Fully Fuzzy Solution of Transportation Problem in Fuzzy Environment

Anju Raj¹, Jitendra Singh²

¹Rishi Bhartrihi Matsya University Alwar, Rajasthan, India
anju1993jinkiruri[at]gmail.com

²Department of Mathematics, R. R. College Alwar, Rajasthan, India
jsambesh[at]gmail.com

Abstract: The fully fuzzy transportation problem is a most important special type of transportation problem in optimization theory that crude material of the organization is shipped from source to destination by various transport movements such as load flights, trucks, product trains. For the better approach to create and deliver multi-level values and services to the customers as per customer's requirement, the fully fuzzy optimal cost and time are the best concept in fuzzy transportation problem. In this paper, we propose a fully fuzzy initial basic feasible solution of transportation problem in fuzzy environment in which all variables and parameters are addressed as a fuzzy number. Usually, many researchers are transformed fuzzy transportation model into crisp model for solving fuzzy transportation problems. Rashmi Singh, Vipin Saxena (2018) [4] gave a comparative study for solving fully fuzzy linear optimization problems by using intuitionistic fuzzy optimization method. R. Ezzati A, E. Khorrab B, R. Enayati (2015) [7] proposed a new algorithm for solving fully fuzzy linear optimization problems by using the multi-objective linear optimization techniques. K. Jaikumar (2016) [10] developed an approach for solving fully fuzzy transportation problem. Rashmi Singh, Vipin Saxena (2017) [18] proposed a new ranking based fuzzy approach for solving fuzzy transportation problem. M. Premkumar and M. Kokila (2017) [17] developed a method for solving fuzzy transportation problem of symmetric trapezoidal with alpha cut and ranking technique. M. Ramesh kumar and S. Subramanian (2018) [11] developed a method for solving a special type of fuzzy transportation problems by using ranking function with triangular fuzzy numbers. K. Balasubramanian and S. Subramanian (2018) [3] proposed a new approach for solving fuzzy transportation problem. P. Uma Maheswari and K Ganesan (2018) [13] used a simple approach for solving fuzzy transportation problem under fuzzy environment in which all parameters (transportation costs, supplies, demands) are represented as pentagonal fuzzy numbers. D. Stephen Dinagar and B. Christopar Raj (2019) [5] developed a method for solving fully fuzzy transportation problem with generalized quadrilateral fuzzy numbers. Usually, many researchers are transformed fuzzy transportation model into crisp problem and then solved by existence methods but we find out fuzzy solution of transportation model in which the total transportation cost is a minimum with fuzzy number that assumes that the direct route, between a source, and a destination is a minimum cost. The effectiveness of this approach is illustrated by numerical example.

Keywords: Excel Solver Software, Fully Fuzzy Number, Linear Optimization, Fully Fuzzy Transportation Problem, Membership Function

1. Introduction

In the recent scenario of the competitive market, the fully fuzzy transportation problem is a most important special type of problem in optimization theory. Transportation is the movement of people, animals and various goods from one place to another and center of management science and operations research that maintains the economic and social activities. The fuzzy transportation problem is one of the earliest applications of linear optimization that has wide applications in logistics and supply chain for reducing the transportation cost. Different type of efficient approaches has been developed for solving fuzzy transportation problem, permit only those shipments that go directly from a fuzzy supply point to a fuzzy demand point. The most of the existing approaches generated only crisp model for solving the fuzzy transportation problem. During last decade, several researchers have carried out different type of investigations and approaches on fuzzy transportation problems.

2. Literature Survey

3. Problem definition

There are various types of fuzzy numbers in fuzzy set theory defined over the field $\mathbb{R}$ of real numbers and the $n$-dimensional space $\mathbb{R}^n$. This paper sets up the following concepts of fuzzy numbers.

Definition 3.1 [Fuzzy Number]
A fuzzy number is a special case of a convex, normalized fuzzy set of the real line and refers to a connected set of possible values. Each possible value of fuzzy number has its own weight between 0 and 1 and it has the characteristics (a)

A number $\bar{A}$ is normal, convex, upper semi-continuous and $\text{sup}(\bar{A})$ is bounded in $\mathbb{R}$.

