# Analysis of Forced Convection on an Incompressible Fluid Flow in a Rectangular Enclosure

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**Abstract:** ***A numerical analysis of convective heat transfer in a three-dimensional rectangular enclosure is presented in the manuscript, with uniform heat flux from middle of the floor, a centrally placed fan at the ceiling and ventilation holes at the upper parts on the opposite vertical walls. The temperature and velocity profiles as a result of forced convection was investigated. The fluid flow under consideration was assumed to be incompressible, Newtonian and steady.The fan was set to rotate at constant speed. To analyze the flow and heat transfer rates, a complete set of non-dimensionalzed equations governing Newtonian fluid flow with boundary conditions were discretized using the Three-point Central Difference Approximations for a uniform mesh. MATLAB simulation software was used to solve the resulting finite difference equations. The velocity profiles and temperature distributions in a room were shown in tables and graphs for various Reynolds and Richardson numbers and at a constant Prandtl number of 0.71.It was discovered that as the Reynolds number rises, the velocity rises due to increasing inertial forces in comparison to viscous forces.It was also discovered that as the Reynolds number rises, the temperature rises as well. The data also reveal that when the Richardson number increases, the velocity decreases due to the low buoyancy effect at low Richardson numbers. It was also clear that the fan greatly enhances the fluid velocity but lowers the temperature in the enclosure.***

**Keywords:** Steady flow, forced convection, heat flow, rectangular enclosure

**Nomenclature**

| Symbol | Meaning |
|---|---|
| $\eta$ | Momentum diffusivity |
| $C_p$ | Specific heat capacity at constant pressure |
| t | Time in seconds |
| p | Pressure in $N/m^2$ |
| u, v | Velocity in x and y directions respectively |
| x, y | Coordinates in cartesian |
| L | Characteristic length |
| Pr | Prandtl number, a ratio of momentum diffusivity to thermal diffusivity $=\frac{\eta}{\alpha}$ |
| Re | Reynolds number, a ratio of inertial forces to viscous forces $=\frac{UL}{\eta}$ |
| $\propto$ | The thermal diffusivity given by $\frac{k}{\rho C_p}$ |
| K | Thermal conductivity of fluid (W/mk) |
| Θ | Dimensionless temperature |
| Φ | Dissipation function of scalar transport |
| μ | Eddy viscosity |
| ρ | Density |
| q | Heat flux |
| β | Thermal expansion coefficient of air |

## 1. Introduction

Heat transfer by forced convection make use of fan or pump to provide high velocity fluid and as a result, the thermal resistance across the fluid-to-heated-surface boundary decreases, increasing the amount of heat carried away by the fluid. To improve heat exchange, fans or pumps have been included in the design of most electrical equipment. In their operations, this equipmentdissipate heat and hence requires cooling systems that can hasten the mixing of fluid. Convection entails the transfer of heat and represents actual movement of molecules within a fluid. Forced convection; which is motion caused by external force such as a pump or a fan is encountered in our everyday life like in our homes, offices computers, vehicles and Television and all this requires an effective heating and cooling systems.

The specific objectives of the study include:
1) To develop a model for forced convection of an incompressible fluid on a rectangular enclosure;
2) To determine the temperature and velocity distribution in a rectangular enclosure caused by forced convection;

3) To investigate the effect of Richardson number on velocity profiles in a rectangular enclosure caused by forced convection;
4) To investigate the effect of Reynolds number on temperature and velocity profiles in a rectangular enclosure.

## 2. Literature Review

Hyung *et al.* (1995), studied forced convection from isolated heat sources in a channel with a porous medium and results showed that in a view of pressure drop the employment of a thicker and denser porous substrate in electronic cooling is less desirable and that as the thickness of the porous substrate increases the flow rate also increases. Sigey *et al.* (2004), considering a three-dimensional enclosure in the form of a room with a convectional heater built into the walls and a window in the same wall, explored turbulent flow in great detail. By using the Finite Difference Technique, they found out that turbulent natural convection plays a major role in temperature distribution in an enclosure. The room is divided into a number of regions with the near the source regions having high temperatures and those near the windows having low temperatures

