International Journal of Science and Research (IJSR)
ISSN: 2319-7064
SJIF (2020): 7.803

Bivariate Dependence Modelling of a Non-Life Insurance Company: Pair-Copula Construction

Nurulain Farahtulnisa' Rozainee[1], **Isaudin Ismail**[1*]

[1]Department of Mathematics and Statistics, Faculty of Applied Sciences and Technology, Universiti Tun Hussein Onn Malaysia, 84600 Pagoh, Johor, Malaysia
*Corresponding author: isaudin[at]uthm.edu.my

Abstract: ***This study demonstrates the aggregate losses and risk estimation. The purpose of this study is to test the bivariate dependence modelling between losses among different business lines. Subsequently, this research estimate the risk of a non-life insurance company by using Value-at-Risk (VaR) and Tail-Value-at-Risk (TVaR). The risks were calculated by using incurred claims and premiums data from nine business lines based on Malaysia non-life insurance company for loss data. R software, an open source software is used to conduct the analysis in this study. Two models from pair-copula construction were used; C-Vine and D-Vine copula model. D-Vine copula model was found has stronger dependence structure than the C-Vine model based on the value of Loglik, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). Lastly, this paper visualize the trend of incurred claims data of the non-life insurance company. This study believe the trend is based on Malaysian behaviour. This research observe that Motor insurance has the layout claims, which may translate to higher road accidents in Malaysia.***

Keywords: Non-life insurance, Pair-copula, D-Vine, C-Vine, AIC, BIC, Loglik, RStudio

1.Introduction

Insurance is known as protection from risk and based on the principle of exchange which involves two parties, an individual known as a policyholder and insurance company. The insurance company accepts payments in the form of capital and will pay for indemnity towards losses or damages. There are two types of insurance; life insurance and non-life insurance. However, in this research only focus on non-life insurance. Non-life insurance protects against catastrophes and unfortunates towards motor vehicles, marines and aviations, and products or goods owned by the policyholders. Economy and institution can influence the premium of non-life insurance [1]. This research is to recognize the dependence structure from two non-life business lines in Malaysia. The insurance companies have to improve the risk management system and optimize the risk quantification approaches because risk and uncertainty can lead to substantial capital losses. By using VaR and TVaR, this study can estimate the total risks [2].

In the insurance industry, risk dependencies between losses from various lines were recognized as the main factors influencing the insurance company's aggregate losses and involve high dimensional data. Therefore, to overcome this problem, this research used a pair-copula approach to analyze the claims data due to its ability to model high dimensional dependence structure of loss data. This method is very convenience rather than the traditional method as it simplify the complexity model. Hence, VaR and TVaR estimate the total risk from pair-copula models.

This paper aims to test the bivariate dependence modelling between losses by using pair-copula approach. Next, this study want to estimate the risk of a non-life insurance company by using VaR and TVaR. Besides, this research want to investigate the incurred claims of the insurance company to the policyholders. All result in this research, the database obtains a from realistic environment by taking nine losses from non-life insurance company business lines in Malaysia.

2.Materials and Methods

The main issue in modelling a non-life insurance loss is to model the claim data. To address this problem, this study fit the losses by marginal distribution and simulate the data from fitted copulas to estimate non-life insurance companies' risks.

2.1 Pair-copulas construction

The process to perform the analysis from a set of bivariate into multivariate copula is known as pair copula contruction, or more precisely, pair copula densities. It is a product resulting from the decomposition process of copula density [3]. Aas et al. [4] proposed two cases of regular vines copulas which are canonical-vine (C-vine) and drawable-vine (D-vine). The n-dimensional vine is defined by;

1. $(n-1)$ trees (T)
2. The tree j has $n(n-1+j)$ nodes and $(n-j)$ edges.
3. Every edge corresponds to a density of the bivariate copula.
4. Full decomposition is characterized by $\frac{n(n-1)}{2}$ edges and marginal densities of every variable.

The joint copula density may be expressed as a combination of several bivariate pair-copulas. This research specify the copula families and parameters as vectors of length $\frac{n(n-1)}{2}$, where n is the total number of variables. In this study, we carried out nine variables, so, we had 36 pairs of copulas families related to C-vine and D-vine. C-vine and D-vine were approached in this study. The simulation algorithms for them are simple to apply [4]. More related studies using copula approached related to insurance can be seen in [5],[6],[7] and [8].

2.2 Risk estimation

By applying VaR and TVaR method, this paper measure the risk and control the capital requirement in the company which can avoid the company from being insolvent. Tang and Valdez [9] calculate each copula, the distribution of the aggregate loss at the level of the company taking a weighted average of the loss of each business line according to the pre-specified proportion of the premium received. The aggregate loss of non-life risk insurance portfolio can be defined as;

$$S = \sum_{i}^{n} \lambda_{it} S_{it}, \quad (1)$$

where S_{it} is claims yielded from individual risk of business line, i and obtained per semi, t. The weight of the line in the portfolio can be defined as

$$\lambda_{it} = \frac{EP_{it}}{\sum_{i}^{n} EP_{it}}, \quad (2)$$

where EP_{it} is weighted on the premium earned in period, t, compared to premium amounts of risk, i.