Definition 3.2 [Triangular Fuzzy Number]
A fuzzy number $\bar{A} = (a_1, a_2, a_3)$ said to be triangular fuzzy number if its membership function $\mu_{\bar{A}} : \mathbb{R} \rightarrow [0, 1]$ defined as follows:

$$\mu_{\bar{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x < a_2 \\ 1, & x = a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

Definition 3.3 [Trapezoidal Fuzzy Number] A fuzzy number $\bar{A} = (a_1, a_2, a_3, a_4)$ is said to be a trapezoidal fuzzy number if its membership function $\mu_{\bar{A}} : \mathbb{R} \rightarrow [0, 1]$ defined as follows:

$$\mu_{\bar{A}}(x) = \begin{cases} 0, & x \leq a_1 \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \frac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4 \\ 0, & x \geq a_4 \end{cases}$$

Definition 3.4 [Fuzzy Initial Basic Feasible Solution of Fully Fuzzy Transportation Problem]
A set of non-negative values of the decision fuzzy variables ($\tilde{x}_{ij} \geq 0$) that satisfies the total fuzzy supply and total fuzzy demand of fully fuzzy transportation problem is said to be fuzzy initial basic feasible solution.

Definition 3.5 [Fuzzy Optimal Solution of Fully Fuzzy Transportation Problem]
Fuzzy initial basic feasible solution is said to be optimal solution of fully fuzzy transportation problem if it optimizes the fully fuzzy transportation costs.

Definition 3.6 [Balanced Fully Fuzzy Transportation Problem]
A fully fuzzy transportation problem is said to be balanced, if total supply equal to total demand i.e., $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$.

Otherwise fully fuzzy transportation problem is said to be unbalanced. In case of unbalance there are two cases arise such either $\sum_{i=1}^{m} \tilde{a}_i < \sum_{j=1}^{n} \tilde{b}_j$ or $\sum_{i=1}^{m} \tilde{a}_i > \sum_{j=1}^{n} \tilde{b}_j$.

Case I if demand exceeds supply, the constraints of the transportation table will appear as $\sum_{i=1}^{n} \tilde{x}_{ij} < \sum_{j=1}^{m} \tilde{b}_j$, and then add a fully fuzzy dummy source to the problem with fuzzy capacity $\sum_{j=1}^{m} \tilde{b}_j - \sum_{i=1}^{n} \tilde{a}_i$ in order to make it a balance problem we follow as following process

$$\sum_{j=1}^{m} \tilde{x}_{ij} = \tilde{a}_i; \quad i = 1, 2, ..., m$$

and

$$\sum_{i=1}^{n} \tilde{x}_{ij} \leq \tilde{b}_j; \quad j = 1, 2, ..., n$$

$$\Rightarrow \sum_{j=1}^{m} \tilde{x}_{ij} = \tilde{a}_i; \quad i = 1, 2, ..., m$$

and

$$\sum_{i=1}^{n} [\tilde{x}_{ij} + \tilde{s}_{m+1j}] = \tilde{b}_j; \quad j = 1, 2, ..., n$$

$$\Rightarrow \sum_{i=1}^{n} \tilde{s}_{m+1j} = \sum_{j=1}^{m} \tilde{b}_j - \sum_{i=1}^{n} \tilde{a}_i = \text{excess fuzzy demand.}$$

If $\tilde{s}_{m+1}$ denotes the excess fuzzy demand, then the modified fuzzy transportation problem can be represented by...
Minimize \[ Z = \sum_{i=1}^{m} \sum_{j=1}^{m} (\tilde{c}_{ij} \tilde{x}_{ij} + \tilde{c}_{m+1,j} \tilde{s}_{m+1,j}) \]

subject to \[ \sum_{j=1}^{m} \tilde{x}_{ij} = \tilde{a}_i \quad ; \quad i = 1, 2, ..., m + 1 \]

\[ \sum_{i=1}^{m} \tilde{x}_{ij} + \tilde{s}_{m+1,j} = \tilde{b}_j \quad ; \quad j = 1, 2, ..., n \]

and where \( \tilde{c}_{m+1,j} = 0 \) for all \( j \) and

\[ \sum_{i=1}^{m} \tilde{a}_i + \tilde{a}_{m+1} = \sum_{j=1}^{n} \tilde{b}_j \quad \text{or} \quad \tilde{a}_{m+1} = \sum_{j=1}^{n} \tilde{b}_j - \sum_{i=1}^{m} \tilde{a}_i \]

It follows that if \( \sum_{j=1}^{n} \tilde{b}_j > \sum_{i=1}^{m} \tilde{a}_i \), then a dummy row can be added to the transportation table.