Using finite approximations, a study of turbulent mixed convection in air-filled enclosures was carried out byHamid *et al*. (2011). The results showed that as Reynolds increases, the circulation of flow vortices grows and gets stronger, making forced convection more dominant for various values of Richardson number. Natural convection is also a prominent component of heat transfer in a cavity at large Richardson number. Mohd Z. *et al.* (2011) used the finite difference approach to investigate numerical solutions of forced convection boundary layer flow on a horizontal circular cylinder with Newtonian heating, and the results showed that increasing the Prandtl number resulted in a drop in temperature profiles. Ghadimi *et al*. (2012) used numerical simulation of heat transfer to investigate free and forced convection in air flow windows. Air flow influence increased in air flow windows, according to the findings (in both forced and natural convection). It also demonstrates that air flow is related to inlet temperature and flow rate, with the temperature influence being greater than the flow rate effect.

Salim G. (2014) conducted a numerical study of forced convection in a rectangular channel using finite volume elements, and the results showed that the velocity profiles and calculated temperature have a side effect on the input speed limits for two developing layers that extend over a more or less large length depending on the Reynolds number.In a rectangular room, Momanyi J. N. (2015) investigated the effects of forced convection on temperature distribution and velocity profile, with heater placed on opposite walls, two windows on adjacent opposite walls and one fan centrally placed on the ceiling and using finite difference method they found out that forced convection affects velocity profile and temperature profile. Temperature dropped up the room as the fan reduced pressure, resulting in low pressures, with the highest temperature near the heater, followed by the windows, and the lowest near the fan.

The effect of forced convection on temperature distribution and velocity profile in a rectangular room was explored by Atieno E. O *et al* (2019), Using central difference approximations to obtain finite difference equations which are solved using MATLAB computer programme. They discovered that as the Reynolds number rises, the velocity rises as well. Temperature rises as the Reynolds number decreases. Faraz Afshar (2019) also did experimental and numerical study forced convection to increase heat transfer in home radiators by employing solar cells and fans. The problem was simulated using ANSYS – Fluent programme to compare obtained numerical results to those recorded experimentally. They realized that the average heat transfer rate improved by about 21% by using forced convection.

To evaluate velocity, temperature, and heat fluxes, Mahmood A. M et al (2020) conducted an experimental research of forced convection heat transfer in a partially opened box filled with porous media. They found out that the average Nusselt number increase with the increase of Reynold and decreases with the high increase of heat flux. Aadel A. R *et al* (2020) did experimental and numerical study of forced convection heat transfer in different internally ribbed tubes configuration using a Nano fluid. By using the finite volume element, they found out that the Nusselt number increase as the Nano fluid concentration increases. Adhikari R.C *et al* (2020) conducted experimental and numerical studies of forced convection heat transfer from fins at low Reynolds number. It was found that heat transfer rate decreased linearly with the increase in channel length but remained approximately constant

## 3. Statement of the Problem

Forced convection arising from constant heating and cooling is encountered in a number of practical occurrences such as use of convectional heaters and cooling fans. A rapid technological advance in the construction industries requires that design and operation problems are resolved quickly in order to keep industries competitive in terms of energy efficiency and obtaining optimum temperatures in any enclosure. There is need to vary the geometry and position of heating and cooling systems in order to find out the most economical and efficient geometry and position to use otherwise there may be a lot of wastage which may lead to economic stagnation In the past studies have been conducted in a rectangular enclosure with different positions of the heating source, fan, windows and ventilation holes and results have been obtained on their effects on both temperature and velocity profile As a result, a model that can assist in the solution of fluid properties in forced convection is required. The model in this research allows for determination of temperature distribution and velocity profile of air in the rectangular enclosure and heaters and fan placed at the Centre of the enclosure

