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Fully Fuzzy Solution of Transportation Problem in Fuzzy Environment

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Abstract: ***The fully fuzzy transportation problem is a most important special type of transportation problem in optimization theory that crude material of the organization is shipped from source to destination by various transport movements such as load flights, trucks, product trains. For the better approach to create and deliver multi-level values and services to the customers as per customer's requirement, the fully fuzzy optimal cost and time are the best concept in fuzzy transportation problem. In this paper, we propose a fully fuzzy initial basic feasible solution of transportation problem in fuzzy environment in which all variables and parameters are addressed as a fuzzy number. Usually, many researchers are transformed fuzzy transportation model into crisp problem and then solved by existence methods but we find out fuzzy solution of transportation model in which the total transportation cost is a minimum with fuzzy number that assumes that the direct route, between a source, and a destination is a minimum cost. The effectiveness of this approach is illustrated by numerical example.***

Keywords: Excel Solver Software, Fully Fuzzy Number, Linear Optimization, Fully Fuzzy Transportation Problem, Membership Function

1. Introduction

In the recent scenario of the competitive market, the fully fuzzy transportation problem is a most important special type of problem in optimization theory. Transportation is the movement of people, animals and various goods from one place to another and center of management science and operations research that maintains the economic and social activities. The fuzzy transportation problem is one of the earliest applications of linear optimization that has wide applications in logistics and supply chain for reducing the transportation cost. Different type of efficient approaches has been developed for solving fuzzy transportation problem, permit only those shipments that go directly from a fuzzy supply point to a fuzzy demand point. The most of the existing approaches generated only crisp model for solving the fuzzy transportation problem. During last decade, several researchers have carried out different type of investigations and approaches on fuzzy transportation problems.

2. Literature Survey

Various efficient approaches have been developed when variables are known exactly when our human systems and activities are not certain they are not clear but they are certain up to tolerance level the first attempt was made by L. A. Zadeh (1965) [] and formulated a system in which sets are represented by membership functions by using fuzzy sets and systems. P. Pandian and G. Natarajan (2010) [15] proposed a new algorithm of fuzzy zero-point method for solving fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers and obtained optimal solution of this problem in the form of trapezoidal fuzzy numbers. Ibrahim A. Baky (2010) [2] presented two fuzzy goal-programming approaches for multi-level multi-objective linear optimization and this model is developed to minimize the group regret of degree of satisfactions of all the decision makers, and to achieve the highest degree of each of the defined membership function goals to the extent. Amit Kumar, Jagdeep Kaur, Pushpinder Singh (2011) [9] proposed a new method for find out the fuzzy optimal solution of fuzzy linear optimization problem. Pandit P. (2013) [16] proposed a new method for multi-objective linear optimization problem in which all parameters are involved as a fuzzy number. Ali Ebrahimnejad (2013) [6] gave a review with some extensions on solving transportation problems with triangular fuzzy numbers. S. K. Bharati and and S. R. Singh (2014) [4] gave a comparative study for solving multi objective linear programming problems by using intuitionistic fuzzy optimization method. R. Ezzati A, E. Khorram B, R. Enayati (2015) [7] proposed a new algorithm for solving fully fuzzy linear optimization problems by using the multi-objective linear optimization techniques. K. Jaikumar (2016) [10] developed an approach for solving fully fuzzy transportation problem. Rashmi Singh, Vipin Saxena (2017) [18] proposed a new ranking based fuzzy approach for solving fuzzy transportation problem. M. Premkumar and M. Kokila (2017) [17] developed a method for solving fuzzy transportation problem of symmetric trapezoidal with alpha cut and ranking technique. M. Ramesh kumar and S. Subramanian (2018) [11] developed a method for solving a special type of fuzzy transportation problems by using ranking function with triangular fuzzy numbers. K. Balasubramanian and S. Subramanian (2018) [3] proposed a new approach for solving fuzzy transportation problem. P. Uma Maheswari and K Ganesan (2018) [13] used a simple approach for solving fuzzy transportation problem under fuzzy environment in which all parameters (transportation costs, supplies, demands) are represented as pentagonal fuzzy numbers. D. Stephen Dinagar and B. Christopar Raj (2019) [5] developed a method for solving fully fuzzy transportation problem with generalized quadrilateral fuzzy

