Proof of $1/0 = \infty$

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Main Proof of 1/0 = ∞

We have,

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots\infty$$

[From Theory of Binomial Index]

Now, putting x = -1 & n = -1, i.e.,

$$(1-1)^{-1} = 1 + \frac{(-1)1}{1!} + \frac{(-1)(-2)1^2}{2!} + \frac{(-1)(-2)(-3)1^3}{3!} + \ldots\infty$$

$$\therefore \frac{1}{0} = 1 + 1 + 1 + 1 + \ldots\infty$$

$$\therefore \frac{1}{0} = \infty$$

Hence Proved

Note: This Proof Includes claiming that the limit of x should be $|x|\leq 1$ in Binomial Expansion for any index.

∴ We can say that,

$$\frac{1}{0} = \infty \;\&\; \frac{-1}{0} = -\infty$$

Supporting Proofs

Proof of Binomial Expansion for any Index (With the help of Maclaurin's Series):

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}(x)^n$$

$f(x) = f(0) + f`(0)x + f``(0)x^2/2! + \ldots\infty$

Now, let $(1+x)^n$, $n \in Q$

$\therefore\; f(x) = (1+x)^n \implies f(0) = 1$

$f`(x) = n(1+x)^{n-1} \implies f`(0) = n(1+0) = n$

$f``(x) = n(n-1)(1+x)^{n-2} \implies f``(0) = n(n-1)$

$f```(0) = n(n-1)(n-2)$ & So on…

$$\therefore\; f(x) = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots\infty$$

Here, if **|x|>1**, then fractional power for any **-ve** number in the base will not be possible, i.e. like for x = -2,

$(1-2)^{1/2}$ or $(1-100)^{3/2}$ will not be possible for real numbers, its possible in **Complex Region**.

So, they might have taken condition that $|x|\leq 1$.

- But, in $(1+x)^n$, if we put x=1 or -1, it satisfies all the values & $(1-1)^{-1}$ gives the proof of $1/0 = \infty$.
  $\implies$The condition should be $|x|\leq 1$.
- Also, 1/0 itself is a proof of $1/0 = \infty$, if we see the graph of 1/x.

Conditions to Satisfy |x|≤1:

For x = 1 & x = -1:

- $(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots\infty$
  Now, it is true for $|x|<1$.
  Now For x = 1:
- $(1+1)^1 = 1 + \frac{1(1)}{1!} + \frac{1(1-1)1^2}{2!} + \cdots\infty$
  $= 1 + 1 + 0 + 0 + \ldots\infty$
  $= 2$
  $\therefore (1+1)^1 = 2$ *Satisfied*
- $(1+1)^{-1} = 1 + \frac{(-1)1}{1!} + \frac{(-1)(-1-1)1^2}{2!} + \cdots\infty$
  $= 1 - 1 + 1 - 1 + \ldots\infty$
  $\therefore (1+1)^{-1} = S_1 = 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \ldots\infty$
  Now, $1 - S_1 = 1 - (1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \ldots\infty)$
  $= 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \ldots\infty$
  $= S_1$

$\therefore 1 - S_1 = S_1 \Longrightarrow 1 = 2S_1$

$\therefore S_1 = ½$ → Result (1)

$\therefore (1+1)^{-1} = ½$ *Satisfied*

- $(1-1)^1 = 1 + \frac{1(-1)}{1!} + \frac{1(1-1)1^2}{2!} + \cdots\infty$

  $= 1 - 1 + 0 + 0 + \ldots\infty$

  $= 0$

  $\therefore (1-1)^1 = 0$ *Satisfied*

- $(1-1)^{1/2} = 1 + \frac{(1/2)(-1)}{1!} + \frac{(1/2)(-1/2)(-1)^2}{2!} + \frac{(1/2)\left(-\frac{1}{2}\right)(-3/2)(-1)^3}{3!} + \cdots\infty$

  Now, it will become 0, after round about 50 terms, derived from calculations.