## 4. Mathematical Formulation

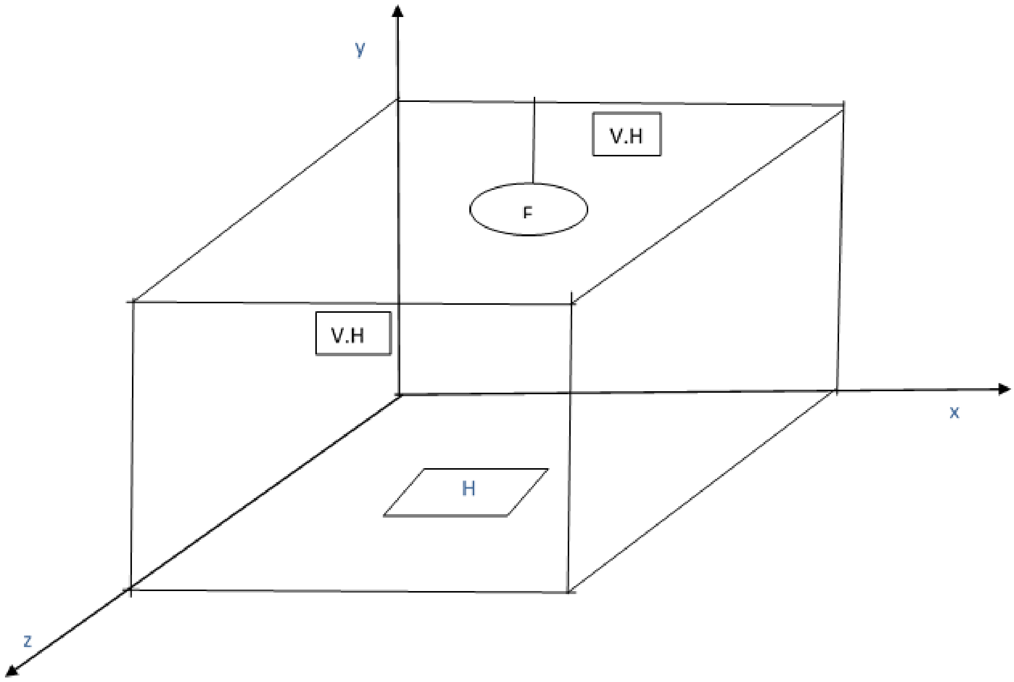


**Figure 1:** Geometry of the problem

The base is to be heated (H) slightly which will change the room temperature causing a change in density of the fluid. The ventilation holes (V.H) will be introducing cold air into the enclosure. The fan (F) which is to be set at an appropriate constant speed is to be majorly responsible for forced convection since it will be pushing cold air over the warm heated air.

## 5. Governing Equations

From the universal laws we get the following governing equations

**a) Continuity Equation**

This is a mathematical equation of the principle of mass conservation. The law of conservation of mass states that the rate of increase of mass within a controlled volume is equal to the net rate of influx through the controlled surface

$$\frac{\partial \rho}{\partial t}\frac{\partial(\rho u_j)}{\partial x_j} = 0 \quad (1)$$

**b) Momentum Equations**

Derived from Newton's second law of motion which states that the totals of a system's body and surface forces equals the system's rate of change in linear momentum.

**i. X- Component**

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad (2)$$

**ii. Y- component**

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2}\right) + g\beta(T - T_0) \quad (3)$$

Where p is the thermodynamic pressure, μ is the kinematic viscosity and $g\beta(T - T_0)$ is the body force due to gravity

**c) The Energy Equation**

This is derived from the first law of thermodynamics, which states that the rate of energy gain in a system is proportional to the heat given to it and the work done on it.

$$\rho c_p(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\emptyset \quad (4)$$

Equations (2), (3) and (4) can be non- dimensionalized to get

**i. Momentum in x direction**

$$u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = -\frac{\partial p'}{\partial x'} + \frac{1}{Re}\left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2}\right) \quad (5)$$

**ii. Momentum in y direction**

$$u'\frac{\partial v'}{\partial x'} + v'\frac{\partial v'}{\partial y'} = -\frac{\partial p'}{\partial y'} + \frac{1}{Re}\left(\frac{\partial^2 v'}{\partial x'^2} + \frac{\partial^2 v'}{\partial y'^2}\right) + Ri\theta \quad (6)$$

**d) Energy equation**

$$\left[u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right] = \frac{1}{RePr}\left[\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right] \quad (7)$$

Equations (5), (6) and (7) can be discretised using Three Point Central Difference Approximations as shown below.