Numbers. P. Anukokila & B. Radhakrishnan (2019) [1] proposed a goal programming approach for solving fully fuzzy fractional transportation problem. In this paper, we proposed a fuzzy level approach for solving fully fuzzy transportation problem by assuming that a decision maker is uncertain about the precise fully fuzzy values of the transportation costs, availability, and demand of the product. The objective of using fully fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation costs, while satisfying fuzzy supply. All parameters of the product may be triangular fuzzy numbers, trapezoidal fuzzy numbers, pentagonal fuzzy numbers etc.

3. Problem definition

There are various types of fuzzy numbers in fuzzy set theory defined over the field R of real numbers and the n-dimensional space R^n. This paper sets up the following concepts of fuzzy numbers.

Definition 3.1 [Fuzzy Number]
A fuzzy number is a special case of a convex, normalized fuzzy set of the real line and refers to a connected set of possible values. Each possible value of fuzzy number has its own weight between 0 and 1 and it has the characteristics (a) A number $\overline{A}$ is normal, convex, upper semi-continuous and sup ($\overline{A}$) is bounded in R.

Definition 3.2 [Triangular Fuzzy Number]
A fuzzy number $\overline{A} = (a_1, a_2, a_3)$ said to be triangular fuzzy number if its membership function $\mu_{\overline{A}} : R \rightarrow [0, 1]$ defined as follows:

$$\mu_{\overline{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, a_1 \le x < a_2 \\ 1, \quad x = a_2 \\ \frac{a_3 - x}{a_3 - a_2}, a_2 \le x \le a_3 \\ 0, otherwise \end{cases}$$

Definition 3.3 [Trapezoidal Fuzzy Number] A fuzzy number $\overline{A} = (a_1, a_2, a_3, a_4)$ is said to be a trapezoidal fuzzy number if its membership function $\mu_{\overline{A}} : R \rightarrow [0, 1]$ defined as follows:

$$\mu_{\overline{A}}(x) = \begin{cases} 0, \quad x \le a_1 \\ \frac{x - a_1}{a_2 - a_1}, a_1 \le x \le a_2 \\ 1, \quad a_2 \le x \le a_3 \\ \frac{a_4 - x}{a_4 - a_3}, a_2 \le x \le a_3 \\ 0, \quad x \ge a_4 \end{cases}$$

Definition 3.4 [Fuzzy Initial Basic Feasible Solution of Fully Fuzzy Transportation Problem]
A set of non-negative values of the decision fully fuzzy variables ($\tilde{x}_{ij} \ge 0$) that satisfies the total fuzzy supply and total fuzzy demand of fully fuzzy transportation problem is said to be fuzzy initial basic feasible solution.

Definition3.5 [Fuzzy Optimal Solution of Fully Fuzzy Transportation Problem]
Fuzzy initial basic feasible solution is said to be optimal solution of fully fuzzy transportation problem if it optimizes the fully fuzzy transportation costs.

Definition 3.6 [Balanced Fully Fuzzy Transportation Problem]
A fully fuzzy transportation problem is said to be balanced, if total supply equal to total demand i.e., $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$,

Otherwise fully fuzzy transportation problem is said to be unbalanced. In case of unbalance there are two cases arises such either $\sum_{i=1}^{m} \tilde{a}_i < \sum_{j=1}^{n} \tilde{b}_j$ or $\sum_{i=1}^{m} \tilde{a}_i > \sum_{j=1}^{n} \tilde{b}_j$.