  $\therefore (1-1)^{1/2} = 0$ *Satisfied*

- $(1+1)^{1/2} =$

  $1 + \frac{(1/2)1}{1!} + \frac{(1/2)(-1/2)1^2}{2!} + \frac{(1/2)\left(-\frac{1}{2}\right)(-3/2)1^3}{3!} + \cdots\infty$

  $= 1 + 0.5 - 0.125 + 0.0625 - 0.0390625 + \ldots\infty$

  $\approx$ Remains near the value of 1.41

  $\therefore (1+1)^{1/2} \approx 1.41$ *Satisfied*

- $(1+1)^{-1/2} = 1 + \frac{(-1/2)1}{1!} + \frac{(-1/2)(-3/2)1^2}{2!} + \frac{(-1/2)\left(-\frac{3}{2}\right)(-5/2)1^3}{3!} + \cdots\infty$

  $= 1 - 0.5 + 0.375 - 0.3125 + 0.2734375 - \ldots\infty$

  $\approx$ Remains near to 0.7 after number of terms

  $\therefore (1+1)^{-1/2} \approx 0.7\ (= 1/\sqrt{2})$ *Satisfied*

- $(1+1)^{3/2} = 1 + \frac{(3/2)1}{1!} + \frac{(3/2)(1/2)1^2}{2!} + \frac{(3/2)\left(\frac{1}{2}\right)(-1/2)1^3}{3!} + \cdots\infty$

  $= 1 + 1.5 + 0.375 - 0.0625 + 0.0234375 - \ldots\infty$

  $\approx$ Remains exactly constant near 2.82 & doesn't change

  $\therefore (1+1)^{3/2} \approx 2.82\ (=2\sqrt{2})$ *Satisfied*

- $(1+1)^{-3/2} = 1 + \frac{(-3/2)1}{1!} + \frac{(-3/2)(-5/2)1^2}{2!} + \frac{(-3/2)\left(\frac{-5}{2}\right)(-7/2)1^3}{3!} + \cdots\infty$

  $= 1 - 1.5 + 1.875 - 2.1875 + 2.4609375 - \ldots\infty$

  $\approx$ Will slowly decrease to 0.35355 and will become constant to it, after number of terms

  $\therefore (1+1)^{-3/2} \approx 0.3535$ *Satisfied*

- $(1+1)^{-2} = 1 + \frac{(-2)1}{1!} + \frac{(-2)(-3)1^2}{2!} + \frac{(-2)(-3)(-4)1^3}{3!} + \cdots\infty$

  $= 1 - 2 + 3 - 4 + 5 - 6 + \ldots\infty$

  $\therefore S_2 = 1 - 2 + 3 - 4 + 5 - 6 + \ldots\infty$

  $S_2 = \quad 1 - 2 + 3 - 4 + 5 - 6 + \ldots\infty$

  $\therefore 2S_2 = 1 - 1 + 1 - 1 + \ldots\infty$ [∵ Adding S_2 twice]

  $\therefore 2S_2 = ½$ [From Result (1) (Pg. 03)]

  $\therefore S_2 = 1/4$

  $\therefore (1+1)^{-2} = 1/4$ *Satisfied*

- $(1-1)^{-1} = 1 + \frac{(-1)1}{1!} + \frac{(-1)(-2)1^2}{2!} + \frac{(-1)(-2)(-3)1^3}{3!} + \ldots\infty$

  $\therefore \frac{1}{0} = 1 + 1 + 1 + 1 + \ldots\infty$

  $\therefore \frac{1}{0} = \infty$ *Satisfied*

- $(1-1)^{-1/2} = 1 + \frac{(-1/2)(-1)}{1!} + \frac{(-1/2)(-3/2)(-1)^2}{2!} + \frac{(-1/2)\left(-\frac{3}{2}\right)(-5/2)(-1)^3}{3!} + \cdots\infty$

