Numerical Investigation of Hydromagnetic Natural Convection Flow of Heat Absorbing Fluid Past an Impulsively Moving Infinite Vertical Plate in the Presence of Uniform Transverse Magnetic Field

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Abstract: ***The study of hydromagnetic natural flow of convection of heat absorbing fluid past a spontaneously moving infinite vertical plate in the occurrence of unchanging magnetic field are considered. The momentum and energy incomplete differential equations controlling the hydromagnetic fluid currents were solved by applying the method of Central finite difference. A resulting set of partial differential equations were solved using MATLAB in computer programmer. The study determined the impacts of magnetic path parameter, Prandtl number and Grashof number on speed profiles and temperature distribution. The findings were presented in graphical and tabular form to show the impacts of the non-dimensional parameters arising in the fluid flow.***

Nomenclature

T	Temperature of a fluid [K]
t	Time [s]
Gr	Thermal Grashof Number
g	Acceleration due to gravity [ms^{-2}]
M^2	Magnetic Parameter
Pr	Prandtl number
Gm	Mass Grashof Number

1. Introduction

The Magneto fluid dynamics also known as Magnetohydrodynamics MHD or hydromagnetic is the study of the magnetic behaviors and properties of electrically conducting fluids. Illustrations of such magnetofluids comprise of electrolyte, water salt, metal liquids, and plasmas. The term "magnetohydrodynamics" was generated from magneto, denoting magnetic paths, hydro – denoting water and dynamics denoting movement. The path of MHD was introduced by Alfven Hanns that made him be the winner of 1970 Nobel Price. The essential concepts supporting the MHD is that the magnetic path is able to induce a flow of current in a mobile conductive fluid that in turn will polarize the fluid and mutually alters the magnetic path.

MHD currents uses the momentum, energy and mass equations. MHD movement is linked to Engineering challenges like electronic equipment, cooling of nuclear reactors, liquid metals and plasma confinement. The experimental/theoretical investigations of natural convention movement of incompressible and viscous fluid in a vertical conduit are of countless significance since transient movement is anticipated at the commencement time of numerous engineering devices such as nuclear reactors, pumps, flow meters, accelerators, generators etc. Considering this fact, a number of academicians researched transient natural flow of convection of incompressible and viscous fluid within the parallel plate vertical conduit in various boundary and initial conditions.

2. Literature Review

Odongo *et al* (2021) examined the impacts of natural convection and angle of inclination on velocity profile for MHD where the Runge Kutta Fourth Order scheme was applied to solve the generated ordinary differential equations by applying the similarity transform techniques. The results demonstrated that as the Hartman number and the Grashof number increases, the fluid velocity decreases..

Amos *et al* (2020) conducted research on the magnetohydrodynamic mobile convective mass and heat transfer of dissipative Casson fluid with thermal conductivity and variable viscosity impacts. The outcome demonstrated that a heightened elongating index(n) upsurges the friction of the skin and lowers mass and energy accordingly. The phenomenon relocation results in a reduction in the thermal progression whereas the gradient of the temperature reached its maximum within a variation of (0.4 – 0.6) of the Casson parameter.

Bulinda *et al* (2020) studied Magnetohydrodynamics mobile convention movement of non-compressible liquids over a wavy effervescent bottommost surface with mass, heat and hall currents transfer. The wavy patterns are sinusoidal in nature. The outcome demonstrated that an upsurge in magnetic parameters results in a reduction in the magnitude of secondary profiles and primary profiles. Additionally, a bigger amount of the Prandtl quantity has the propensity of reducing the fluid temperature within the layer of boundary together with the thickness of thermal boundary.

Prabhakar (2019) studied the impacts of thermal diffusion and radiation on a shaky hydro-magnetic natural convection mass and heat transmission flow of incompressible, viscous, chemically and rotating fluid beyond an infinite perpendicular flat plate in the existence of heat sink has been put forward. It was reported that velocity and fluid temperature is heightened boundary film as the radiation parameter rises. The fluid concentration and velocity tend to reduce with rising Schmidt quantity and chemical parameter reaction while the inverse impact is seen with rising Soret quantity. The effect of the physical parameters on the frictional skin demonstrates quite the reserve impact to that of the velocity of fluid.

Sreevani (2018) investigated MHD impacts on the 3-dimensional squeezing transfer of an electrically conducting in a spinning network and its mass and heat transfer features. It was established that the transverse and axial velocities are heightened due to increase in Schmidt number.

Raj *et al* (2018) examined heterogeneous- homogeneous heat absorption and chemical reaction impacts on a 2dimensional steady hydromagnetic Newtonian nano liquid current within an endlessly extending sheet It was established that a practical magnetic field has an impeding effect on the nanofluid species and velocity concentration. Whereas, it does not present any important impact on the nanofluid temperature whereas the heterogeneous and homogeneous tends to reduce the concentration species.

