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21-Parameter Poincare Group in Six-dimensional Space-Time

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Abstract: ***The homogeneous Lorentz transformations in terms of 6x6 antisymmetric transformation matrix have been derived in six-dimensional space-time. The infinitesimal generators of Lorentz group and their commutation relations have been identified in vector representation of generators. The commutation relations and invariant operators of Homogeneous Lorentz group have been shown to confine in their representation space. The commutation relations of generator matricesfor twenty-one parameter inhomogeneous Lorentz Group in six-spacedo inherit space-time subspaces, which are necessarily structured in six-space. Constructing operators for Poincare Group, it has been shown that the operator relations choose specificspace-time selection and unitary representation of operators in six-dimensional space time, $D(3\oplus 3)$is same as in observable four-dimensional Minkowski space [R^4] $\equiv D(3\oplus 1)$.***

Keywords: Six-dimensional space-time, Extended relativity, Lorentz group

1. Introduction

Most advance theories of theoretical physics talk about higher-dimensions through different theoretical perspectives. In terms of observable parameters, space-time structure is the most fundamental basis to investigate nature of all possible events in extended relativistic formulations [1-5] including superluminal phenomena. The concept of multi-dimensional structure of space-timehas drawn large interest in literature in connection with particle stability at high energies [6], unification of fundamental forces [7], current superstring theories [8],and dyonic black hole solutions [9].The most symmetric, minimal pseudo-Euclidian space-time, with equal number of space and time axes, is six-dimensional space-time [10-15]. Significant role of space-time symmetry has been observed in Classical [16, 17] and quantum electrodynamics [18] developed in six-space. Through charge-field interaction in six-space, we have shown [19] that considerable amount of energy is required, if time orientation of an event to be changed to other one. For observational reality, We have defined two space-time dependent structural mappings [20] to switch from six-dimensional phenomena to observable four-dimensional Minkowski space [R^4] $\equiv D(3\oplus 1)$, or to superluminal tachyonic space [T^4] $\equiv D(1\oplus 3)$.

In the present paper, constructing mathematical background of six-dimensional space-time with signature $g_{\mu\nu}$(-1, -1, -1, 1 1 1), we have derived homogeneous Lorentz transformations in terms of 6x6 antisymmetric transformation matrix. This matrix may be expressed in four 3x3 sub-matrices, which together preserve space-time rotations in six-space. The infinitesimal generators of Lorentz group have been identified with their vector representation. The commutation relations and invariant operators of Homogeneous Lorentz group have been shown to confine in their representation space. The commutation relations and operators for twenty-one parameter inhomogeneous Lorentz Group in six-space have been derived with their representation subspaces. Schodinger picture of invariance is expressed for operators and thus the transition from classical to quantum is easily possible in six-space.

2. Lorentz Transformations in Six-Dimensional Space-Time

In six-dimensional space-time$D(3\oplus 3)$, a point $P\{x^{\mu}\}$ is represented by three space and three time orthogonal coordinates, comprised of a spatial vector $\vec{r}$ and a temporal vector $\vec{t}$. such that,

$$D(3\oplus 3) \equiv (\overrightarrow{x_r}, \overrightarrow{x_t}) \text{ or} \tag{1}$$

$$D(3\oplus 3) \equiv \{x^{\mu}\} \equiv (x_1, x_2, x_3, x_4, x_5, x_6) \tag{2}$$

$$\text{and } [R^3] \perp [T^3]. \tag{3}$$

A position vector$\{x^{\mu}\}$is specified by following six-component column vector;

$$\{x^{\mu}\} \equiv \begin{pmatrix} \vec{r} \\ \vec{t} \end{pmatrix} \equiv (x^1, x^2, x^3, x^4 = t^1, x^5 = t^2, x^6 = t^3)^{T} \tag{4}$$