## 6. Method of Solution

**Horizontal Velocity**

To investigate the impact of Re on horizontal velocity profiles, Equation (5) is discretized Using a central difference numerical scheme, $U_x$, $U_y$, $U_{xx,}$ $U_{yy}$,and$P_x$is replaced by central difference approximation. In equation (5) Substituting these approximations yields

$$U\frac{U_{i+1,j} - U_{i-1,j}}{2\Delta x} + V\frac{U_{i,j+1} - U_{i,j+1}}{2\Delta y} = -\frac{P_{i+1,j} - P_{i-1,j}}{2\Delta x} + \frac{1}{Re}\left[\frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{(\Delta x)^2} + \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta y)^2}\right] \quad (8)$$



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**Vertical Velocity:**

In equation(6),$V_x$, $V_y$, $V_{xx}$, $V_{yy}$ *and* $P_y$is replaced by central difference approximation to get

$$U\frac{V_{i+1,j}-V_{i-1,j}}{2\Delta x}+V\frac{V_{i,j+1}-U_{i,j-1}}{2\Delta y}=-\frac{P_{i,j+1}-P_{i,j-1}}{2\Delta y}+\frac{1}{Re}\left[\frac{V_{i+1,j}-2V_{i,j}+V_{i-1,j}}{(\Delta x)^2}+\frac{V_{i,j+1}-2V_{i,j}+U_{i,j-1}}{(\Delta y)^2}\right]+Ri\theta \qquad (9)$$

**Temperature Distribution**

To investigate the impact of Re on temperature profiles, the equation (7) is discretized. using a mixed numerical technique we obtain

$$\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}+U\frac{\theta_{i+1,j}-\theta_{i-1,j}}{2\Delta x}+\frac{\theta_{i,j+1}-\theta_{i,j-1}}{2\Delta y}=\frac{1}{RePr}\left[\frac{\theta_{i+1,j}-2\theta_{i,j}+\theta_{i-1,j}}{(\Delta x)^2}+\frac{\theta_{i,j+1}-2\theta_{i,j}+\theta_{i,j-1}}{(\Delta y)^2}\right] \qquad (10)$$

Equations (8), (9) and (10) are combined with boundary conditions are solved by a MATLAB program to generate numerical results below

## 7. Results and Discussion

### a) Effects of Reynolds number on horizontal velocity profile

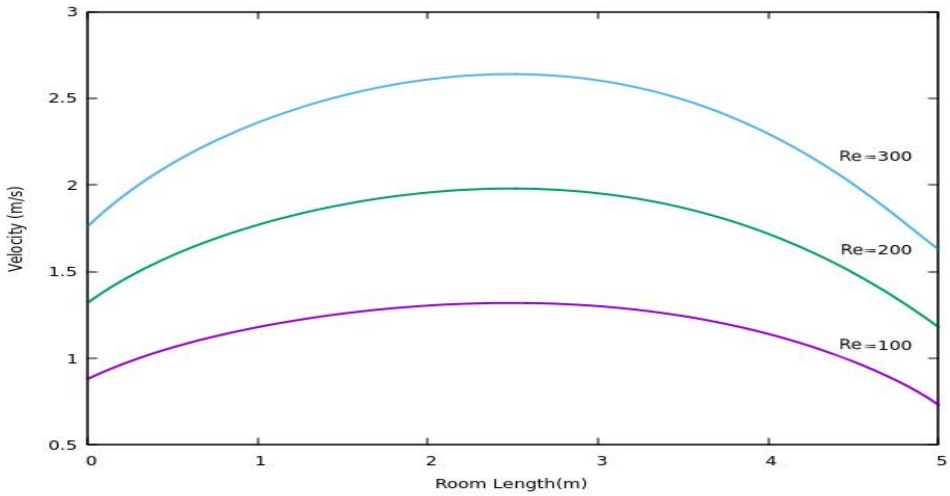


**Figure 2:** Effects of Reynolds number on horizontal velocity profile

From Figure2, as Re increases, the horizontal velocity increase, due to higher inertial forces as Re increase.A symmetrical profile is also observed with axis being at the Centre of the room wherethe highest velocity is attained, due to the fan that enhances the velocity as well asthe heating which reduces the density of the fluid