  $\approx \infty$

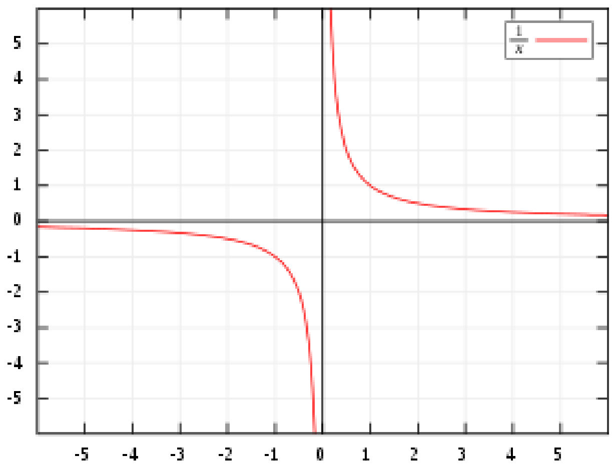
  $\therefore (1-1)^{-1/2} \approx \infty$ *Satisfied*

Now, according to result of $(1+1)^{-2}$, all the results for **negative integral power** of (1+1) i.e., for power of 2, can be proved. And for positive integral power, Binomial Theorem is always applicable & can be proved by the same method of Binomial Index.

Therefore, now, basically we can say that, the condition or limit of x as $|x| \le 1$ also satisfies the Binomial Expansion for any index. Hence, Proof of $1/0 = \infty$ can be also given.

Graphical Interpretation

→The Graph of 1/x itself explains that $1/0 = \infty$



Here, the graph of $1/x$ does not intersect at $x=0$. But approaches higher values.

Now, ∞ is something greater than any highest imagination. It is higher than any highest number one can imagine.

∞ is larger than limit of Imagination.

So, basically in this graph, we can deduce that, $f(x) = 1/x$ intersects $x = 0$ at ∞ OR approaches ∞, if $x>0$ and $f(x)$ approaches to $-\infty$ if $x<0$ (i.e., $-1/0 = -\infty$).

Theoretically & Practically "1/0"

→ $1/4 = 0.25$ i.e., "4^{th} part of 1".

Similarly, ½ = 0.5 i.e., "2^{nd} part of 1".

Also, by definition, "$1/x$ is the x^{th} part of 1".

→ Now, Similarly, if we apply $1/0.5 = 2$, then we can say "2 is a number whose $(0.5)^{th}$ part is 1". Also, for $1/0.25 = 4$, we can say, "4 is a number whose $(0.25)^{th}$ part is 1".

→ Now, $1/x$, $x \in N$, then this is applicable in real life or it makes practical sense.
Like if we divide 1 pen in 2 parts, it makes 0.5(half) pen i.e., ½ pen = 0.5 pen.
→ But, in the same way, if we apply this to $1/x$, $x \in Q$, then it doesn't make any sense or it is impractical.
Like, if we divide 1 pen in 0.25 parts, it wont make 4 pens i.e., 1/(0.25) pen = 4 pens.
→ So, as we can see, $1/x$, $x \in Q$ doesn't make any sense in practical life.
Similarly, if we do $1/0 = \infty$, it means "0^{th} part of ∞ is 1".
It shows that ∞ is too large that its 0^{th} part is 1.
→ So, if we see, $1/0$ is not undefined. It has some meaning like ½, ¼, 1/0.5, etc.
→Therefore, basically, we can conclude that $1/x$, $x \in Q$ & $1/0$ both of them don't (and also do) have a practical meaning. But they do possess a theoretical meaning.

→ At last, I want to conclude that behaviour of 0 & ∞ is highly similar upto great extent in Mathematics.

($0*x = 0 \Longrightarrow \infty*x = \infty$, $x-0 = x \Longrightarrow \infty-x = \infty$,

$x+0 = x \Longrightarrow \infty+x = \infty$, $0/x = 0 \Longrightarrow \infty/x = \infty$)

So, as operations with 0 are considered in Mathematics, operations of ∞ should also be considered.

And Proof of $1/0 = \infty$ should be considered.

Being inexperienced, I will very highly value any piece of advice you give me.