Nyariki *et al* (2017) studied instable the flow of magneto hydrodynamics Couette of an electrically conducting flow of liquid in 2-parallel endless plates within a variable that is inclined within a magnetic path and immovable relative to the mobile permeable plate with pressure. It was realized that a rise in the pressure results into a reduction in the magnetic path, temperature and velocity profiles whereas pressure has a hindering effect on the velocity of the fluid. As the magnetic parameter and the angle of inclination increases, the induced magnetic path and velocity also increases.

Ahmad *et al* (2015) examined the MHD heat and flow transfer via a permeable medium over a stretching/shrinking surface with force. The key results of their research were that the component of viscosity for the flow of fluid over shrinking surface reduced with rising values of the suction parameter and magnetic parameter, however the reverse impacts were established for flow over stretching surface. Similarly, the temperature function reduced with rising values of suction parameter and Prandtl number. The outcome holds for the flow of liquid over shrinking/stretching surface; however, the distribution of temperature was greater for flow over stretching surface compared to the shrinking surface.

Kiragu *et al* (2015) researched on the natural convention of a non-compressible fluid flow beyond the infinite perpendicular flat plate with constant velocity suction. They examined the impacts of Grashof parameter on speed profiles. The outcomes were found and presented in graphs and tables. The findings demonstrated that a rise in Grashof parameter results into a rise in the speed profile close to the plate, however, as the distance lengthens from plate, the speed profile reduces.

Seth *et al* (2013) researched the impacts of rotation and thermal radiation on nonstable hydromagnetic mobile convection flow beyond a spontaneously moving perpendicular plate having the ramped temperature in permeable medium. The solution was found by Laplace change approach. The answers gotten from various values of heat sink parameters and Prandtl number (Pr) demonstrated that a rise within the parameters of the heat sink reduces both the Nusselt number and the skin friction.

The specific objectives of the study are:
1) To develop mathematical modelling of the hydromagnetic natural heat flow convection absorbing fluid beyond a spontaneously flowing vertical plate.
2) To determine the impacts of varying Grashof quantity and magnetic field parameter on the velocity profiles
3) To determine the impacts of varying the Prandtl number on temperature distribution

3. Mathematical Formulation

Consider unsteady movement of heat-absorbing, electrically conducting, incompressible, and the flow of viscous fluid beyond an infinite perpendicular plate in the existence of a homogeneous magnetic path. X-axis is extrapolated on perpendicular plates in the ascending trajectory and y-axis normal to the plate in the liquid.

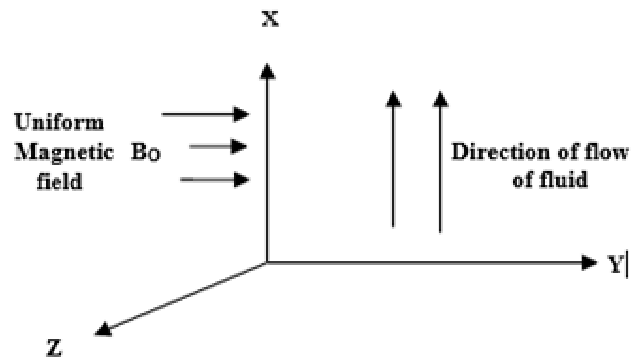

Figure 1: Flow Configuration

Fluid is infused by a homogeneously transverse magnetic path B_0 applied parallel to y-axis. Originally, that is at time $t \leq 0$ **,** both the plate and fluid are at rest and at a unchanging temperature T_∞ . At time $t > 0$ plate begins to move in x-direction with unvarying velocity U_0. By the time $0 < t \leq t_0$ the plate temperature is lowered or raised to $T_\infty + (T_w - T_\infty) t / t_0$and then it is kept at unvarying temperature T_w that is when (t_0 being the characteristic time. It is presumed that the induced magnetic path created by the moving fluid is ignored, compared to the applied one.

The components of velocity are taken as u and v in the x and y directions correspondingly. Based on the Boussinesq's calculation within the boundary layer, the steady, laminar, two-dimensional boundary layer flow under consideration is governed by the equations of continuity, momentum, and energy respectively as follows; Seth et *al* (2013):

Continuity equation

The equation shows that the flow is continuous

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=0 \qquad (1)$$

4. Momentum Equation

The momentum equation, which is derived from Newton's Second Law, connects the sum of forces acting on a fluid element to its acceleration or rate of change of momentum.

