delta t_{1}^{8}}{8} \right) \\ &\quad + C_{2} \left[\alpha \left(\frac{T}{2} - t_{1} \right) + \frac{\beta}{2} \left(\frac{T^{2}}{3} - t_{1}^{2} \right) + \frac{\gamma}{3} \left(\frac{T^{3}}{4} - t_{1}^{3} \right) + \frac{\delta}{4} \left(\frac{T^{4}}{5} - t_{1}^{4} \right) \\ &\quad + \frac{\xi}{5} \left(\frac{T^{5}}{6} - t_{1}^{5} \right) \right] + \frac{C_{2}}{T} \left\{ \frac{\alpha t_{1}^{2}}{2} + \frac{\beta t_{1}^{3}}{3} + \frac{\gamma t_{1}^{4}}{4} + \frac{\delta t_{1}^{5}}{5} + \frac{\delta t_{1}^{6}}{6} \right\} + \\ \frac{c_{3}\theta_{0}}{T} \left(\frac{\alpha t_{1}^{2}}{2} + \frac{\beta t_{1}^{3}}{3} + \frac{\gamma t_{1}^{4}}{4} + \frac{\delta t_{1}^{5}}{5} + \frac{\delta t_{1}^{6}}{6} \right) (20) \\ \text{For least average entire cost , put $\frac{dC(t_{1})}{2t} = 0 \\ \text{we get }, \frac{D(t)}{T} \left\{ \frac{\theta_{0}m}{6} t_{1}^{3} + \frac{(h\theta_{0} + m)}{2T} t_{1}^{2} + (h + C_{2} + C_{3}\theta_{0}) t_{1} - \\ C_{2}T \right\} = 0 \text{ implies} \\ \left\{ \frac{\theta_{0}m}{6T} t_{1}^{3} + \frac{(h\theta_{0} + m)}{2T} t_{1}^{2} + \frac{(h + C_{2} + C_{3}\theta_{0})}{T} t_{1} - C_{2} \right\} = 0 \\ \end{split}$$$

Which is cubic in t_1 with constant term negative. so, it has at least 1 positive root say t_1^* and $\frac{d^2 C(t_1^*)}{dt_1^{*2}} > 0$. So optimum value of t_1 is t_1^* . Hence the optimum value of S is $S^* = (\alpha t_1^* + \frac{\beta t_1^{*2}}{2} + \frac{\gamma t_1^{*3}}{3} + \frac{\delta t_1^{*4}}{4} + \frac{\epsilon t_1^{*5}}{5}) + \theta_0 (\frac{\alpha t_1^{*2}}{2} + \frac{\beta t_1^{*3}}{3} + \frac{\gamma t_1^{*4}}{4} + \frac{\delta t_1^{*5}}{5}) + \theta_0 (\frac{\alpha t_1^{*2}}{2} + \frac{\beta t_1^{*3}}{3} + \frac{\gamma t_1^{*4}}{4} + \frac{\delta t_1^{*5}}{5}) + \theta_0 (\frac{\alpha t_1^{*2}}{2} + \frac{\beta t_1^{*3}}{3} + \frac{\gamma t_1^{*4}}{4} + \frac{\delta t_1^{*5}}{5}) + \theta_0 (\frac{\alpha t_1^{*2}}{2} + \frac{\beta t_1^{*3}}{3} + \frac{\gamma t_1^{*4}}{4} + \frac{\delta t_1^{*5}}{5}) + \theta_0 (\frac{\alpha t_1^{*2}}{2} + \frac{\beta t_1^{*3}}{3} + \frac{\gamma t_1^{*4}}{4} + \frac{\delta t_1^{*5}}{5}) + \theta_0 (\frac{\alpha t_1^{*2}}{2} + \frac{\beta t_1^{*3}}{3} + \frac{\gamma t_1^{*4}}{4} + \frac{\delta t_1^{*4}}{4} + \frac{\delta t_1^{*3}}{5}) + \theta_0 (\frac{\alpha t_1^{*2}}{2} + \frac{\beta t_1^{*3}}{3} + \frac{\gamma t_1^{*4}}{4} + \frac{\delta t_1^{*4}}{4} + \frac{\delta t_1^{*4}}{5} + \frac{\delta t_1^{*4}}{6})$

Minimum value of C(t₁) is

$$\begin{split} C\left(t_{1}^{*}\right) &= \frac{h}{T} \left(\frac{\alpha t_{1}^{*2}}{2} + \frac{\beta t_{1}^{*3}}{3} + \frac{\gamma t_{1}^{*4}}{4} + \frac{\delta t_{1}^{*5}}{5} + \frac{\delta t_{1}^{*6}}{6}\right) + \\ &\frac{\theta_{0}h}{2T} \left(\frac{\alpha t_{1}^{*3}}{3} + \frac{\beta t_{1}^{*4}}{4} + \frac{\gamma t_{1}^{*5}}{5} + \frac{\delta t_{1}^{*6}}{6} + \frac{\delta t_{1}^{*7}}{7}\right) \\ &+ \frac{m}{2T} \left(\frac{\alpha t_{1}^{*3}}{3} + \frac{\beta t_{1}^{*4}}{4} + \cdots + \frac{\delta t_{1}^{*7}}{7}\right) \\ &+ \frac{\theta_{0}m}{6T} \left(\frac{\alpha t_{1}^{*4}}{4} + \frac{\beta t_{1}^{*5}}{5} + \cdots + \frac{\delta t_{1}^{*8}}{8}\right) \\ &+ C_{2} \left[\alpha \left(\frac{T}{2} - t_{1}^{*}\right) + \frac{\beta}{2} \left(\frac{T^{2}}{3} - t_{1}^{*2}\right) + \frac{\gamma}{3} \left(\frac{T^{3}}{4} - t_{1}^{*3}\right) + \frac{\delta}{4} \left(\frac{T^{4}}{5} - t_{1}^{*5}\right)\right] + \frac{C_{2}}{T} \left\{\frac{\alpha t_{1}^{*2}}{2} + \frac{\beta t_{1}^{*3}}{3} + \frac{\gamma t_{1}^{*4}}{4} + \frac{\delta t_{1}^{*5}}{5} + \frac{\delta t_{1}^{*5}}{6}\right) \end{split}$$

Which gives optimal value of total average cost per unit time.

4. Conclusion

Here, an Inventory model has been created for items depleted due to demand as well as Deterioration by taking demand as a biquadratic polynomial function of time , constant deterioration rate and linear carrying cost. I have obtained minimum total average cost. This model can be extended further for other values of demand and carrying cost.

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