### b) Effects of Reynolds number on vertical velocity of fluid flow

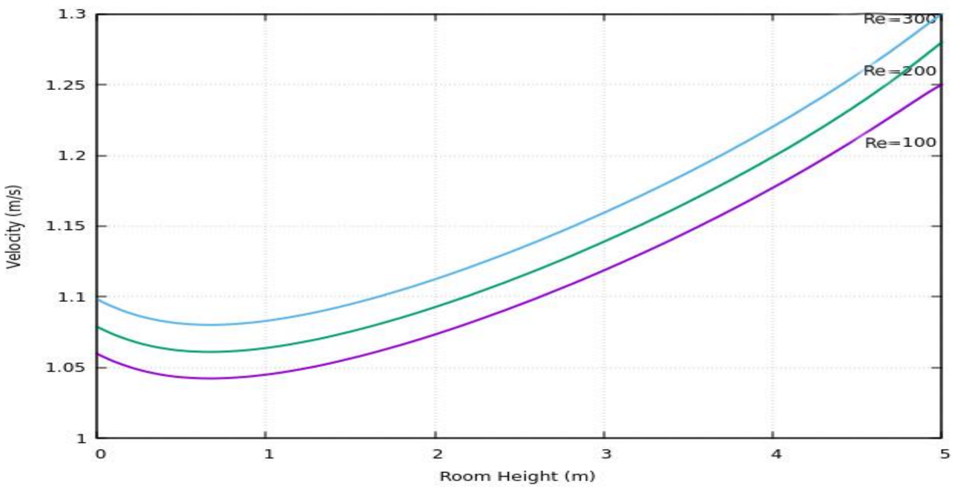


**Figure 3:** Graph of vertical velocity against room height at varying Reynolds number

From Figure3 above, the vertical velocity increase as Re number increases. This is due to high inertial forces at high Re number. The velocity is also seen to increase as the height of the room increases. This is because as air increases above the ground, it approaches the fan which greatly enhances their velocity.At a given Reynolds number the velocity starts at a slightly higher value due to the heat from the heater. The lowest velocity is attained at some height from the ground since in this region the air is relatively cold and therefore heavy as it is far from the heat.

### c) Effects of Richardson number on vertical velocity of fluid flow

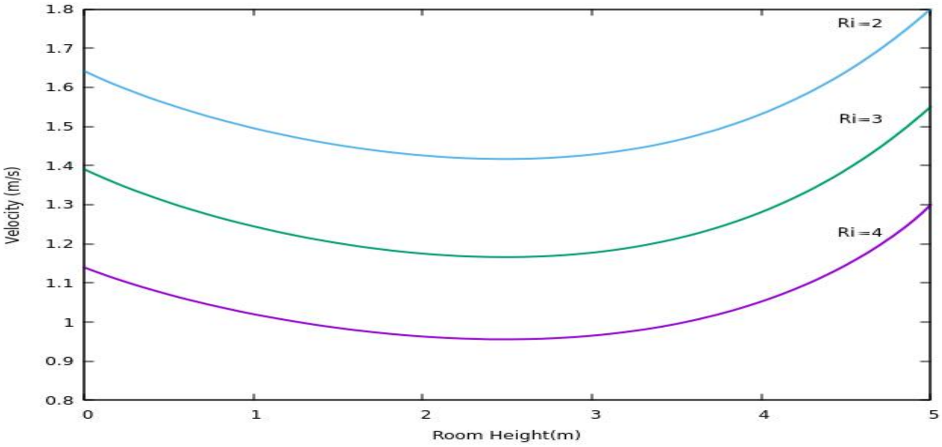


**Figure 4:** Graph of vertical velocity against room height at varying Rinumber

From Figure4 above, the velocity increases as Ri decreases. This is due to low buoyancy effects at low Ri, since the flow is majorly forced convection. The velocity profiles show a symmetrical pattern about the central part of the room where the lowest velocity is attained. This is because at this position the effect of fan and heat are minimal