Momentum equation in the x direction

$$\rho\frac{\partial u}{\partial t}=\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+g\beta\rho\left(T-T_\infty\right)-\sigma\mu_e^2H^2u \qquad (2)$$

Momentum equation in the y direction

$$\frac{\partial v}{\partial t}=\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\sigma\mu^2H_o^2v}{\rho} \qquad (3)$$

Energy equation

The Energy Equation, which involves energy, heat transmission, and work, is based on the First Law of Thermodynamics.

$$\frac{\partial T}{\partial t}=\frac{\kappa}{\rho Cp}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-\frac{Q_o}{\rho Cp}\left(T-T_\infty\right) \qquad (4)$$

where $u, K, Q_o, \rho, \upsilon, \sigma, k, C_p$ are respectively, velocity of the fluid, permeability of porous medium, heat absorption coefficient, density, kinetic coefficient of viscosity, electrical conductivity, thermal conductivity and specific heat at constant pressure.

5. Specific Momentum Equations

Equations (2) and (3), in non-dimensional form, become

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}-\frac{M^2u}{k_1}+GrT+Gm\phi \qquad (5)$$

$$\frac{\partial v}{\partial t}=\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}-\frac{M^2v}{k_1} \qquad (6)$$

Specific Energy Equation

In non-dimensional form, we obtain;

$$\frac{\partial\theta}{\partial t}=\frac{\kappa}{\rho c_p}\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)-\phi\theta \qquad (7)$$

6. Method of Solution

The solutions were determined using Forward Time Central Difference method. The resulting linear equations was tackled using the central difference approximations that entails choosing a homogenous mesh which comprises of a web of rectangles of breadth Δx and Δy.

7. Discretization of momentum equations

We consider momentum equations in both vertical and horizontal directions.

a) Vertical velocity

We consider the fluid velocity along the vertical direction of the plates. For the Forward Time central space scheme (FTCS), the values u_t is substituted by forward difference approximation while u_{xx} and u_{yy} are substituted by central difference approximation. When these values are substituted into partial derivatives appearing in (5), we get Forward Time central space scheme as follows;

$$\left[\frac{U_{i,j}^{n+1}-U_{i,j}^{n}}{\Delta t}\right]=\left[\frac{U_{i+1,j}^{n}-2U_{i,j}^{n}+U_{i-1,j}^{n}}{(\Delta x)^2}\right]+\left[\frac{U_{i,j+1}^{n}-2U_{i,j}^{n}+U_{i,j-1}^{n}}{(\Delta y)^2}\right]-\frac{M^2}{k_1}\left[\frac{U_{i+1,j}^{n}+U_{i,j}^{n}}{2}\right]+GrT+Gm\phi \qquad (8)$$

The effect of M, Gr and Gm on the velocity of the fluid are investigated. Given that $\Delta t=0.01$ and $\Delta x=\Delta y=0.05$, k_1 = 0.2, 4,5, and $T=\phi=1$,equation 8 reduces to;

$$\left(4.03+0.05M^2\right)U_{i+1,j}^{n}-14U_{i,j}^{n}+4U_{i-1,j}^{n}=U_{i,j}^{n+1}-4U_{i,j+1}^{n}-4U_{i,j-1}^{n}-0.01G_r-0.01G_m \qquad (9)$$

Taking i = 1,2,3,4,5,6,7…….10 , n = 0 and j =1 ,a system of linear algebraic equations are formed .

$$\left.\begin{aligned}
&\left(4.03+0.05M^2\right)U_{2,1}^{0}-14U_{1,1}^{0}+4U_{0,1}^{0}=U_{1,1}^{0}-4U_{1,2}^{0}-4U_{1,0}^{0}-0.01Gr-0.01Gm\\
&\left(4.03+0.05M^2\right)U_{3,1}^{0}-14U_{2,1}^{0}+4U_{1,1}^{0}=U_{2,1}^{0}-4U_{2,2}^{0}-4U_{2,0}^{0}-0.01Gr-0.01Gm\\
&\left(4.03+0.05M^2\right)U_{4,1}^{0}-14U_{3,1}^{0}+4U_{2,1}^{0}=U_{3,1}^{0}-4U_{3,2}^{0}-4U_{3,0}^{0}-0.01Gr-0.01Gm\\
&\left(4.03+0.05M^2\right)U_{5,1}^{0}-14U_{4,1}^{0}+4U_{3,1}^{0}=U_{4,1}^{0}-4U_{4,2}^{0}-4U_{4,0}^{0}-0.01Gr-0.01Gm\\
&\left(4.03+0.05M^2\right)U_{6,1}^{0}-14U_{5,1}^{0}+4U_{4,1}^{0}=U_{5,1}^{0}-4U_{5,2}^{0}-4U_{5,0}^{0}-0.01Gr-0.01Gm\\
&\left(4.03+0.05M^2\right)U_{7,1}^{0}-14U_{6,1}^{0}+4U_{5,1}^{0}=U_{6,1}^{0}-4U_{6,2}^{0}-4U_{6,0}^{0}-0.01Gr-0.01Gm\\
&\left(4.03+0.05M^2\right)U_{8,1}^{0}-14U_{7,1}^{0}+4U_{6,1}^{0}=U_{7,1}^{0}-4U_{7,2}^{0}-4U_{7,0}^{0}-0.01Gr-0.01Gm\\
&\left(4.03+0.05M^2\right)U_{9,1}^{0}-14U_{8,1}^{0}+4U_{7,1}^{0}=U_{8,1}^{0}-4U_{8,2}^{0}-4U_{8,0}^{0}-0.01Gr-0.01Gm\\
&\left(4.03+0.05M^2\right)U_{10,1}^{0}-14U_{9,1}^{0}+4U_{8,1}^{0}=U_{9,1}^{0}-4U_{9,2}^{0}-4U_{9,0}^{0}-0.01Gr-0.01Gm\\
&\left(4.03+0.05M^2\right)U_{11,1}^{0}-14U_{10,1}^{0}+4U_{9,1}^{0}=U_{10,1}^{0}-4U_{10,2}^{0}-4U_{10,0}^{0}-0.01Gr-0.01Gm
\end{aligned}\right\} \qquad (10)$$