In case II, if excess supply available, then the constraints of the transportation table will appear as \( \sum_{i=1}^{m} \tilde{a}_i > \sum_{j=1}^{n} \tilde{b}_j \), and add fully fuzzy dummy destination to the problem with fuzzy demand \( \sum_{i=1}^{m} \tilde{a}_i - \sum_{j=1}^{n} \tilde{b}_j \) in order to make it a balance problem we follow as following process

\[ \sum_{j=1}^{m} \tilde{x}_{ij} \leq \tilde{a}_i \quad i = 1, 2, ... m \]

\[ \text{and} \quad \sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j \quad j = 1, 2, ... n \]

\[ \sum_{j=1}^{n} \tilde{x}_{ij} + \tilde{s}_{i,n+1} = \tilde{a}_i \]

\[ \sum_{i=1}^{m} \tilde{x}_{ij} + \tilde{s}_{i,n+1} = \tilde{a}_i \quad ; \quad j = 1, 2, ..., n + 1 \]

\( \sum_{i=1}^{m} \tilde{s}_{i,n+1} = \sum_{i=1}^{m} \tilde{a}_i - \sum_{j=1}^{n} \tilde{b}_j = \) excess supply available

If \( \tilde{b}_{n+1} \) denotes the excess supply available, then the modified transportation problem can be presented as follows

\[ \text{Min} Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \left( \tilde{c}_{ij} \tilde{x}_{ij} + \tilde{c}_{i,n+1} \tilde{s}_{i,n+1} \right) \]

subject to

\[ \sum_{j=1}^{n} \tilde{x}_{ij} + \tilde{s}_{i,n+1} = \tilde{a}_i \quad ; \quad i = 1, 2, ..., m \]

\[ \sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j \quad ; \quad j = 1, 2, ..., n + 1 \]

where \( \tilde{c}_{i,n+1} = 0 \ (i = 1, 2, ..., m, j) \) and

\[ \sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j + \tilde{b}_{n+1} \quad \text{or} \quad \tilde{b}_{n+1} = \sum_{i=1}^{m} \tilde{a}_i - \sum_{j=1}^{n} \tilde{b}_j \]

It follows that, if \( \sum_{i=1}^{m} \tilde{a}_i > \sum_{j=1}^{n} \tilde{b}_j \), then a dummy column can be added to the transportation table.

4. Methodology

The fully fuzzy transportation problems deal with the transportation of a single product from several sources to several sinks. Let us consider \( S_1, S_2, S_3, ..., S_m \) sources with \( \tilde{a}_i \) fuzzy supplies, to be allocated among \( n \) destinations \( D_1, D_2, D_3, ..., D_n \) with \( \tilde{b}_j \) fuzzy demand. The problem is to determine the transportation schedule to minimize the total transportation cost and satisfy the fuzzy supply and fuzzy demand conditions. The fully fuzzy transportation problem can be defined as follows:

\[ \text{Min} Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij} \]

Subject to

\[ \sum_{j=1}^{n} \tilde{x}_{ij} \leq \tilde{a}_i, \quad \sum_{i=1}^{m} \tilde{x}_{ij} \geq \tilde{b}_j, \quad \sum_{j=1}^{n} \tilde{b}_j = \sum_{i=1}^{m} \tilde{a}_i, \]

\( \tilde{x}_{ij} \geq 0, \ i = 1, 2, ..., m; \ j = 1, 2, ..., n \)

Where \( \tilde{c}_{ij} \) is the fully fuzzy transportation cost matrix from \( i \)th source to the \( j \)th destination and fuzzy transportation supply \( \tilde{a}_i \) and demand \( \tilde{b}_j \) quantities are fully fuzzy number. The corresponding fuzzy levels of supply \( \tilde{a}_i \) and demand \( \tilde{b}_j \) are equal. In this problem the transportation cost, supply and demand may be considered different type of fully fuzzy numbers as triangular fuzzy numbers, trapezoidal fuzzy numbers, and pentagonal fuzzy numbers etc. A general fuzzy transportation problem in transportation cost matrix form is presented by