**d) Velocity profiles at the top of the room at various Reynolds number.**

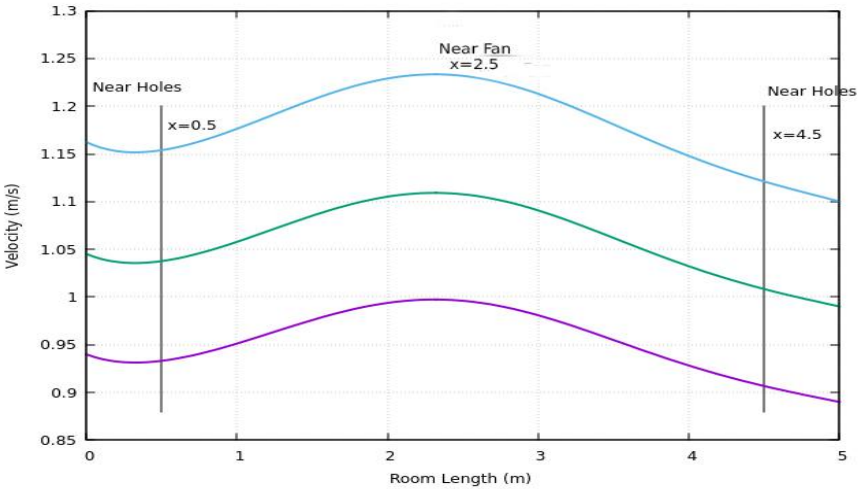


**Figure 5:** Graph of horizontal velocity against Room Length at the top of the room across holes and fan

From Figure 5 above, the velocity near the ventilation hole is low. This due to the high density of air as it is cold. As the air approaches the Centre of the room the velocity increases for each value of Re. This is due to the effect of the fan as it accelerates the air particles. The same observation is made at the other end of the room where another ventilation hole is placed

**e) Velocity Profile at Different Positions along the X-axis**

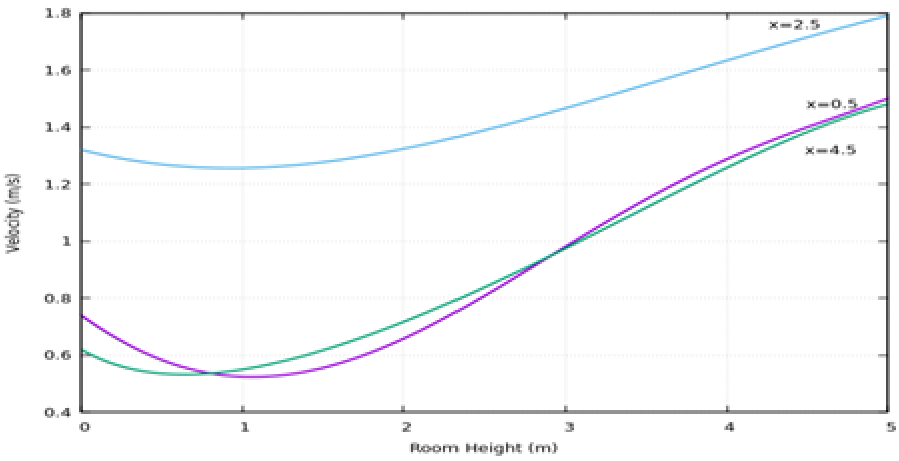


**Figure 6:** Graph of vertical velocity against room length along X- axis

From Figure6 above, the velocity profile is highest at around the Centre of the room. This can be attributed to the heating from below as well as the effect of the fan. At the either side of the room the velocity profile is low and almost geometrically identical. This is because they are equidistantly slightly far from heat and the fan.