Paper ID: SR22328150026 DOI: 10.21275/SR22328150026

With initial conditions $U_{i,0}^{0} = U_{i,j}^{0} = 1$,and boundary conditions $U_{1,1}^{1} = 0$,a matrix form is obtained from the algebraic equations (10).

$$\begin{bmatrix} -14 & (4.03+0.05M^2) & 0 & L & 0 & 0 \\ 4 & -14 & (4.03+0.05M^2) & O & M & 0 \\ M & 4 & O & O & 0 & M \\ 0 & 0 & O & -14 & (4.03+0.05M^2) & 0 \\ 0 & L & O & 4 & -14 & (4.03+0.05M^2) \\ 0 & 0 & K & 0 & 4 & -14 \end{bmatrix} \begin{bmatrix} U^0{}_{1,1} \\ U^0{}_{2,1} \\ U^0{}_{3,1} \\ U^0{}_{4,1} \\ U^0{}_{5,1} \\ U^0{}_{6,1} \\ U^0{}_{7,1} \\ U^0{}_{8,1} \\ U^0{}_{9,1} \\ U^0{}_{10,1} \end{bmatrix} = \begin{bmatrix} -8-0.01Gr-0.01Gm \\ -8-0.01Gr-0.01Gm \\ -8-0.01Gr-0.01Gm \\ -8-0.01Gr-0.01Gm \\ -8-0.01Gr-0.01Gm \\ -8-0.01Gr-0.01Gm \\ -8-0.01Gr-0.01Gm \\ -8-0.01Gr-0.01Gm \\ -8-0.01Gr-0.01Gm \\ -8-0.01Gr-0.01Gm \end{bmatrix} \quad (11)$$

Solving the above matrix equation (11) using MATLAB, we get the vertical velocity profiles for varying M^2, Gr and Gm numbers

b) Horizontal velocity

We consider the fluid velocity along the horizontal direction of the plates For the Forward Time central space scheme (FTCS), the values v_t is substituted by forward difference approximation while v_{xx} and v_{yy} are substituted by central difference approximation. When these values are substituted into partial derivatives , we get Forward Time central space scheme as follows;

$$\left[\frac{V_{i,j}^{n+1} - V_{i,j}^{n}}{\Delta t}\right] = \left[\frac{V_{i+1,j}^{n} - 2V_{i,j}^{n} + V_{i-1,j}^{n}}{(\Delta x)^2}\right] + \left[\frac{V_{i,j+1}^{n} - 2V_{i,j}^{n} + V_{i,j-1}^{n}}{(\Delta y)^2}\right] - \frac{M^2}{k_1}\left[\frac{V_{i+1,j}^{n} + V_{i,j}^{n}}{2}\right] \quad (12)$$

The effect of M on the fluid horizontal velocity was investigated. Given that $\Delta t = 0.01$ and $\Delta x = \Delta y = 0.05$, and k_1 =4.0,4.5,5.0, equation 12 reduces to;

$$\left(1250 - M^2\right)V_{i,j+1}^{n} + \left(308.5 - M^2\right)V_{i,j}^{n} + V_{i,j-1}^{n} = 312.5V_{i,j}^{n+1} - 1250V_{i+1,j}^{n} - 1250V_{i-1,j}^{n} \quad (13)$$