<table>
<thead>
<tr>
<th>Sources</th>
<th>( D_1 )</th>
<th>( D_2 )</th>
<th>( D_3 )</th>
<th>( D_n )</th>
<th>Fuzzy Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O_1 )</td>
<td>( \tilde{c}_{11} )</td>
<td>( \tilde{c}_{12} )</td>
<td>( \tilde{c}_{1m} )</td>
<td>( \tilde{a}_1 )</td>
<td></td>
</tr>
<tr>
<td>( O_2 )</td>
<td>( \tilde{c}_{21} )</td>
<td>( \tilde{c}_{22} )</td>
<td>( \tilde{c}_{2m} )</td>
<td>( \tilde{a}_2 )</td>
<td></td>
</tr>
<tr>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td></td>
</tr>
<tr>
<td>( O_m )</td>
<td>( \tilde{c}_{m1} )</td>
<td>( \tilde{c}_{m2} )</td>
<td>( \tilde{c}_{mn} )</td>
<td>( \tilde{a}_m )</td>
<td></td>
</tr>
</tbody>
</table>

| Fuzzy Demand | \( \tilde{b}_1 \) | \( \tilde{b}_2 \) | \( \cdots \) | \( \tilde{b}_n \) | \( \sum_{j=1}^{n} \tilde{b}_j = \sum_{i=1}^{m} \tilde{a}_i \) |

In this section, we applied following steps to proposed fuzzy level approach for initial fuzzy basic feasible solution of
fully fuzzy transportation problem. All parameters of transportation cost matrix; source and destination are taken as fully fuzzy numbers:

Step 1: First of all, find following mathematical formulation of fully fuzzy transportation problem

$$\text{Min} \ Z = \sum_{j=1}^{m} \sum_{i=1}^{n} \tilde{C}_{ij} \tilde{x}_{ij}$$

Subject to $\sum_{j=1}^{m} \tilde{x}_{ij} \leq \tilde{a}_{i}$, $\sum_{i=1}^{n} \tilde{x}_{ij} = \tilde{b}_{j}$, $\sum_{j=1}^{m} \tilde{b}_{j} = \sum_{i=1}^{n} \tilde{a}_{i}$,

$$\tilde{x}_{ij} \geq 0, \ i = 1,2,..m; \ j = 1,2,..n$$

$$\sum_{j=1}^{m} (\tilde{x}_{ij})^{L1} \leq (\tilde{a}_{i})^{L1}, \sum_{j=1}^{m} (\tilde{x}_{ij})^{L2} \leq (\tilde{a}_{i})^{L2} \ldots \ldots \ldots \ldots \sum_{j=1}^{m} (\tilde{x}_{ij})^{Lk} \leq (\tilde{a}_{i})^{Lk},$$

$$\sum_{i=1}^{n} (\tilde{x}_{ij})^{L1} \geq (\tilde{b}_{j})^{L1}, \sum_{i=1}^{n} (\tilde{x}_{ij})^{L2} \geq (\tilde{b}_{j})^{L2} \ldots \ldots \ldots \ldots \sum_{i=1}^{n} (\tilde{x}_{ij})^{Lk} \geq (\tilde{b}_{j})^{Lk},$$

$$\sum_{j=1}^{m} (\tilde{b}_{j})^{L1} = \sum_{i=1}^{n} (\tilde{a}_{i})^{L1}, \sum_{j=1}^{m} (\tilde{b}_{j})^{L2} = \sum_{i=1}^{n} (\tilde{a}_{i})^{L2} \ldots \ldots \sum_{j=1}^{m} (\tilde{b}_{j})^{Lk} = \sum_{i=1}^{n} (\tilde{a}_{i})^{Lk}, \tilde{x}_{ij} \geq 0$$

where $\tilde{x}_{ij} = ((\tilde{x}_{ij})^{L1}, (\tilde{x}_{ij})^{L2}, \ldots (\tilde{x}_{ij})^{Lk}) \forall \tilde{C}_{ij} = ((\tilde{C}_{ij})^{L1}, (\tilde{C}_{ij})^{L2}, \ldots (\tilde{C}_{ij})^{Lk})$, are fuzzy numbers