Where T denotes the transpose. The quadratic invariance between two inertial reference frames is defined as:

$$|ds|^2 = -|d\vec{r}|^2 + |d\vec{t}|^2 = g_{\mu\nu}x^{\mu}x^{\nu} \tag{5}$$

Where Greek indices μ,νvary from 1 to 6, and $g_{\mu\nu}$(-1, -1, -1, 1 1 1)is the reference metric describing space-time manifold. The formal expression of transcendent Lorentz Transformation is independent of any space direction hence, an inertial observer has equal freedom to choose any of the axes. The relative motion between two inertial frames, say Kand K$'$,may be expressed in six-component form. We may define a unit time vector $\vec{\alpha}$, in the time field of the particle [16] such that, the components of velocity vector $\vec{v}$ for a moving particle, are integrated with the components of the unit time vector $\vec{\alpha}$. The six-velocity vector in six-space is defined as;

$$\{ v^{\mu} \} = dx^{\mu}/ d\tau = \gamma(v) [\vec{v}, \vec{\alpha}]^{T} \tag{6}$$

With $\gamma(v)$= dt / dτ. The time vector is directed tangentially to the time trajectory of the particle and infinitesimal increments $\overrightarrow{dt}$ and $\overrightarrow{d\tau}$are measured along the time curve of the particle in moving and instantaneous frames, respectively. Let us consider another frame of reference K$''$, with six-velocity $\{v_\nu\}$ with respect to frame K$'$. The velocity addition law, for these inertial frames may be expressed in terms of six-velocity vector, as,

$$\Lambda\{v_\mu\}.\,\Lambda\{v_\nu\} = \Lambda\{v_\nu\}.\,\Lambda\{v_\mu\} = \Lambda\{v_\lambda\} \tag{7}$$

$$\text{Where, } \{v_\lambda\} = [\{v_\nu\} + \{v_\mu\}] \,/\, [\,1 + \{v_\nu\}^{\mathrm{T}}.\{v_\mu\}\,] \tag{8}$$

In general, the matrices $\Lambda\{v_\mu\}$ and $\Lambda\{v_\nu\}$ do not commute and hence Lorentz transformation between the first and third frames involves a combined boost and rotation, i.e. Thomas rotation, therefore Thomas precession will exist in six-space kinematics. The Lorentz Transformations between prime and non-prime frame K' and K, in $D(3\oplus 3)$ may be expressed as,

$$\{x^\mu\} \rightarrow \{X^\mu\}' = \Lambda_{\mu\nu}\{x^\nu\} \tag{9}$$

Where $\Lambda_{\mu\nu}$ is 6x6 transformation matrix, such that,

$$\Lambda g_{\mu\nu}\Lambda^T = \pm g_{\mu\nu} \tag{10}$$

light speed is invariant in $D(3\oplus 3)$. The condition for isotropy of the space-time leads to the following condition;

$$(\det.\Lambda)^2 = 1 \text{ or } \det.\Lambda = \pm 1. \tag{11}$$

Thus, as per positive or negative value of the determinant, transformation matrix has two disconnected space-time signatures. Each disconnected group has two distinct possibilities, mathematically;

$$\Lambda_t{}^t \geq +1. \text{ or } \Lambda_t{}^t \leq -1 \tag{12}$$

Where 't' has three choices of time dimensions (= 4,5,6). Thus each subgroup has three identical, equivalent self-sufficient group, associated with each choice of time dimension in $D(3\oplus 3)$ space-time.In explicit form, transformation matrixΛmay be written as four 3x3 sub-matrices A,P,Q,R –

$$\Lambda_{\mu\nu} = \begin{bmatrix} \mathbf{A} & \mathrm{P} \\ \mathrm{Q} & \mathrm{R} \end{bmatrix} \tag{13}$$

The sub-matrices of transformation matrix satisfy;

$$\mathrm{AA^T - PP^T = A^TA - Q^TQ = 1} \tag{14}$$
$$\mathrm{RR^T - QQ^T = R^TR - P^TP = 1} \tag{15}$$
$$\mathrm{AQ^T - PR^T = A^TP - Q^TR = 0} \tag{16}$$

The components of 3x3 sub-matrices A, P, Q and R, together express 6x6Lorentz Transformations and satisfy necessary matrix relations to constitute elements of proper Lorentz transformations. The various relativistic effects like length contraction, time dilation and velocity addition law may be visualized in $D(3\oplus 1)$ in four-dimensional observable space-time or in $D(1\oplus 3)$ i.e. superluminal space-time, under structural mappings [20].