**f) Velocity Profile at Different Positions along Y-axis**

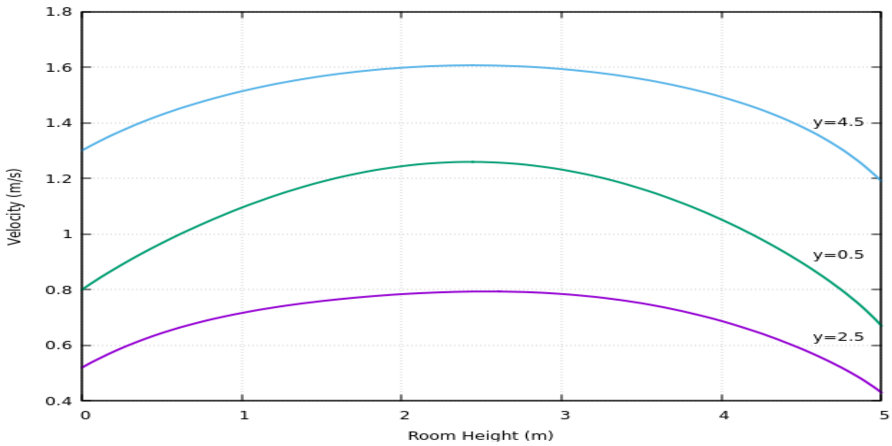


**Figure 7:** Graph of horizontal velocity against room length along y-axis

From Figure7 above, the velocity profile is highest at the topmost part of the room, followed by the bottom part of the room and is lowest at the central part. This is because of the effect of fan which contributes a lot to fluid velocity followed by the heater. The velocity increases as it approaches the Centre where the fan and heater are placed and starts dropping again as the fluid cools down and the distance from the fan increases.

**g) Effects of Reynolds number on temperature of fluid**

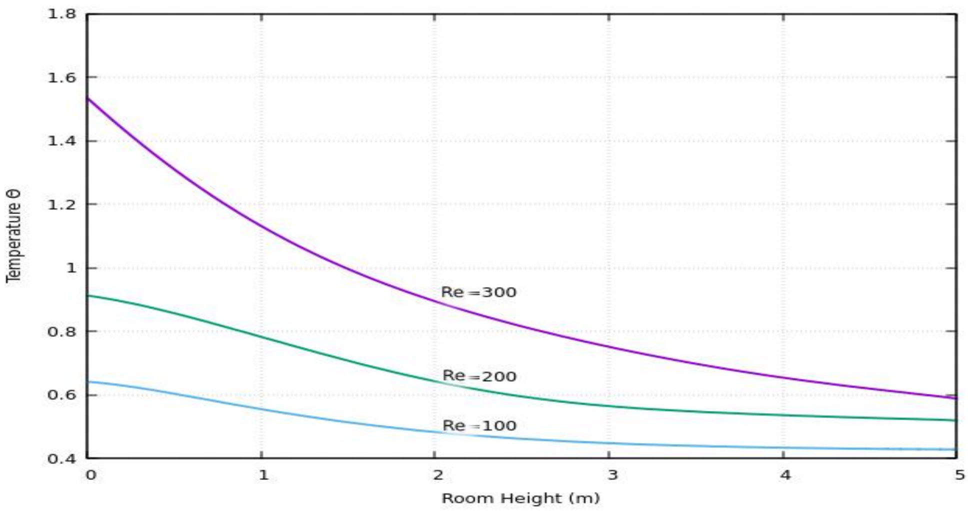


**Figure 8:** Graph of temperature against room height at varying Re number

The temperature is highest at high Reynold number and lowest at low Reynold number, as shown in Figure 8. This is because heat transfer is greater at high Re numbers than at low Re numbers. For high Re number inertial forces are predominant hence transferring more heat energy As the room height increase the temperature decay exponentially approaching a constant value for each Re number due to increase in distance from the heat source as well cooling from the fan and the ventilation holes

## 8. Conclusion and Recommendation

From the results obtained, it is evident that Reynolds number affects the horizontal velocity of the fluid across the room due to changes in inertial forces which leads to change momentum (velocity). Variation of the Richardson number leads to a change in the vertical velocity. This is because the flow is forced convection and hence the buoyancy effect has low effect. Thus, the velocity increases as the Richardson number decreases. It is also observed that temperature decreased along the height of the room. The reduction of pressure by the fan led to low temperature. The temperature is observed to be highest near the region close to the heater followed by the regions near ventilation holes and then the region near the fan had the lowest temperature. The temperature is highest at high Reynold number and lowest at low Reynold number.

**Recommendations**

1) Investigate forced convection when the fan is rotated at varying speed
2) Investigate forced convection in cylindrical enclosure using cylindrical coordinates



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3) Investigate forced convection for compressible fluids in cylindrical enclosure
4) Use of other methods of solution to carry out the same research
5) For commercial and domestic users positioning the fan and heater at the middle enhances homogeneous heating and cooling in the room hence economical

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