Step 2: Separate all corresponding fuzzy levels in objective function and constraints of the fully fuzzy transportation problem:

$$\text{Min} \ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \{ (\tilde{C}_{ij})^{L1}(\tilde{C}_{ij}), (\tilde{C}_{ij})^{L2}(\tilde{C}_{ij}), (\tilde{C}_{ij})^{L3}(\tilde{C}_{ij}) \} $$

Step 3: Apply following fuzzy level approach in the fully fuzzy transportation problem

$$\text{Min} \ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij}^{L1} \tilde{x}_{ij} + \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij}^{L2} \tilde{x}_{ij} + \ldots + \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij}^{Lk} \tilde{x}_{ij}$$

Subject to $\sum_{j=1}^{m} (\tilde{x}_{ij})^{L1} \leq (\tilde{a}_{i})^{L1}, \sum_{j=1}^{m} (\tilde{x}_{ij})^{L2} \leq (\tilde{a}_{i})^{L2} \ldots \ldots \ldots \ldots \sum_{j=1}^{m} (\tilde{x}_{ij})^{Lk} \leq (\tilde{a}_{i})^{Lk},$

$$\sum_{i=1}^{n} (\tilde{x}_{ij})^{L1} \geq (\tilde{b}_{j})^{L1}, \sum_{i=1}^{n} (\tilde{x}_{ij})^{L2} \geq (\tilde{b}_{j})^{L2} \ldots \ldots \ldots \ldots \sum_{i=1}^{n} (\tilde{x}_{ij})^{Lk} \geq (\tilde{b}_{j})^{Lk},$$

$$\sum_{j=1}^{m} (\tilde{b}_{j})^{L1} = \sum_{i=1}^{n} (\tilde{a}_{i})^{L1}, \sum_{j=1}^{m} (\tilde{b}_{j})^{L2} = \sum_{i=1}^{n} (\tilde{a}_{i})^{L2} \ldots \ldots \sum_{j=1}^{m} (\tilde{b}_{j})^{Lk} = \sum_{i=1}^{n} (\tilde{a}_{i})^{Lk},$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij}^{L1} \tilde{x}_{ij} \geq \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij}^{L1} (\tilde{x}_{ij})^{L1}, \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij}^{L2} \tilde{x}_{ij} \geq \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{C}_{ij}^{L2} (\tilde{x}_{ij})^{L2}, \tilde{x}_{ij} \geq 0$$

where $\tilde{x}_{ij} = ((\tilde{x}_{ij})^{L1}, (\tilde{x}_{ij})^{L2}, \ldots (\tilde{x}_{ij})^{Lk}) \forall \tilde{C}_{ij} = ((\tilde{C}_{ij})^{L1}, (\tilde{C}_{ij})^{L2}, \ldots (\tilde{C}_{ij})^{Lk}), are fuzzy numbers

Step 4: Solve this problem by using any software until all fuzzy demand and fuzzy supply are satisfied.

5. Results & Discussion

A company has two fuzzy sources $O_1$ & $O_2$ and the destinations $D_1, D_2 & D_3$. The fuzzy transportation costs for unit quantity of the product from $i$th source to $j$th destination is summarizing in the following matrix and fuzzy availability of the product at source are $(75, 95, 125)$, $(45, 65, 95)$and the fuzzy demand of the product at destinations are $(35, 45, 65)$, $(25, 35, 45)$, $(60, 80, 110)$ respectively.

Hence, the fully fuzzy transportation problem summarizes as follows:

<table>
<thead>
<tr>
<th>Table 2: Fully fuzzy transportation problem</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sources</strong></td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>$O_1$</td>
</tr>
<tr>
<td>$O_2$</td>
</tr>
<tr>
<td><strong>Fuzzy Demand</strong></td>
</tr>
</tbody>
</table>

Step 1: First of all, find following mathematical formulation of fully fuzzy transportation problem
Min \( Z = (15, 25, 35)x_{11} + (55, 65, 85)x_{12} + (85, 95, 105)x_{13} + (65, 75, 85)x_{21} + (80, 90, 110)x_{22} + (30, 40, 50)x_{23} \)

subject to \((x_{11} + x_{12} + x_{13}) \leq (75, 95, 125), (x_{21} + x_{22}) \leq (45, 65, 95), (x_{11} + x_{12}) \geq (35, 45, 65), (x_{12} + x_{22}) \geq (25, 35, 45), (x_{13} + x_{22}) \geq (60, 80, 110), x_i \geq 0 \)

