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A Preliminary Concept on Graph Coloring

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Abstract: ***This paper introduces the new study about combining the concept of Coloring with Fractal Graphs. Fractal Graph is the famous graph which has self-similarity property included it. It is developed in many areas and has much application in various engineering fields like computer science, Physical, Medical etc. Coloring is the advanced tool of Graph Theory. Its application is growing in various fields. Coloring the vertices is nothing but give the color to each vertices. Adjacent vertices are not colored by the same. Terminal vertices of each edge are colored by different colors like green, blue, yellow, red etc. Chromatic Number is the number of colors needed to the graph for coloring the vertices. It is denoted by CN (G). This paper analyses the chromatic number of various graphs Null Graph, Trivial Graph, Complete Graph, Star Graph, and Wheel Graph and also self-similarity fractal graph like Cantor Set Von Koch Curve. This paper found that the constant ratio of number of colors needed for coloring the above graphs.***

Keywords: Graph, Coloring of graphs, Vertex degrees

AMS Classification key: 05C, 05C15, 05C07

1. Introduction

Coloring [1] is an advanced tool of graph theory. It is the most development concept of Research Area. It is applied in many areas such as computer applications, clustering, Data mining, image capturing, Networking, Resource allocation, Process scheduling etc. It is nothing but an easiest way of labelling graph components such as vertices, edges, or may be regions of a planar graph. It has differentiated into three categories such as vertex coloring [8], Edge coloring and Region coloring. This paper finds the chromatic number in various graphs especially self-similarity Fractal Graphs.

2. Preliminaries

2.1 Vertex Coloring [22]

It is nothing but an assignment of colors to the non-adjacent vertices of the given Graph. It means that the terminal vertices of the edge should not be assigned by the same color.

2.2 Edge Coloring [12]

It is the way of assigning colors to the non-adjacent edges of the given graph.

2.3 Region Coloring

It is an assignment of colors to the different region of the planar graph [21]. These regions are converted into graph. The regions are partitioned into different connected graph. The edges of the regions are collected. Different edges enclosed the region are assigned by same color.

2.4 Chromatic Number [13]

The minimum number of colors required to color the non-adjacent vertices of the graph is called as chromatic number. It is denoted by CN (G).

2.5 Fractal Graph [3]

Fractals can be analyzed according to the characteristics and property of self-similarity of Graphs. It has accurately or loosely similar to a part of itself. A small particle has projected into the whole of the body of the object. It has seen in natures like clouds, mountains, leaves, and etc. Hausdroff dimension [18] is the measurement of roughness or complicated Fractal Graphs. It has well shape accomplished structure at random scales. Fractal Graphs are not easily described in Proper Euclidean Geometric Language [4]. A fractal dimension [5] is measured by the ratio of configuring out the complexity of a system given its measurement.

3. Calculating Chromatic Number to Various Types of Graphs

In this paper calculates the chromatic number for Various Graph [2] Such as Null Graph, Trivial Graph, Complete Graph, Regular Graph, Star Graph, Cycle Graph and Most World level Famous Fractal Graph like Cantor Set and Von Koch Curve. Chromatic Number depends on their structures and implementation of the corresponding graphs.

3.1 Null Graph [6]

It has single vertex or more than one vertex without edge. This graph chromatic number is equivalent to number of vertices of the given graph. Each vertex is not adjacent itself. Each vertex has colored by different colors. Therefore CN (G) = Number of Vertices V (G).

3.2 Trivial Graph

This graph has single vertex only. It has colored by single color only ie. CN (G) =1.

3.3 Complete Graph [7]

Each vertex of Complete Graph is adjacent with other vertices of the given graph. A graph with n vertices has n-1 degree. It is denoted by K_n. Each vertex is colored by different colors [24]. A complete Graph has n Chromatic Number ie CN (G) =n.

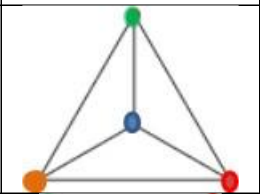
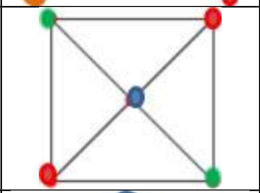
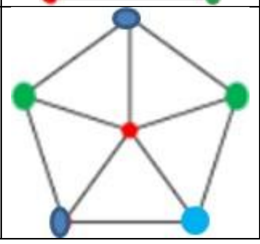
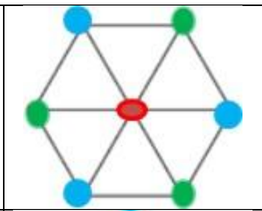
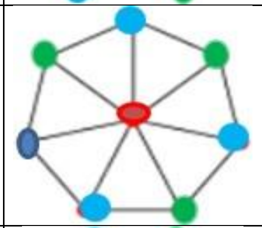
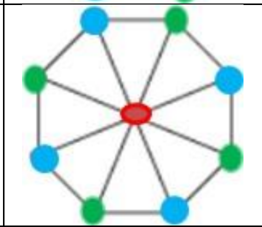
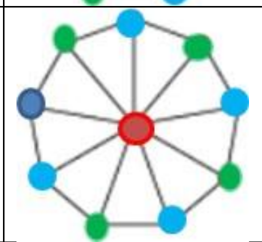
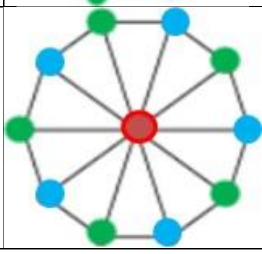
3.4 Wheel Graph