Case I if demand exceeds supply, the constraints of the transportation table will appear as $\sum_{i=1}^{m} \tilde{a}_i < \sum_{j=1}^{n} \tilde{b}_j$, and then add a fully fuzzy dummy source to the problem with fuzzy capacity $\sum_{j=1}^{n} \tilde{b}_j - \sum_{i=1}^{m} \tilde{a}_i$ in order to make it a balance problem we follow as following process

$$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i; \quad i = 1,2, \dots, m$$

$$and \sum_{i=1}^{m} \tilde{x}_{ij} \le \tilde{b}_j; \quad j = 1,2, \dots, n$$

$$\Rightarrow \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i \quad ; \quad i = 1,2, \dots, m$$

$$and \sum_{i=1}^{m} [\tilde{x}_{ij} + \tilde{s}_{m+1,j}] = \tilde{b}_j; \quad j = 1,2, \dots, n$$

$$\Rightarrow \sum_{j=1}^{n} \tilde{s}_{m+1,j} = \sum_{j=1}^{n} \tilde{b}_j - \sum_{i=1}^{m} \tilde{a}_i = \text{excess fuzzy demand}.$$

If $\tilde{a}_{m+1}$ denotes the excess fuzzy demand, then the modified fuzzy transportation problem can be represented by

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{m} \left(\tilde{c}_{ij}\,\tilde{x}_{ij} + \tilde{c}_{m+1,j}\,\tilde{s}_{m+1,j}\right)$$

$$\text{subject to} \quad \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i \quad ; \quad i = 1,2,\dots,m+1$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} + \tilde{s}_{m+1},j = \tilde{b}_j ; \quad j = 1,2,\dots,n$$

and

where $\tilde{c}_{m+1,j} = 0$ for all j and

$$\sum_{i=1}^{m} \tilde{a}_i + \tilde{a}_{m+1} = \sum_{j=1}^{n} \tilde{b}_j \quad \text{or} \quad \tilde{a}_{m+1} = \sum_{j=1}^{n} \tilde{b}_j - \sum_{i=1}^{m} \tilde{a}_i$$

It follows that if $\sum_{j=1}^{n} \tilde{b}_j > \sum_{i=1}^{m} \tilde{a}_i$, then a dummy row can be added to the transportation table

In case II, if excess supply available, then the constraints of the transportation table will appear as $\sum_{i=1}^{m} \tilde{a}_i > \sum_{j=1}^{n} \tilde{b}_j$,and add fully fuzzy dummy destination to the problem with fuzzy demand $\sum_{i=1}^{m} \tilde{a}_i - \sum_{j=1}^{n} \tilde{b}_j$ in order to make it a balance problem we follow as following process

$$\sum_{j=1}^{n} \tilde{x}_{ij} \le \tilde{a}_i, \quad i = 1,2,\dots m$$

$$and \sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j \quad j = 1,2,\dots n$$

$$\sum_{j=1}^{n} \tilde{x}_{ij} + \tilde{s}_{i,n+1} = \tilde{a}_i$$

$$\sum_{i=1}^{m} \left[\sum_{j=1}^{n} \tilde{x}_{ij} + \tilde{s}_{i,n+1}\right] = \sum_{i=1}^{m} \tilde{a}_i$$

$\sum_{i=1}^{m} \tilde{s}_{i,n+1} = \sum_{i=1}^{m} \tilde{a}_i - \sum_{j=1}^{n} \tilde{b}_j =$ excess supply available

If $\tilde{b}_{n+1}$ denotes the excess supply available, then the modified transportation problem can be presented as follows

$$Min\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} \left(\tilde{c}_{ij}\,\tilde{x}_{ij} + \tilde{c}_{i,n+1}\tilde{s}_{i,n+1}\right)$$

subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij} + \tilde{s}_{i,n+1} = \tilde{a}_i ; \quad i = 1,2,\dots,m$$

$$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{b}_j \quad ; \quad j = 1,2,\dots,n+1$$

where $\tilde{c}_{i,n+1} = 0\ (i = 1,2,\dots, x_{ij} \ge 0$ for all i, j where $\tilde{c}_{i,n+1} = 0(i = 1,2,\dots,m)$ and

$$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j + \tilde{b}_{n+1} \quad \text{or} \quad \tilde{b}_{n+1} = \sum_{i=1}^{m} \tilde{a}_i - \sum_{j=1}^{n} \tilde{b}_j$$

It follows that, if $\sum_{i=1}^{m} \tilde{a}_i > \sum_{j=1}^{n} \tilde{b}_j$, then a dummy column can be added to the transportation table.