The sum of them is equal to one. Finally, the risk measure is applied to the estimation, to evaluate risk capital under the analysis of non-linear dependence [10]. VaR and TVaR are the tools to measure the risk capital of capital requirements. The TVaR is the expected result, conditional upon the distribution's loss exceeding the VaR, where distribution support is continuous. The general formula to calculate VaR is;

$$VaR_{1-\alpha}\left(X\right) = \inf\left\{x \in \mathbb{R} : P\left(X \le x\right) \ge 1-\alpha\right\} \quad (3)$$

$$TVaR_{\alpha}\left(X\right) = VaR_{\alpha}\left(X\right) + \frac{1}{1-\alpha} E\left[\left(X - VaR_{\alpha}\left(\cdot \right.\right.\right. \quad (4)$$

To get VaR and TVaR estimates of aggregate loss, S at confidence interval level α combining copulas, the researcher proceed to simulate N losses using the bivariate model as follows;

1. Fitting marginal distribution of losses.
2. Transform each variable into uniform distribution.
3. Fitting the suitable copula for each pair transformed data vectors and conclude the estimation of parameters by maximum likelihood function estimation (MLE).
4. Calculate simulated losses by using the fitted pair-copula. Conclude the estimation of VaR and TVaR.

3.Results and Discussion

Data used in this study are sourced from the Bank Negara Malaysia (BNM) official website. The frequency of the data are semiannual, collected from 2009 to 2020 and extracted from the financial statements published by the Bank Negara Malaysia website. The data set applied to estimate the model consists of incurred claims and premiums of the insurance company, derived from nine lines of business: Marine, Aviation and Transit (MAT), Contractors' all risks and Engineering (CE), Fire, Medical expenses and personal accident (MEPA), Motor Act Cover (MA), Motor Others (MO), Liability (LB), Workmen's compensation and employers' liability (WCEL) and Miscellaneous (Mis). We discuss the results in 3.1, 3.2 and 3.3.

3.1 Pair-copulas

Table 1: Estimated parameters for nine-dimensional C-Vine

	Bivariate copulas
AIC	-191.32
BIC	-152.71
LogLik	129.66

Table 2: Estimated parameters for nine-dimensional D-Vine

	Bivariate copulas
AIC	-200.42
BIC	-157.27
LogLik	138.21

This research fit CD-Vine copulas types to our data set and decide Vine copula structure's accuracy in terms of one or more criteria. Hence, this study compute the information criterions such as Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) at the fitted models. Furthermore, this article also look at the Loglik for each model to confirm our appropriate model choice. Based on Table 1 and 2, this study can see the value Loglik for C-Vine is 129.66 while for D-Vine is 138.21. By comparing the result based on Table 1 and 2, found that the D-vine copula model has a better fit for data than the C-vine copula model because it has a higher value of LogLik and a lower value of AIC and BIC. This model provides a more precise description of claim amount dependence. Hence, (MAT, WCEL, LB, Mis, MO, MEPA, CE, MA, Fire) were selected as the best appropriate permutation for the first tree of D-vine copula model, since it comprises the largest possible dependence.

3.2 Risk Estimation

Table 2: Results VaR and TVaR estimates

Methods		95.5%	97.5%	98%	99%	99.5%
DVine	VaR	0.0457	0.0259	0.0211	0.0102	0.0053
	TVaR	0.5311	0.5204	0.5184	0.5133	0.5108
Indep	VaR	0.0460	0.0244	0.0206	0.0106	0.0058
	TVaR	0.5241	0.5136	0.5116	0.5066	0.5041

Table 2 reveals VaR and TVaR estimation results from simulating the risk based on fitted D-Vine copulas and the independence hypotheses to make the comparison. Throughout this approach, this research realised that the value of VaR and TVaR is calculated at a lower risk than that of an independent one. The value of VaR was marginally lower than that of the TVaR. This clarifies that VaR is not a coherent measurement, while TVaR does. In reality, TVaR provides for a more conservative amount of capital requirement because it is a coherent measure. Since this is the case, the regulator should reconsider the use of TVaR as the risk measure.