3. Infinitesimal Generators and Commutation Relations

The infinitesimal generators of Lorentz group, are derived from rotational invariance in two coordinates, in respective dimensions, chosen by μ, ν such that;

$$\mathrm{M}_{\mu\nu} = \frac{d}{d\emptyset}\Lambda_{\mu\nu}(\emptyset)\Big|_{\emptyset=0} \tag{17}$$

Where $\emptyset$ is the angle of rotation between the coordinates. The infinitesimal generators corresponding to spatial rotation with choice μ, ν = i, j, k and their permutations only, are represented by,

$$\mathrm{M}_{\mu\nu} = \begin{cases} 1 \text{ for} \mathrm{M}_{jk} = -\mathrm{M}_{kj} \\ 0 \text{ for } \mu,\nu \neq j,k \end{cases}. \tag{18}$$

Similarly, infinitesimal generators for choice μ, ν = l, m, n and their permutations only, correspond to temporal rotation as-

$$\mathrm{M}_{\mu\nu} = \begin{cases} 1 \text{ for} \mathrm{M}_{mn} = -\mathrm{M}_{nm} \\ 0 \text{ for } \mu,\nu \neq m,n \end{cases} \tag{19}$$

The infinitesimal generators for boosts are represented by,

$$\mathrm{M}_{\mu\nu} = \begin{cases} 1 \text{ for} \mathrm{M}_{km} = -\mathrm{M}_{mk} \\ 0 \text{ for } \mu,\nu \neq k,m \end{cases}, \tag{20}$$

Where the $\mathrm{M}_{\mu\nu}$ is 6x6 linearly independent matrix with fifteen independent components satisfying $\mathrm{M}_{\mu\nu} = -\mathrm{M}_{\nu\mu}$. We may define specific vector forms for these generators as;

$$\overrightarrow{\mathrm{M^S}} \equiv (\mathrm{M}_{12}, \mathrm{M}_{23}, \mathrm{M}_{31}) \equiv (\mathrm{M}_3, \mathrm{M}_1, \mathrm{M}_2) \tag{21}$$
$$\overrightarrow{\mathrm{M^T}} \equiv (\mathrm{M}_{45}, \mathrm{M}_{56}, \mathrm{M}_{64}) \equiv (\mathrm{M}_6, \mathrm{M}_4, \mathrm{M}_5) \tag{22}$$

Where superscript S or T denotes spatial and temporal coordinate space respectively. The infinitesimal generators for space-time rotations form a dyadic, which, with equivalence of time coordinates, may be represented in following vector form[21] for each value of l (=4,5,6) and k=1,2,3 ;

$$\overrightarrow{\mathrm{N^D}} \equiv (\mathrm{N}^4)_1, (\mathrm{N}^4)_2, (\mathrm{N}^4)_3 \tag{23}$$

In terms of permutations, the relations for infinitesimal generators may also be written as;

$$(\overrightarrow{\mathrm{M^S}})_{\mathrm{l}} = (1/2)\, \epsilon_{ijk}\, (\mathrm{M^S})^{jk}\,;\ (\,i,j,k = 1,2,3\,) \tag{24}$$
$$(\overrightarrow{\mathrm{M^T}})_{\mathrm{l}} = (1/2)\, \epsilon_{lmn}\, (\mathrm{M^T})^{mn};\ (\,l,m,n = 4,5,6\,) \tag{25}$$
$$\overrightarrow{\mathrm{N^D}} \equiv (\mathrm{N^l})_k\,. \tag{26}$$

The general commutation relations for the generator matrices are;

$$[\,(\overrightarrow{\mathrm{M^S}})_i, (\overrightarrow{\mathrm{M^S}})_j\,] = \epsilon_{ijk}\, (\overrightarrow{\mathrm{M^S}})^k \tag{27}$$
$$[\,(\overrightarrow{\mathrm{M^T}})_l\,, (\overrightarrow{\mathrm{M^T}})_m\,] = \epsilon_{lmn}\, (\overrightarrow{\mathrm{M^T}})^n \tag{28}$$
$$[\,(\overrightarrow{\mathrm{M^S}})_i, (\overrightarrow{\mathrm{N^l}})_j\,] = \epsilon_{ijk}\, (\overrightarrow{\mathrm{N^l}})^k \tag{29}$$
$$[\,(\overrightarrow{\mathrm{N^l}})_i, (\overrightarrow{\mathrm{N^l}})_j\,] = -\,\epsilon_{ijk}\, (\overrightarrow{\mathrm{M^S}})^k \tag{30}$$