4. Methodology

The fully fuzzy transportation problems deal with the transportation of a single product from several sources to several sinks. Let us consider $S1, S2, S3, \dots, Sm$ are m sources with $\tilde{a}_i$ fuzzy supplies, to be allocated among n destinations $D1, D2, D3, \dots, Dn$ with $\tilde{b}_j$ fuzzy demand. The problem is to determine the transportation schedule to minimize the total transportation cost and satisfy the fuzzy supply and fuzzy demand conditions. The fully fuzzy transportation problemcan be defined as follows:

$$MinZ = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij}\tilde{x}_{ij}$$

$$Subjectto \ \sum_{j=1}^{n}\tilde{x}_{ij} \le \tilde{a}_i, \ \sum_{i=1}^{m}\tilde{x}_{ij} \ge \tilde{b}_j, \ \sum_{j=1}^{n}\tilde{b}_j = \sum_{i=1}^{m}\tilde{a}_i,$$

$$\tilde{x}_{ij} \ge 0, \ i = 1,2,\dots m; \ j = 1,2,\dots n$$

Where $\tilde{c}_{ij}$ is the fully fuzzy transportation cost matrix from ith source to the jth destination and fuzzy transportation, supply $\tilde{a}_i$ and demand $\tilde{b}_j$ quantities are fully fuzzy number. The corresponding fuzzy levels ofsupply $\tilde{a}_i$ and demand $\tilde{b}_j$ are equal. In this problem the transportation cost, supply and demand may be considered different type of fully fuzzy numbers as triangular fuzzy numbers, trapezoidal fuzzy numbers, and pentagonal fuzzy numbers etc. A general fuzzy transportation problem in transportation cost matrix form is presented by

Table 1: Fully Fuzzy Transportation Problem

Sources →/ *Destinations* ↓	D_1	D_2	…………	D_n	*Fuzzy Supply*
O_1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	………….	$\tilde{c}_{1n}$	$\tilde{a}_1$
O_2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	………….	$\tilde{c}_{2n}$	$\tilde{a}_2$
.	.	.	.	.	….
O_M	$\tilde{c}_{m1}$	$\tilde{c}_{m2}$	………….	$\tilde{c}_{mn}$	$\tilde{a}_m$
Fuzzy Demand	$\tilde{b}_1$	$\tilde{b}_2$	………….	$\tilde{b}_n$	$\sum_{j=1}^{n}\tilde{b}_j = \sum_{i=1}^{m}\tilde{a}_i$

In this section, we applied following steps to proposed fuzzy level approach for initial fuzzy basic feasible solution of fully fuzzy transportation problem. All parameters of transportation cost matrix; source and destination are taken as fully fuzzy numbers:

Step1: First of all, find following mathematical formulation of fully fuzzy transportation problem

$$Min.Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij}\tilde{x}_{ij}$$

$$Subject\ to\ \sum_{j=1}^{n}\tilde{x}_{ij} \le \tilde{a}_i, \sum_{i=1}^{m}\tilde{x}_{ij} \ge \tilde{b}_j, \sum_{j=1}^{n}\tilde{b}_j = \sum_{i=1}^{m}\tilde{a}_i,$$

$$\tilde{x}_{ij} \ge 0,\ i=1,2,...m;\ j=1,2,....n$$

Step 2: Separate all corresponding fuzzy levels in objective function and constraints of the fully fuzzy transportation problem:

$$Min.Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\{(\tilde{C}_{ij}\tilde{x}_{ij})^{L1}, (\tilde{C}_{ij}\tilde{x}_{ij})^{L2}, (\tilde{C}_{ij}\tilde{x}_{ij})^{L3}, ...\}$$