3.3 Incurred claims vs Period

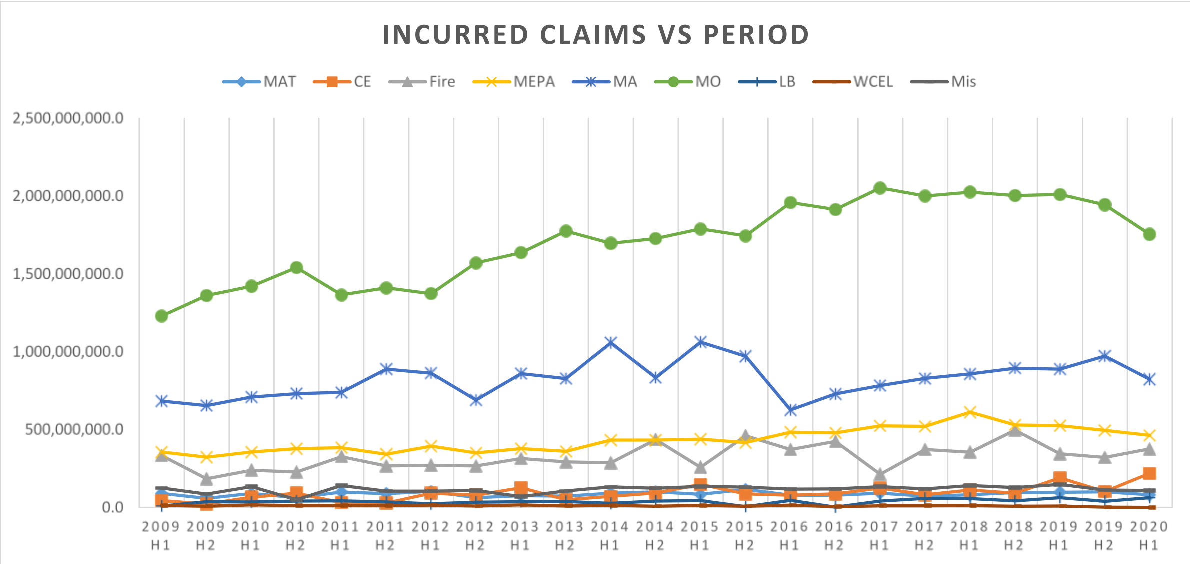

Figure 1: Incurred claims vs. Period

Figure 1 reveals that Motor Others (MO) has the highest value of incurred claims among Malaysians and followed by Motor Act Cover (MA). In contrast, Workmen's Compensation and Employers' Liability has the lowest value. This trend occurred because a few factors according to the behaviour of Malaysians. This showed us the road traffic accident rate in Malaysia is high.

4.Conclusion

This research investigates the bivariate dependence modelling between losses of Malaysia insurance company. This study conduct empirical analysis in two stages. First and foremost, this research test the modelling dependence between losses by using bivariate pair-copulas from pair-copula construction models namely C-Vine copula and D-Vine copula models. The result suggests D-Vine copula model has the highest dependence structure and therefore it is a suitable to model dependence structure of losses. For the second stage, this paper focus on Malaysia's non-life insurance company's risk estimation by calculating the VaR and TVaR risk measures. This study found that the value of TVaR provides a better estimation on the risk the non-life insurance company and a better understanding regarding bivariate dependence modelling. This approach can be continued by measuring the simulated aggregate losses and concluding the value of capital requirements. The result will be more visualized rather than just estimates the risk for the company. Finally, this research visualize the trend of incurred claims data and we observe that motor insurance has the highest value of incurred claims as compared to other types of insurance. This outcome suggests the possibility of high road traffic accidents recorded in Malaysia.

References

[1] Tian, L., Jiang, S.J., Pan, G., & Zhang, N. (2018). Non-life insurance price dynamics: Evidence from the Chinese insurance market. Economic Research-Ekonomska Istrazivanja, 31(1), 171-187.

[2] Mejdoub, H., & Arab, M. B. (2018). Impact of dependence modelling of non-life insurance risks on capital requirement: D-Vine Copula approach. Research in International Business and Finance, 45(C): 208-218.

[3] Aas, K. (2016). Pair-copula constructions for financial applications: A review. Econometrics, 4(4): 43.

[4] Aas, K., Czado, C., Frigessi, A., & Bakken, H. (2009). Pair-copula constructions of multiple dependence. Insurance: Mathematics and Economics, 44(2): 182-198.

[5] Frees, E. W., & Wang, P. (2006). Copula credibility for aggregate loss models. Insurance: Mathematics and Economics, 38(2), 360-373.

[6] Eling, M., & Toplek, D. (2009). Modeling and management of nonlinear dependencies-copulas in dynamic financial analysis. In Journal of Risk and Insurance, vol. 76.pp.651-681.

[7] Zhao, X. B., & Zhou, X. (2010). Applying copula models to individual claim loss reserving methods. Insurance: Mathematics and Economics, 46(2), 290-299.

[8] Shi, P., & Frees, E. W.(2011). Dependent loss reserving using copulas. ASTIN Bulletin, 41(2), 499-486.

[9] Tang, A., & Valdez, E. A. (2011). Economic Capital and the Aggregation of Risks using Copulas. Proceedings of the 28th International Congress of Actuaries, Paris, France. Available at http://www.ica2006.com/282.html

[10] Diers, D., Eling, M., & Marek, S. D. (2012). Dependence modeling in non-life insurance using the Berstein copula. Insurance: Mathematics and Economics, 50(3): 430-436.