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$$[\ (\overrightarrow{N^{l}})_{i},\ (\overrightarrow{N^{m}})_{i}\ ] = -\in_{lmn}\ (\overrightarrow{M^{T}})^{n} \qquad (31)$$
$$[\ (\overrightarrow{M^{T}})_{l},\ (\overrightarrow{N^{i}})_{m}] = \in_{lmn}\ (\overrightarrow{N^{i}})^{n} \qquad (32)$$

The commutations, equation (27) and (28) preserve the invariance of spatial and temporal degrees of freedom and form the subalgebra isomorphic to the rotation group R(θ). Commutation (29) represents rotation and boost to generate boost in $[R^3] \in D(3 \oplus 3)$, whereas commutation (30) successive boosts to have resultant in $[R^3] \in D(3 \oplus 3)$, sub-space. The commutation (31) and (32) carry the same successive events but express themselves in $[T^3] \in D(3 \oplus 3)$ sub-space.

These infinitesimal generators lead to following operators;

$$\hat{P}_k = (1/2)\ [(M^S)_k + (N^m)_k] \qquad (33)$$
$$\hat{Q}_k = (1/2)\ [(M^S)_k - (N^m)_k] \qquad (34)$$
$$\hat{R}_m = (1/2)\ [(M^T)_m + (N^k)_m] \qquad (35)$$
$$\hat{S}_m = (1/2)\ [(M^T)_m - (N^k)_m] \qquad (36)$$

Where sub- or super- script k = 1,2,3 stands for spatial coordinates and m=4,5,6 corresponds to temporal coordinates. These operators satisfy following commutation relations;

$$[\hat{P}_k, \hat{Q}_k] \equiv [\ \hat{R}_m, \hat{S}_m\ ] \equiv 0 \qquad (37)$$
$$[\ \hat{P}_i, \hat{P}_j\ ] \equiv \underline{i}\ \in^{ijk}\ \hat{P}_k \qquad (38)$$
$$[\ \hat{Q}_i, \hat{Q}_j\ ] \equiv \underline{i}\ \in^{ijk}\ \hat{Q}_k \qquad (39)$$
$$[\ \hat{R}_l, \hat{R}_m] \equiv \underline{i}\ \in^{lmn}\ \hat{R}_n \qquad (40)$$
$$[\ \hat{S}_l, \hat{S}_m] \equiv \underline{i}\ \in^{lmn}\ \hat{S}_n \qquad (41)$$

The invariant operators from these relations may be defined as,

$$\hat{E} = P^2 + Q^2 \equiv (1/2)\ [\{(M^S)_k\}^2 + \{(N^m)_k\}^2] \qquad (42)$$
$$\hat{F} = P^2 - Q^2 \equiv (M^S).(N^m)_k. \qquad (43)$$
$$\hat{G} = R^2 + S^2 \equiv (1/2)\ [\{(M^T)_m\}^2 + \{(N^k)_m\}^2] \qquad (44)$$
$$\hat{H} = R^2 - S^2 \equiv (M^T).(N^k)_m \qquad (45)$$

Each operator commutes with all infinitesimal generator matrices and hence represents invariant operators of the group. It's worthwhile to notice that operators represented by equations (42) to (45) are invariant, provided all space and time coordinates are equivalent in nature and description.

4. Poincare Groupin Six-dimensional space-time

The best endorsement of space-time structure is expressed in terms of Inhomogeneous Lorentz Group. let us introduce a six translational vector along the axes. The Inhomogeneous Lorentz transformations in six-space may be represented by,

$$\{x^{\mu}\} = \Lambda_{\mu\nu}\{x^{\nu}\} - \{a_{\nu}\} \qquad (46)$$

Where $\{a_{\nu}\}$ is a six-column vector in six-space. The IHLG, with six space-time translational generators appears as twenty-one parametergroup, comprised of fifteen generators of the 6DHLG and six space-time translational generators. The Poincare group, therefore, carries (i) the generators of infinitesimal translation along six-coordinate axes; ($\hat{P}_S, \hat{P}_T$) (ii) three-component spatialand three-component temporal rotations in respective spaces($\hat{R}_S, \hat{R}_T$) ; and (iii) nine boosts for space-time rotations. These twenty-one generators may be shown to satisfy following commutation relations;