With initial conditions $V_{i,j}^{0} = 0$ and boundary conditions $V_{i,j}^{n} = 1$ and taking i=1,2……10 n = 0 and j =1, we form the following matrix equation;

$$\begin{bmatrix} (308-M^2) & (1250-M^2) & 0 & L & 0 & 0 \\ 1250 & (308-M^2) & (1250-M^2) & O & M & 0 \\ M & 1250 & O & O & 0 & M \\ 0 & 0 & O & (308-M^2) & (1250-M^2) & 0 \\ 0 & L & O & 1250 & (308-M^2) & (1250-M^2) \\ 0 & 0 & K & 0 & 1250 & (308-M^2) \end{bmatrix} \begin{bmatrix} V^0{}_{1,1} \\ V^0{}_{2,1} \\ V^0{}_{3,1} \\ V^0{}_{4,1} \\ V^0{}_{5,1} \\ V^0{}_{6,1} \\ V^0{}_{7,1} \\ V^0{}_{8,1} \\ V^0{}_{9,1} \\ V^0{}_{10,1} \end{bmatrix} = \begin{bmatrix} 312.5 \\ 312.5 \\ 312.5 \\ 312.5 \\ 312.5 \\ 312.5 \\ 312.5 \\ 312.5 \\ 312.5 \\ 312.5 \end{bmatrix} \quad (14)$$

Solving the above matrix equation (14) using MATLAB, we get the horizontal fluid velocity profile for varying M^2 parameter

C. Energy equation

Discretizing the energy equation (7) becomes

$$\frac{\theta_{i,j}^{n+1} - \theta_{i,j}^{n}}{\Delta t} = \frac{1}{\Pr}\left(\frac{\theta_{i+1,j}^{n} -{}_{i,j}^{n}\theta + \theta_{i-1,j}^{n}}{(\Delta x)^2} + \frac{\theta_{i,j+1}^{n} - \theta_{i,j}^{n} + \theta_{i,j-1}^{n}}{(\Delta y)^2}\right) - \phi\left(\frac{\theta_{i+1,j}^{n} + \theta_{i,j}^{n}}{2}\right) \quad (15)$$

Multiply by Δt

$$\frac{\theta_{i,j}^{n+1} - \theta_{i,j}^{n}}{1} = \frac{\Delta t}{\Pr}\left(\frac{\theta_{i+1,j}^{n} -{}_{i,j}^{n}\theta + \theta_{i-1,j}^{n}}{(\Delta x)^2} + \frac{\theta_{i,j+1}^{n} - \theta_{i,j}^{n} + \theta_{i,j-1}^{n}}{(\Delta y)^2}\right) - \phi\Delta t\left(\frac{\theta_{i+1,j}^{n} + \theta_{i,j}^{n}}{2}\right) \quad (16)$$

Re-arranging (16)

$$\frac{\theta_{i,j}^{n+1} - \theta_{i,j}^{n}}{1} = \frac{1}{\Pr}.\frac{\Delta t}{(\Delta x)^2}\left(\frac{\theta_{i+1,j}^{n} -{}_{i,j}^{n}\theta + \theta_{i-1,j}^{n}}{1} + \frac{\theta_{i,j+1}^{n} - \theta_{i,j}^{n} + \theta_{i,j-1}^{n}}{1}\right) - \frac{\phi\Delta t}{2}\left(\frac{\theta_{i+1,j}^{n} + \theta_{i,j}^{n}}{1}\right) \quad (17)$$

Taking $\phi = 4$, $\Delta x = \Delta y = 0.5$ and $\Delta t = 0.01$

$$\frac{\theta_{i,j}^{n+1} - \theta_{i,j}^{n}}{1} = \frac{1}{\Pr}\cdot\frac{\Delta t}{(0.5)^2}\left(\frac{\theta_{i+1,j}^{n} -_{i,j}^{n}\theta + \theta_{i-1,j}^{n}}{1} + \frac{\theta_{i,j+1}^{n} - \theta_{i,j}^{n} + \theta_{i,j-1}^{n}}{1}\right) - \frac{4\times 0.01}{2}\left(\frac{\theta_{i+1,j}^{n} + \theta_{i,j}^{n}}{1}\right) \quad (18)$$

Which gives

$$(1.02 - 25\Pr)\theta_{i,j}^{n} - 0.08\theta_{i+1,j}^{n} - \theta_{i-1,j}^{n} = -25\Pr\theta_{i,j}^{n+1} \quad (19)$$

Taking i =1,2,3,4,5,6…….10 , n = 0 and j =1 in (19), the following systems of linear algebraic equations are formed.