This graph looks like a wheel. All Vertices are adjacent with the center vertex through the edges. It gets from the cycle graph [9]. All the vertices of the cycle graph are incident with the center vertex. It is denoted by W_n. This graph has n vertices and 2n-1 edges. In the First Iteration of wheel graph has 4 vertices and 6 edges. In the First Iteration of Wheel Graph [11] have 4 vertices and also adjacent. It is even. It is required four different colors. The Second Iteration has 5 vertices and 8 edges. The Third Iteration has 6 vertices and 10 edges. In the upcoming iteration, it has increased by one and number of edges is increased by two. The chromatic number are calculated by the following formulae

$$CN(G) = \begin{cases} 4 \text{ if no. of vertices is even} \\ 3 \text{ if no. of vertices is odd} \end{cases}$$

The following table shows Chromatic number in some of the iteration of wheel graph

Table 1: Graph Coloring of Wheel Graph

S. No	Iteration	Graph	No. of Vertices	Chromatic Number CN (G)
1	1		4	4
2	2		5	3
3	3		6	4
4	4		7	3
5	5		8	4
6	6		9	3
7	7		10	4
8	8		11	3

3.5 Star Graph

This is the very interested type of Graph [14]. Here single vertex is connected with all the other vertices in the given graph. n vertex star graph is denoted S_n. n-1 vertices are adjacent with nth vertex (called as center node). These n-1 vertices are not adjacent themselves. It looks like wheel graph but it has loosened its boundary edges [15]. Center Node only has degree n-1. Remaining vertices has single degree. It may be considered as terminal nodes. Coloring of this type of graph is very easiest one. It needs only two colors for coloring this type of graphs [17]. Here Chromatic number [20] is two only.

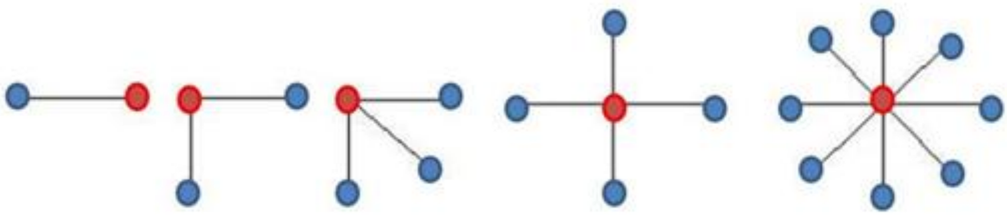


Figure 1: Graph Coloring of Star Graph

3.6 Cantor Set

It is the basic type of Fractal Graphs. Fractal Graph [10] is the famous graph in Graph Theory. Nowadays it has placed a major role in wild wide applications. It has more contribution in Computer Science, Animation, Artificial Creations, Data Structures, and Architecture and so on. Cantor set [16] is the basic level of Fractal Graphs. It is started at a single line. It has unit interval [0, 1]. A single line has splitted into three segments with the interval 1/3. Middle segment has eliminated. In the first iteration, a single edge has partitioned into two edges. The same process is applied in each and every edges of the graph. Each edge has partitioned into two edges. In the upcoming iteration, number of edges has increased into twice.

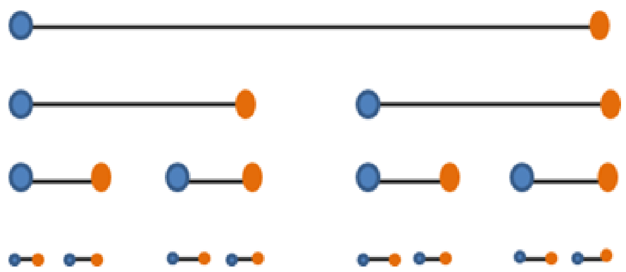


Figure 2: Graph Coloring of Von Koch Curve

4. Conclusion

Coloring has world wide application in our life. Vertex coloring is a hard combinatorial optimization problem. Many applications involving vertex coloring are evaluating the minimum number of colors required. In this paper finds the Chromatic Number of the various graphs like Trivial Graph, Null Graph, Complete Graph, Cycle Graph, Wheel Graph and special fractal Graphs like Cantor set and Von Koch Curve. The formulae of Chromatic number are common for all the iteration of the corresponding graph. It is very useful to find chromatic number in a simple way of very large iteration of Fractal Graph**.** Vertex coloring has involved many application like scheduling, chemical combination and computer optimization etc.

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