$$[\ \hat{P}_T, \hat{P}_S] = 0\ ;\ [\ \hat{P}_T, \hat{R}_S\ ] \equiv 0 \qquad (47)$$
$$[\ \hat{P}_S, \hat{R}_S\ ] = 0\ ;\ [\ \hat{P}_i, \hat{P}_j\ ]_S = 0\ ;\ [\ \hat{P}_m, \hat{P}_n]_T = 0 \qquad (48)$$
$$[\ \hat{R}_i, \hat{R}_j\ ]_S = [\ (N^i)_l, (N^j)_l\ ] = [(N^l)_i, (N^l)_j] = -\underline{i}\in_{ijk}\ (\overrightarrow{R^S})^k \qquad (49)$$
$$[\ \hat{R}_l, \hat{R}_m\ ]_T = [\ (N^j)_l, (N^j)_m] = [\ (N^l)_j, (N^m)_j\ ] = -\underline{i}\in_{lmn}\ (\overrightarrow{R^T})^n \qquad (50)$$
$$[\ (\hat{R}_S)_i, (N^l)_j] = -\underline{i}\in_{ijk}\ (N^l)_k \qquad (51)$$
$$[\ (\hat{R}_T)_l, (N^i)_m] = -\underline{i}\in_{lmn}\ (N^i)_n \qquad (52)$$
$$[\ (\hat{R}_S)_i, (N^i)_j] = [\ (\hat{R}_T)_l, (N^i)_m] = 0 \qquad (53)$$
$$[\ (N^m)_i, (\hat{P}_T)_n\ ] = \underline{i}\delta_{mn}\ (\hat{P}_S)_I \qquad (54)$$
$$[\ (N^i)_m, (\hat{P}_S)_j\ ] = \underline{i}\delta_{ij}\ (\hat{P}_T)_m \qquad (55)$$

Where $\in_{ijk}$ and $\in_{lmn}$ are Levi-Civita three index symbols in three dimentions; δ_{ij}andδ_{mn} are Kroneker delta symbols; i, j, k = 1, 2, 3 and l, m, n = 4, 5, 6 ; S and T stand for space and time dimensions; summation convention follows and $\underline{i}$represents complex quantity as in usual $D(3 \oplus 1)$ dimensions.

In general, the infinitesimal transformations associated with any of twenty-one generators can be written as-

$$\Lambda = (I + \mathrm{i} \in \mathrm{L}) \qquad (56)$$

Where $\in$ is infinitesimal and L refers to any of the generators. In Schrodinger picture of the invariance, the infinitesimal operators may also be represented as;

$$\hat{P}_{\mu} = -\underline{i}\ \partial_{\mu}\ .\hat{I} \qquad (57)$$
$$(\hat{R}_S)_i = (M^S)_i - \underline{i}\ (\ \mu_r \mathrm{x}\ \partial_r\ )_i.\hat{I} \qquad (58)$$
$$(\hat{R}_T)_l = (M^T)_l - \underline{i}\ (\ \mu_t\ \mathrm{x}\ \partial_t\ )_l.\hat{I} \qquad (59)$$
$$(N^l)_i = (N^l)_i + \underline{i}\ (\ \mu_t\ \partial\mu_i + \ \mu_r \partial\mu_l).\hat{I} \qquad (60)$$

Where $\hat{P}_{\mu}$issix-translational, $(\hat{R}_S)_i$is spatial-rotational, $(\hat{R}_T)_l$ is temporal-rotational and $(N^l)_i$is nine-componentdyadic representing all possible boost in six-space. The symbol ∂_{μ} represents derivative$\partial / \partial\, x_{\mu}$ and ∂_r and ∂_t are derivatives in spatial and temporal dimensions, and $\hat{I}$ is identity matrix.