$$\left.\begin{aligned}
(1.02 - 25\Pr)\theta_{1,1}^{0} - 0.08\theta_{2,1}^{0} - \theta_{0,1}^{0} &= -25\Pr\theta_{1,1}^{1}\\
(1.02 - 25\Pr)\theta_{2,1}^{0} - 0.08\theta_{3,1}^{0} - \theta_{1,1}^{0} &= -25\Pr\theta_{2,1}^{1}\\
(1.02 - 25\Pr)\theta_{3,1}^{0} - 0.08\theta_{4,1}^{0} - \theta_{2,1}^{0} &= -25\Pr\theta_{3,1}^{1}\\
(1.02 - 25\Pr)\theta_{4,1}^{0} - 0.08\theta_{5,1}^{0} - \theta_{3,1}^{0} &= -25\Pr\theta_{4,1}^{1}\\
(1.02 - 25\Pr)\theta_{5,1}^{0} - 0.08\theta_{6,1}^{0} - \theta_{4,1}^{0} &= -25\Pr\theta_{5,1}^{1}\\
(1.02 - 25\Pr)\theta_{6,1}^{0} - 0.08\theta_{7,1}^{0} - \theta_{5,1}^{0} &= -25\Pr\theta_{6,1}^{1}\\
(1.02 - 25\Pr)\theta_{7,1}^{0} - 0.08\theta_{8,1}^{0} - \theta_{6,1}^{0} &= -25\Pr\theta_{7,1}^{1}\\
(1.02 - 25\Pr)\theta_{8,1}^{0} - 0.08\theta_{9,1}^{0} - \theta_{7,1}^{0} &= -25\Pr\theta_{8,1}^{1}\\
(1.02 - 25\Pr)\theta_{9,1}^{0} - 0.08\theta_{10,1}^{0} - \theta_{8,1}^{0} &= -25\Pr\theta_{9,1}^{1}\\
(1.02 - 25\Pr)\theta_{10,1}^{0} - 0.08\theta_{11,1}^{0} - \theta_{9,1}^{0} &= -25\Pr\theta_{10,1}^{1}
\end{aligned}\right\} \quad (20)$$

With initial and boundary conditions $\theta_{0,1}^{0} = 1$ and $\theta_{1,1}^{1} = 0$ respectively , we get the matrix equation from as;

$$\begin{pmatrix}
1.02-25\Pr & -0.08 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-1 & 1.02-25\Pr & -0.08 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -1 & 1.02-25\Pr & -0.08 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -1 & 1.02-25\Pr & -0.08 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & 1.02-25\Pr & -0.08 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -1 & 1.02-25\Pr & -0.08 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1 & 1.02-25\Pr & -0.08 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1.02-25\Pr & -0.08 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1.02-25\Pr & -0.08\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1.02-25\Pr
\end{pmatrix}
\begin{pmatrix}\theta_{1,1}^{0}\\ \theta_{2,1}^{0}\\ \theta_{3,1}^{0}\\ \theta_{4,1}^{0}\\ \theta_{5,1}^{0}\\ \theta_{6,1}^{0}\\ \theta_{7,1}^{0}\\ \theta_{8,1}^{0}\\ \theta_{9,1}^{0}\\ \theta_{10,1}^{0}\end{pmatrix}
=
\begin{pmatrix}1\\0\\0\\0\\0\\0\\0\\0\\0\\0\end{pmatrix} \quad (21)$$

Solving the above matrix equation (21) using . We investigate the effects of Pr = 0.7, 0.9, 1.0 on the speed fluid and the distribution of temperature for Prandtl number of various values.

8. Results and Discussion

The section presents the findings of the numerical investigation obtained from (11), (14) and (21) of the effects of varying of Grashof number, magnetic field parameter and Prandtl number on velocity and temperature distribution profiles. They are given in tabular and graphical forms. Further, physical observations are made on the data and discussed by giving scientific explanation.

a) Magnetic field parameter on vertical velocity

Results for varying Magnetic field parameter were obtained numerically from equation (11) and presented in the figure below.

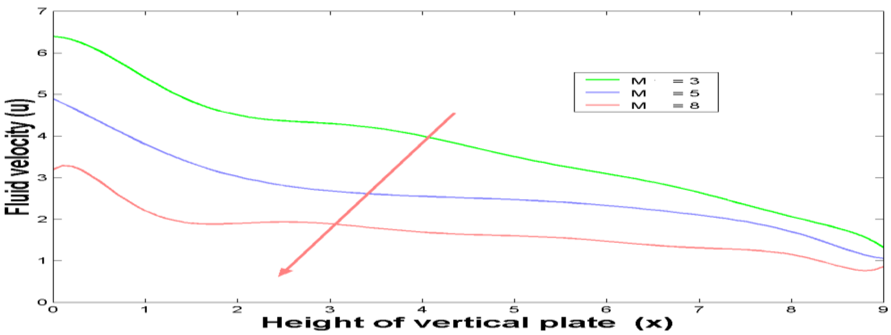

Figure 2: Velocity versus distance of the plate when varying the Magnetic parameter

From Fig 2,the velocity of the fluid decreases when the height of the plate is increased also the velocity of the fluid is higher closer to the boundary surface and decreases as it travels away from the boundary.This because an enhanced magnetic field parameter gives a decrease in velocity since Lorentz force acted against the fluid flow is introduced into an electrically conducting fluid due to magnetic field. This offers resistance to the flow thus the velocity is reduced.

b) Magnetic field parameter on Horizontal velocity.