$$Subject\ to\ \sum_{j=1}^{n}(\tilde{x}_{ij})^{L1} \le (\tilde{a}_i)^{L1}, \sum_{j=1}^{n}(\tilde{x}_{ij})^{L2} \le (\tilde{a}_i)^{L2}, ...\sum_{j=1}^{n}(\tilde{x}_{ij})^{Lk} \le (\tilde{a}_i)^{Lk},$$

$$\sum_{i=1}^{m}(\tilde{x}_{ij})^{L1} \ge (\tilde{b}_j)^{L1}, \sum_{i=1}^{m}(\tilde{x}_{ij})^{L2} \ge (\tilde{b}_j)^{L2}, ...\sum_{i=1}^{m}(\tilde{x}_{ij})^{Lk} \ge (\tilde{b}_j)^{Lk},$$

$$\sum_{j=1}^{n}(\tilde{b}_j)^{L1} = \sum_{i=1}^{m}(\tilde{a}_i)^{L1}, \sum_{j=1}^{n}(\tilde{b}_j)^{L2} = \sum_{i=1}^{m}(\tilde{a}_i)^{L2}, ...\sum_{j=1}^{n}(\tilde{b}_j)^{Lk} = \sum_{i=1}^{m}(\tilde{a}_i)^{Lk}, \tilde{x}_{ij} \ge 0$$

$$where\ \tilde{x}_{ij} = ((\tilde{x}_{ij})^{L1}, (\tilde{x}_{ij})^{L2}, ...(\tilde{x}_{ij})^{Lk})\ \forall\ \tilde{C}_{ij} = ((\tilde{C}_{ij})^{L1}, (\tilde{C}_{ij})^{L2}, ...(\tilde{C}_{ij})^{Lk}), are\ fuzzy\ numbers$$

Step 3: Apply following fuzzy level approach in the fully fuzzy transportation problem

$$MinZ = \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{C}^{L1}{}_{ij}(\tilde{x}_{ij})^{L1} + \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{C}^{L2}{}_{ij}(\tilde{x}_{ij})^{L2} + ... + \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{C}^{Lk}{}_{ij}(\tilde{x}_{ij})^{Lk}$$

$$Subject\ to\ \sum_{j=1}^{n}(\tilde{x}_{ij})^{L1} \le (\tilde{a}_i)^{L1}, \sum_{j=1}^{n}(\tilde{x}_{ij})^{L2} \le (\tilde{a}_i)^{L2}, ...\sum_{j=1}^{n}(\tilde{x}_{ij})^{Lk} \le (\tilde{a}_i)^{Lk},$$

$$\sum_{i=1}^{m}(\tilde{x}_{ij})^{L1} \ge (\tilde{b}_j)^{L1}, \sum_{i=1}^{m}(\tilde{x}_{ij})^{L2} \ge (\tilde{b}_j)^{L2}, ...\sum_{i=1}^{m}(\tilde{x}_{ij})^{Lk} \ge (\tilde{b}_j)^{Lk},$$

$$\sum_{j=1}^{n}(\tilde{b}_j)^{L1} = \sum_{i=1}^{m}(\tilde{a}_i)^{L1}, \sum_{j=1}^{n}(\tilde{b}_j)^{L2} = \sum_{i=1}^{m}(\tilde{a}_i)^{L2}, ..., \sum_{j=1}^{n}(\tilde{b}_j)^{Lk} = \sum_{i=1}^{m}(\tilde{a}_i)^{Lk},$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{C}^{L2}{}_{ij}(\tilde{x}_{ij})^{L2} \ge \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{C}^{L1}{}_{ij}(\tilde{x}_{ij})^{L1}, ..., \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{C}^{Lk}{}_{ij}(\tilde{x}_{ij})^{Lk} \ge \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{C}^{L(k-1)}{}_{ij}(\tilde{x}_{ij})^{L(k-1)}, \tilde{x}_{ij} \ge 0$$

$$where\ \tilde{x}_{ij} = ((\tilde{x}_{ij})^{L1}, (\tilde{x}_{ij})^{L2}, ...(\tilde{x}_{ij})^{Lk})\ \forall\ \tilde{C}_{ij} = ((\tilde{C}_{ij})^{L1}, (\tilde{C}_{ij})^{L2}, ...(\tilde{C}_{ij})^{Lk}), are\ fuzzy\ numbers$$

Step 4: Solve this problem by using any software until all fuzzy demand and fuzzy supply are satisfied.