5. Discussion

The relativistic kinematics with quadratic invariance, equation (5), and temporal degrees of freedom in six-space, may be well constructed in terms of six-vectors (6), as four vectors contribute to express four-dimensional observational world. The proper, orthochronous Lorentz transformations with$\Lambda = +1$, and$\Lambda_t{}^t \geq +1$, have three identical, equivalent sub-groups associated with each choice of time dimension in $D(3 \oplus 3)$ space-time. The six-restricted sub-groups of SLG will have one to one correspondence and complete equivalence except the "chosen" time vector. The Lorentz transformation matrix structure is enlarged to 6x6 antisymmetric matrix, equation (13), connecting $[R^3]$,$[T^3]$, representation subspaces with space-time mixed rotations to count nine possible boosts. The fifteen rotations representing three pure rotations in $[R^3]$ subspace, three pure rotations in $[T^3]$, subspace and nine boosts in [R-T] subspace, may be expressed in terms of four 3x3 sub-matrices. These sub-matrices are connected to each other and obey relations (14,15,16) to preserve nature of Lorentz transformations and represent unique and true mixing of space-time coordinates.

The vector representation of the infinitesimal generators, equations (21,22,23), enables one to articulate the various commutation relations for infinitesimal generators represented by equations (24), (25) and (26). These relations lead to commutations, expressed by equations (27) to (32), confine themselves in respective representation of space-time structure. e.g. the commutation relation (27) expresses spatial rotation and resultant of two rotations is in spatial coordinates. So $[R^3]$subspace of $D(3 \oplus 3)$ confines spatial rotations. The same is true for (28), where rotation is confined in temporal space $[T^3]$ of $D(3 \oplus 3)$.The various sub group transformations of SLG, separately or combinedly, may be used to construct corresponding operators associating various internal particle symmetries. Interestingly, space-time symmetry in six-space does not require new operators to explain the transformations of associated group. Moreover, the space inversion, time inversion and space-time inversion operators need not be studied separately, as space and time coordinates are symmetrized in contrast to conventional four-dimensional observable reality.

The corresponding operators in six-space are represented by equations (33), (34), (35) and (36). These operators preserve symmetry in spatial and temporal subspaces. Spatial operator P,Q commute with each other (37) and so does R, S operators; whereas their self-commutation (38), (39), (40), (41) leads to rotational symmetry in respective space (or time). The invariant operators $\hat{E}, \hat{F}$ equations (42), (43) may be assigned to $D(3 \oplus 1)$ i.e. usual $[R^4]$ space-time and similarly operators $\hat{G}, \hat{H}$ equations (44),(45) to $D(1 \oplus 3)$ i.e. tachyonic space-time$[T^4]$, respectively. The eigen values represented by these operators $\hat{E}, \hat{F}$(or $\hat{G}, \hat{H}$) i.e. [e(e+1), f(f+1)] (or [g(g+1),h(h+1)]), respectively, may be used to characterize the representation of the group. As the number of operators is same in both $D(3 \oplus 3)$ and $D(3 \oplus 1)$ the unitary representation will be same and as such even in a higher dimensional structure no new quantum number is needed. The HLG in $D(3 \oplus 1)$ has six-parameters only and if we assign the same number of parameters to tachyonic HLG i.e. $D(1 \oplus 3)$ then $D(3 \oplus 1) \cup D(1 \oplus 3)$ will have n$\leq$ 12 rotations only. Even in other higher dimensional extended theories (e.g. in complex C^3 space [22], and also inD^8 space [15]), to discuss group formation one has to fix other constraints also. This loss of parameters in respective subspaces, therefore, suggests that six-dimensional space-time is self-sufficient, real description of space-time to unify all relativistic phenomena.

The equivalence of all six-real coordinates suggests fifteen rotational and six translational transformations (46) to constitute twenty-one parameter Poincare group. The commutations of these twenty-one parameters do carry descriptions for both sub- and superluminal world. The translational parameters $\hat{P}_S, \hat{P}_T$ commute with each other (equation (47), and with rotations$\hat{R}_S, \hat{R}_T$ equation (48) and with boosts (50).Equations (49), (50) characterize rotational invariance and commutation (51), (52) represent successive rotation and boost in spatial and temporal dimensions. The equations (57),(58),(59),(60) represent Schodinger picture of invariance, where operators are expressed in terms of infinitesimal rotations and space-time derivatives with identity operator $\hat{I}$.The formalism now can easily be extended to quantum mechanics as we know that every observable quantity is associated with a corresponding operator.

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