The numerical investigation results for varying Magnetic parameter obtained from equation (14) are presented in Fig.3

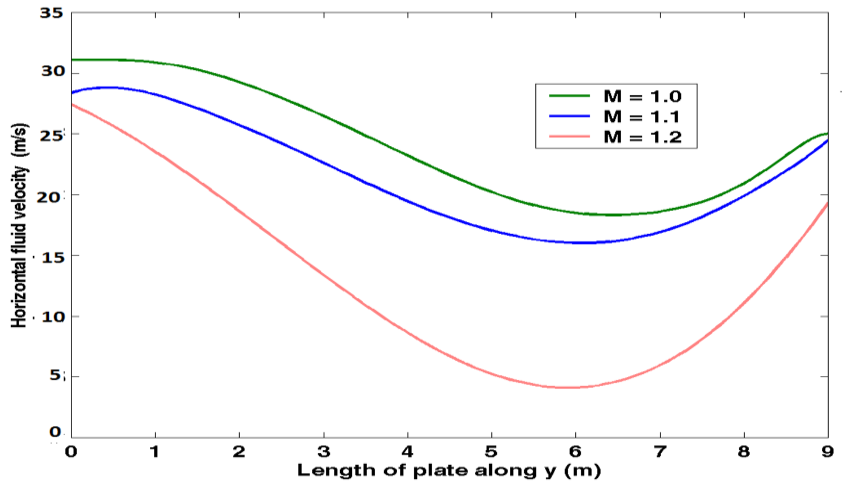

Figure 3: Horizontal velocity graph versus Plate distance with varying the Magnetic parameter

From Fig 3, the velocity of the fluid reduces with an increase in the height of vertical plate but rises steadily due to minimal interference of the magnetic field on the flow field. This implies that the fluid velocity reduces when the values of the magnetic field parameter is enhanced

c) Effects of Grashof number on vertical velocity

The effects of Varying Grashof number were numerically investigated from equation (11) and the findings are presented in Fig 4

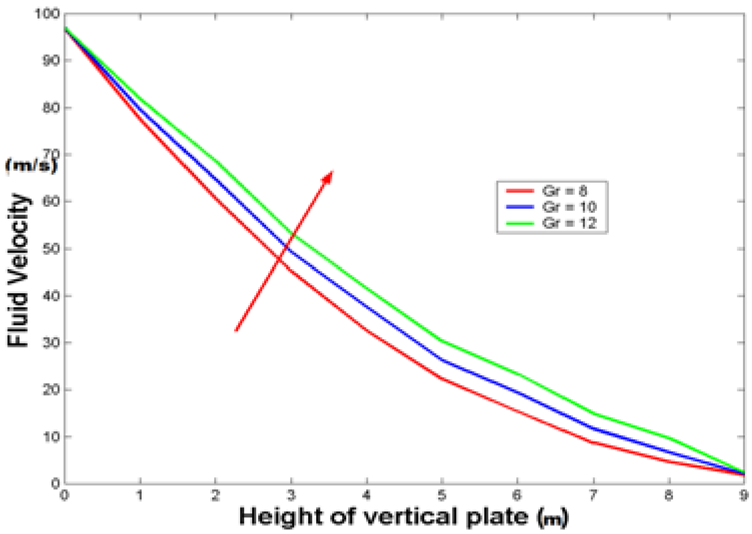

Figure 4: Velocity versus Plate distance when varying Grashof number

From Fig 4, the Grashof number is directly proportional to the velocity profile since fluid with greater Grashof number flows at a faster velocity than a fluid with low Grashof number. This is because the Grashof number gives the ratio of the buoyancy forces to viscous forces therefore a fluid which is less viscous possess greater Grashof number .It implies that a greater Grashof number is as a result of viscous forces being smaller than the buoyancy forces the fluid moves with a lot of ease since the retarding force is minimized

d) Effects of mass Grashof number on vertical velocity

The numerical investigation results for varying mass Grashof number obtained from equation (11) .