5. Results & Discussion

A company has two fuzzy sources O_1& O_2 and the destinations D_1, D_2& D_3. The fuzzy transportation costs for unit quantity of the product from ithsource to jthdestination is summarizing in the following matrix and fuzzy availability of the product at source are (75, 95, 125), (45, 65, 95)and the fuzzy demand of the product at destinations are (35, 45, 65), (25, 35, 45), (60, 80, 110) respectively. Hence, the fully fuzzy transportation problem summarizes as follows:

Table 2: Fully fuzzy transportation problem

Sources → / *Destinations* ↓	*D₁*	*D₂*	*D₃*	*Fuzzy Supply*
O_1	(15, 25, 35)	(55, 65, 85)	(85, 95, 105)	(75, 95, 125)
O_2	(65, 75, 85)	(80, 90, 110)	(30, 40, 50)	(45, 65, 95)
Fuzzy Demand	(35, 45, 65)	(25, 35, 45)	(60, 80, 110)	

Step1: First of all, find following mathematical formulation of fully fuzzy transportation problem

$$Min\ Z = (15,25,35)\tilde{x}_{11} + (55,65,85)\tilde{x}_{12} + (85,95,105)\tilde{x}_{13} + (65,75,85)\tilde{x}_{21} + (80,90,110)\tilde{x}_{22} + (30,40,50)\tilde{x}_{23}$$
$$subject\,to\ \ (\tilde{x}_{11}) + (\tilde{x}_{12}) + (\tilde{x}_{13}) \le (75,95,125),$$
$$(\tilde{x}_{21}) + (\tilde{x}_{22}) + (\tilde{x}_{23}) \le (45,65,95),$$
$$(\tilde{x}_{11}) + (\tilde{x}_{21}) \ge (35,45,65),$$
$$(\tilde{x}_{12}) + (\tilde{x}_{22}) \ge (25,35,45),$$
$$(\tilde{x}_{13}) + (\tilde{x}_{23}) \ge (60,80,110),\ \tilde{x}_{ij} \ge 0$$

Step 2: Separate all corresponding fuzzy levels in objective function and constraints of the fully fuzzy transportation problem:

$$MinZ = [15(\tilde{x}_{11})^{L1} + 55(\tilde{x}_{12})^{L1} + 85(\tilde{x}_{13})^{L1} + 65(\tilde{x}_{21})^{L1} + 80(\tilde{x}_{22})^{L1} + 30(\tilde{x}_{23})^{L1};\ Z^{L2} = 25(\tilde{x}_{11})^{L2} + 65(\tilde{x}_{12})^{L2} + 95(\tilde{x}_{13})^{L2} +$$
$$75(\tilde{x}_{21})^{L2} + 90(\tilde{x}_{22})^{L2} + 40(\tilde{x}_{23})^{L2}; Z^{L3} = 35(\tilde{x}_{11})^{L3} + 85(\tilde{x}_{12})^{L3} + 105(\tilde{x}_{13})^{L3} + 85(\tilde{x}_{21})^{L3} + 110(\tilde{x}_{22})^{L3} + 50(\tilde{x}_{23})^{L3}]$$
$$Subject\ \ to\ \ (\tilde{x}_{11})^{L1} + (\tilde{x}_{12})^{L1} + (\tilde{x}_{13})^{L1} \le 75; (\tilde{x}_{11})^{L2} + (\tilde{x}_{12})^{L2} + (\tilde{x}_{13})^{L2} \le 95;$$
$$(\tilde{x}_{11})^{L3} + (\tilde{x}_{12})^{L3} + (\tilde{x}_{13})^{L3} \le 125;\ (\tilde{x}_{21})^{L1} + (\tilde{x}_{22})^{L1} + (\tilde{x}_{23})^{L1} \le 45;$$
$$(\tilde{x}_{21})^{L2} + (\tilde{x}_{22})^{L2} + (\tilde{x}_{23})^{L2} \le 65;\ \ (\tilde{x}_{21})^{L3} + (\tilde{x}_{22})^{L3} + (\tilde{x}_{23})^{L3} \le 95;$$
$$(\tilde{x}_{11})^{L1} + (\tilde{x}_{21})^{L1} \ge 35; (\tilde{x}_{11})^{L2} + (\tilde{x}_{21})^{L2} \ge 45; (\tilde{x}_{11})^{L3} + (\tilde{x}_{21})^{L3} \ge 65;$$
$$(\tilde{x}_{12})^{L1} + (\tilde{x}_{22})^{L1} \ge 25; (\tilde{x}_{11})^{L2} + (\tilde{x}_{22})^{L2} \ge 35; (\tilde{x}_{11})^{L3} + (\tilde{x}_{22})^{L3} \ge 45;$$
$$(\tilde{x}_{13})^{L1} + (\tilde{x}_{23})^{L1} \ge 60; (\tilde{x}_{13})^{L2} + (\tilde{x}_{23})^{L2} \ge 80; (\tilde{x}_{13})^{L3} + (\tilde{x}_{23})^{L3} \ge 110; \forall\ \tilde{x}_{ij} \ge 0$$

Step3: Apply fuzzy level approach in the fully fuzzy transportation problem

$$MinZ = 15(\tilde{x}_{11})^{L1} + 55(\tilde{x}_{12})^{L1} + 85(\tilde{x}_{13})^{L1} + 65(\tilde{x}_{21})^{L1} + 80(\tilde{x}_{22})^{L1} + 30(\tilde{x}_{23})^{L1} + 25(\tilde{x}_{11})^{L2} + 65(\tilde{x}_{12})^{L2} + 95(\tilde{x}_{13})^{L2} + 75(\tilde{x}_{21})^{L2}$$
$$+ 90(\tilde{x}_{22})^{L2} + 40(\tilde{x}_{23})^{L2} + 35(\tilde{x}_{11})^{L3} + 85(\tilde{x}_{12})^{L3} + 105(\tilde{x}_{13})^{L3} + 85(\tilde{x}_{21})^{L3} + 110(\tilde{x}_{22})^{L3} + 50(\tilde{x}_{23})^{L3}$$
$$Subject\ \ to\ \ (\tilde{x}_{11})^{L1} + (\tilde{x}_{12})^{L1} + (\tilde{x}_{13})^{L1} \le 75; (\tilde{x}_{11})^{L2} + (\tilde{x}_{12})^{L2} + (\tilde{x}_{13})^{L2} \le 95;$$
$$(\tilde{x}_{11})^{L3} + (\tilde{x}_{12})^{L3} + (\tilde{x}_{13})^{L3} \le 125;\ (\tilde{x}_{21})^{L1} + (\tilde{x}_{22})^{L1} + (\tilde{x}_{23})^{L1} \le 45;$$
$$(\tilde{x}_{21})^{L2} + (\tilde{x}_{22})^{L2} + (\tilde{x}_{23})^{L2} \le 65;\ \ (\tilde{x}_{21})^{L3} + (\tilde{x}_{22})^{L3} + (\tilde{x}_{23})^{L3} \le 95;$$
$$(\tilde{x}_{11})^{L1} + (\tilde{x}_{21})^{L1} \ge 35; (\tilde{x}_{11})^{L2} + (\tilde{x}_{21})^{L2} \ge 45; (\tilde{x}_{11})^{L3} + (\tilde{x}_{21})^{L3} \ge 65;$$
$$(\tilde{x}_{12})^{L1} + (\tilde{x}_{22})^{L1} \ge 25; (\tilde{x}_{11})^{L2} + (\tilde{x}_{22})^{L2} \ge 35; (\tilde{x}_{11})^{L3} + (\tilde{x}_{22})^{L3} \ge 45;$$
$$(\tilde{x}_{13})^{L1} + (\tilde{x}_{23})^{L1} \ge 60; (\tilde{x}_{13})^{L2} + (\tilde{x}_{23})^{L2} \ge 80; (\tilde{x}_{13})^{L3} + (\tilde{x}_{23})^{L3} \ge 110; \forall\ \tilde{x}_{ij} \ge 0$$
$$-15(\tilde{x}_{11})^{L1} - 55(\tilde{x}_{12})^{L1} - 85(\tilde{x}_{13})^{L1} - 65(\tilde{x}_{21})^{L1} - 80(\tilde{x}_{22})^{L1} - 30(\tilde{x}_{23})^{L1} + 25(\tilde{x}_{11})^{L2} + 65(\tilde{x}_{12})^{L2} + 95(\tilde{x}_{13})^{L2} + 75(\tilde{x}_{21})^{L2}$$
$$+ 90(\tilde{x}_{22})^{L2} + 40(\tilde{x}_{23})^{L2} \ge 0;$$
$$-25(\tilde{x}_{11})^{L2} - 65(\tilde{x}_{12})^{L2} - 95(\tilde{x}_{13})^{L2} - 75(\tilde{x}_{21})^{L2} - 90(\tilde{x}_{22})^{L2} - 40(\tilde{x}_{23})^{L2} + 35(\tilde{x}_{11})^{L3} + 85(\tilde{x}_{12})^{L3} + 105(\tilde{x}_{13})^{L3} + 85(\tilde{x}_{21})^{L3}$$
$$+ 110(\tilde{x}_{22})^{L3} + 50(\tilde{x}_{23})^{L3} \ge 0;\ \forall\ \tilde{x}_{ij} \ge 0$$