Step 2: Separate all corresponding fuzzy levels in objective function and constraints of the fully fuzzy transportation problem:

\[
\text{Min}\ \mathbf{Z} = [(15, 25, 35)x_{11} + (55, 65, 85)x_{12} + (85, 95, 105)x_{13} + (65, 75, 85)x_{21} + (80, 90, 110)x_{22} + (30, 40, 50)x_{23}]
\]

Subject to \((x_{11})^1 + (x_{12})^1 + (x_{13})^1 \leq 75; (x_{11})^2 + (x_{12})^2 + (x_{13})^2 \leq 95; (x_{11})^3 + (x_{12})^3 + (x_{13})^3 \leq 125; (x_{21})^1 + (x_{22})^1 + (x_{23})^1 \leq 65; (x_{21})^2 + (x_{22})^2 + (x_{23})^2 \leq 65; (x_{21})^3 + (x_{22})^3 + (x_{23})^3 \leq 65; (x_{11})^1 + (x_{12})^1 + (x_{13})^1 \geq 25; (x_{11})^2 + (x_{12})^2 + (x_{13})^2 \geq 25; (x_{11})^3 + (x_{12})^3 + (x_{13})^3 \geq 25; \forall \ x_i \geq 0 \)

Step 3: Apply fuzzy level approach in the fully fuzzy transportation problem

\[
\text{Min}\ \mathbf{Z} = [(15, 25, 35)x_{11} + (55, 65, 85)x_{12} + (85, 95, 105)x_{13} + (65, 75, 85)x_{21} + (80, 90, 110)x_{22} + (30, 40, 50)x_{23} + 25(x_{11})^2 + 65(x_{12})^2 + 95(x_{13})^2 + 75(x_{21})^2] \]

Subject to \((x_{11})^1 + (x_{12})^1 + (x_{13})^1 \leq 75; (x_{11})^2 + (x_{12})^2 + (x_{13})^2 \leq 95; (x_{11})^3 + (x_{12})^3 + (x_{13})^3 \leq 125; (x_{21})^1 + (x_{22})^1 + (x_{23})^1 \leq 65; (x_{21})^2 + (x_{22})^2 + (x_{23})^2 \leq 65; (x_{21})^3 + (x_{22})^3 + (x_{23})^3 \leq 65; (x_{11})^1 + (x_{12})^1 + (x_{13})^1 \geq 25; (x_{11})^2 + (x_{12})^2 + (x_{13})^2 \geq 25; (x_{11})^3 + (x_{12})^3 + (x_{13})^3 \geq 25; \forall \ x_i \geq 0 \)

Step 4: Solve this problem by using any software and find out the following solution

<table>
<thead>
<tr>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>O1</td>
<td>(35, 45, 65)</td>
<td>(25, 35, 45)</td>
<td>(15, 15, 15)</td>
</tr>
<tr>
<td>O2</td>
<td>(45, 65, 95)</td>
<td>(45, 65, 95)</td>
<td>(45, 65, 95)</td>
</tr>
</tbody>
</table>

Fuzzy optimal solution by proposed approach:

\[
\begin{align*}
\bar{x}_{11} &= ((x_{11})^1, (x_{11})^2, (x_{11})^3) = (35, 45, 65) \\
\bar{x}_{12} &= ((x_{12})^1, (x_{12})^2, (x_{12})^3) = (25, 35, 45) \\
\bar{x}_{13} &= ((x_{13})^1, (x_{13})^2, (x_{13})^3) = (15, 15, 15) \\
\bar{x}_{23} &= ((x_{23})^1, (x_{23})^2, (x_{23})^3) = (45, 65, 95) \\
\bar{x}_{12} &= (0, 0, 0)
\end{align*}
\]

The total fuzzy transportation cost is (4525, 7425, 12425)

6. Conclusion

In this paper, the proposed fuzzy level approach for fully fuzzy transportation is investigated using computational procedure which is very simple and easy to understand, obtained fuzzy optimal solution by proposed approach is always optimal, and need not be checked optimality criteria. We therefore, hope that this approach may be used as an effective tool for direct extension of existing methods to apply on real life of this type of problems for the decision makers.

7. Future scope

This approach can be used for fuzzy assignment problems and fuzzy based non-linear optimization techniques as future research studies.

References