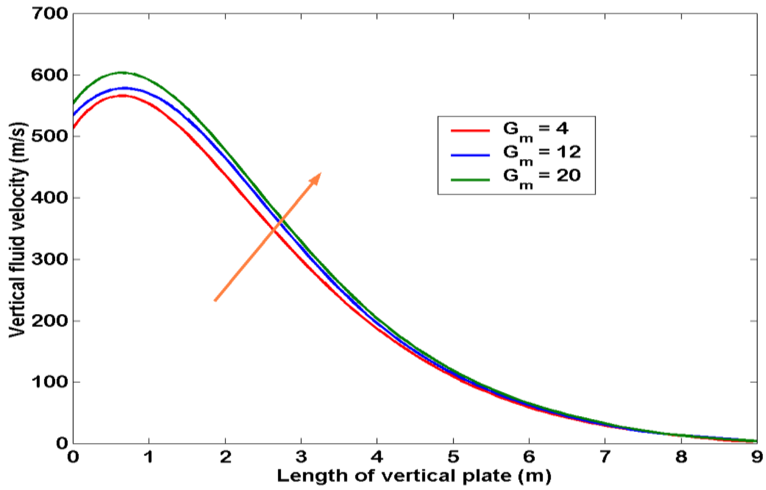

Figure 5: Velocity graph versus distance of the plate with varying the mass Grashof number

From Fig 5, the velocity of the rises steadily at x=0 this is due to the boundary conditions. The fluid velocity then decreases as the fluid moves away from the surface. The velocity of fluid reduces exponentially as it travels far from the surface. The fluid with lower mass Grashof number decays faster than those with higher values therefore fluid velocity is directly proportional to the mass Grashof number

e) Effects of Prandtl number on Temperature distribution

The numerical investigation results for varying Prandtl number obtained from equation (21) are presented in Fig. 6

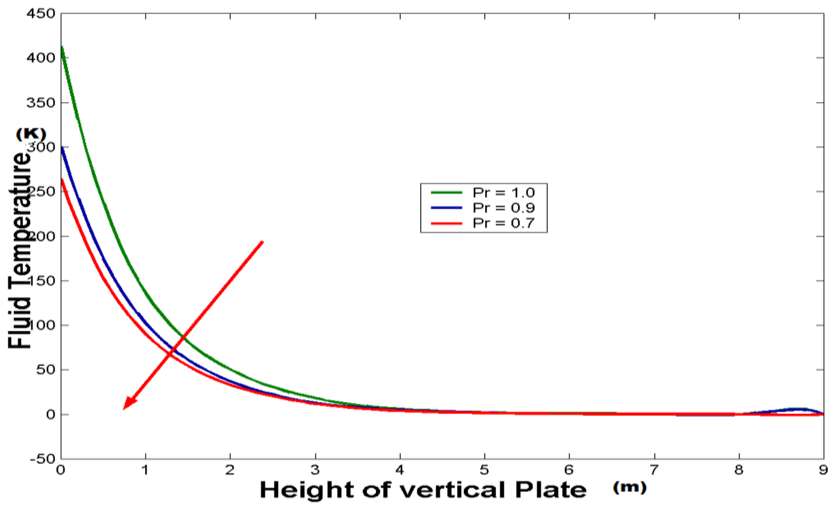

Figure 6: Temperature against Plate distance when varying the Pr number

From Fig 6, fluid temperature reduces exponentially with increase in the height of the vertical plate for all the values of Prandtl number.

It shows that the temperature of fluid is enhanced closer to the plate and it declines as it moves far from the plate with an increase distance of the plate. The rise in the value of the Prandtl number gives a corresponding decrease in the temperature of the fluid. This is because *Prandtl* number compares the relative thickness of the momentum to thermal boundary layers. When the value of *Pr*is of low , thermal diffusivity surpasses momentum diffusivity, temperatures will be great

9. Conclusions and Recommendation

In conclusion, higher value of Prandtl number results to a reduction in the temperature distribution and a decrease in Prandtl number values results to increase in the temperature distribution implying that the momentum diffusivity increases as the heat diffusivity decreases thus the fluid velocity decreases and the fluid flow becomes laminar

A higher value of Grashof number results to an enhanced velocity profile and a reduction in values of Grashof number results to a reduced velocity profiles implying that the fluid is more buoyant hence the fluid experiences a turbulent flow as a result of buoyant forces being higher than the forces of viscosity.

An enhanced Magnetic field parameter values results to a reduced velocity profile.

Recommendation

From this study, we recommend that further work be done on the specific areas.

1) Studying an electrically conducting fluid past an oscillating vertical permeable medium with Pr > 2 in unsteady MHD convective flow.
2) Studying an incompressible electrically conducting fluid past an oscillating perpendicular permeable plate in a permeable medium with a non-homogeneous transverse magnetic path in unsteady MHD convective flow.

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