Step 4: Solve this problem by using any software and find out the following solution

	D_1	D_2	D_3	Supply
O_1	**(35, 45, 65)**	**(25, 35, 45)**	**(15, 15, 15)**	(75, 95, 125)
O_2			**(45, 65, 95)**	(45, 65, 95)
Demand	(35, 45, 65)	(25, 35, 45)	(60, 80, 110)	

Fuzzy optimal solution by proposed approach:

$$\begin{cases} \tilde{x}_{11} = ((\tilde{x}_{11})^{L1}, (\tilde{x}_{11})^{L2}, (\tilde{x}_{11})^{L3}) = (35,45,65) \\ \tilde{x}_{12} = ((\tilde{x}_{12})^{L1}, (\tilde{x}_{12})^{L2}, (\tilde{x}_{12})^{L3}) = (25,35,45) \\ \tilde{x}_{13} = ((\tilde{x}_{13})^{L1}, (\tilde{x}_{13})^{L2}, (\tilde{x}_{13})^{L3}) = (15,15,15) \\ \tilde{x}_{23} = ((\tilde{x}_{23})^{L1}, (\tilde{x}_{23})^{L2}, (\tilde{x}_{23})^{L3}) = (45,65,95) \\ \tilde{x}_{21} = \tilde{x}_{22} = (0,0,0) \end{cases}$$

The total fully fuzzy transportation cost is (4525, 7425, 12425)

6. Conclusion

In this paper, the proposed fuzzy level approach for fully fuzzy transportation is investigated using computational procedure which is very simple and easy to understand, obtained fuzzy optimal solution by proposed approach is always optimal, and need not be checked optimality criteria. We therefore, hope that this approach may be used as an effective tool for direct extension of existing methods to apply on real life of this type of problems for the decision makers.

7. Future scope

This approach can be used for fuzzy assignment problems and fuzzy based non-linear optimization techniques as future research studies.

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