Solving Fuzzy Non Linear Programming Problems using Ranking Function

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Abstract: ***In this study, triangular fuzzy numbers are used as the decision parameters. To solve these fuzzy nonlinear programming problems, we first convert all of the triangular fuzzy numbers into crisp values using the robust ranking method. Next, we obtain a crisp nonlinear programming problem. Finally, we apply the Kuhn-Tucker conditions to obtain the best solution.***

Keywords: Fuzzy Non Linear programming problem, Robust Ranking Method, Triangular Fuzzy Number

1. Introduction

For many years, traditional optimization techniques have been used successfully. The decision of the decision maker's is scientifically based on the availability of information from the mathematical modelling environment. Because of the ambiguities and inexactness of the decision parameters, we have a lot of trouble formulating and addressing real-world problems. These inaccuracies and uncertainties are caused a result of error of estimation, lack of information, imperfect representation of information, subjectivity and human opinion, etc. Thus, the problems of real world optimization cannot be formulated and successfully solved by conventional mathematical techniques. Thus, fuzzy optimization techniques offer a useful and powerful tool for optimization under fuzzy environment.

In 1965, Zadeh introduced Fuzzy set [19]. It plays a crucial role in solving the real world problems. Thereafter, Bellman and Zadeh have applied the decision making concept in fuzzy nature [3]. In 1991, Fuzzy set theory and its applications was discussed by Zimmermann [20].

Rangarajan et al [13] computed Hungarian assignment problems with fuzzy numbers by applying robust ranking techniques to change the fuzzy assignment problem to crisp [6].Arun Pratap Singh [2] presented Allocation of subjects in an educational institution by Robust ranking method [5]. Monalisha Pattnaik [9] applied Robust ranking for two phase fuzzy Optimization.

P. Umamaheswari et al [17] presented fuzzy problem converted into parametric form. Then applying by Kuhn Tucker conditions, the optimal solution of the problem is obtained.

Darunee Hunwisai et al [4],M. S. Annie Christi [1] applied Robust ranking for fuzzy transportation problems [18].Nagoorgani et al [11] discussed an optimality for non-linear minimization fuzzy problem.Sudha et al [15] suggested a new approach for fuzzy neutrosophic quadratic problem.Kokila et al [7] used Robust ranking method for solving Fuzzy Octagonal number .S. U. Malini et al [8] applied Modi method Fuzzy Octagonal number .

In this paper, we solve fuzzy non linear programming problems in which all of the problem's decision parameters are triangular fuzzy numbers. First, we use the robust ranking approach to turn all of the triangular fuzzy numbers into crisp values, and then we get a crisp non linear programming problem, which we solve using the Kuhn Tucker conditions.

The remaining section of this paper as follows;section two introduces the concept of fuzzy sets and related preliminaries and three presents the General form of Fuzzy Non Linear Programming Problems and Kuhn Tucker conditions.Section four investigates a numerical example. Section five, some conclusions are pointed out in the end of this paper based on our discussion.

2. Preliminaries

Definition : 2.1
If X is an universal set and x∈X, then a fuzzy set $\tilde{A}$ defined as a collection of ordered pairs ,
$\tilde{A}$ = {(x, $\mu_{\tilde{A}}$(x)), x ϵ X }
Where $\mu_{\tilde{A}}$(x) is called the membership function that maps X to the membership space M .

Definition : 2.2
A fuzzy number $\tilde{A}$ on R is said to be a triangular fuzzy number if its membership function $\tilde{A}$: R → [0, 1] has the following characteristics:

$$\tilde{A}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, a_2 < x \le a_3 \\ 0, otherwise \end{cases}$$

We denote this triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$.

We use F(R) to denote the set of all triangular fuzzy numbers.

Definition: 2.3
A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is said to be non-negative iff $a_1 \ge 0$

Definition: 2.4
A fuzzy set $\tilde{A}$ defined on X is called a normal fuzzy set if there exist at least one x ∈ X such that $\mu_{\tilde{A}}(x) = 1$

Definition: 2.5
If ã is a fuzzy number, the Robust ranking index is defined by

$$R(\tilde{a}) = 0.5 \int_0^1 (a_\alpha^L, a_\alpha^U)\, d\alpha$$

Where (a_α^L, a_α^U) = {(b-a)α + a, d-(d-c)α } is the α- cut of the fuzzy number $\tilde{a}$.□

Definition : 2.6
Given a fuzzy set '$\tilde{A}$' defined on 'X' and any α ∈ [0,1], the α –cut is denoted by $\tilde{A}$ (α) and is defined as, $\tilde{A}$(α)={x: μ(x)≥α , α ∈ [0,1]}

3. Fuzzy Non Linear Programming Problems:

The fuzzy nonlinear programming problem (FNPP) defined as
Maximize (or) Minimize $\tilde{f}(X) = \tilde{f}(x_1, x_2, \ldots, x_n)$
Subject to $\tilde{g_j}(\tilde{x_j}) \leq$ or ≈ or $\geq \tilde{b_i}$, i = 1,2,3,…,m
& $\tilde{x_j} \geq \tilde{0}$, for j = 1,2,3,…,n

The fuzzy version Kuhn-Tucker conditions given below,

Kuhn - Tucker condition 1: $\nabla f(x) - \sum_{j=1}^{n} \mu_j \nabla g_j(x) = 0$ ----(3.1)
Kuhn - Tucker condition 2: $\mu_j \nabla g_j(x) = 0, \forall\, j = 1,2,3,\ldots,n$ -------------------------- (3.2)
Kuhn - Tucker condition 3: $g_j(x) - b_i \leq 0,\ j = 1,2,3,\ldots,n$ --------------------------- (3.3)

4. Numerical Examples

Consider a fuzzy nonlinear programming problem by [17]

Max $\widetilde{W}$ = (1,3,4) $\tilde{c}_1^2$ + (1,2,3) $\tilde{c}_2^2$

Subject to
(0,1,3) $\tilde{c}_1$ + (2,3,5) $\tilde{c}_2$ ≤ (3,4,6)
(1,2,4) $\tilde{c}_1$ - (0,1,2) $\tilde{c}_2$ ≤ (1,2,5) & $\tilde{c}_1$, $\tilde{c}_2$ ≥ 0

Now convert the given fuzzy problem into crip form using the Robust

Ranking method
Max $\widetilde{W}$ =R (1,3,4) $\tilde{c}_1^2$ + R(1,2,3) $\tilde{c}_2^2$

Subject to
R(0,1,3) $\tilde{c}_1$ + R(2,3,5) $\tilde{c}_2$ ≤ R(3,4,6)
R(1,2,4) $\tilde{c}_1$ - R(0,1,2) $\tilde{c}_2$ ≤ R (1,2,5) &
$R(\tilde{a}) = 0.5 \int_0^1 (a_\alpha^L, a_\alpha^U)\, d\alpha$

Where (a_α^L, a_α^U) = {(b-a)α + a, d-(d-c)α }

Therefore
$R(1,3,4) = 0.5 \int_0^1 \{(3-1)\alpha + 1 + 4 - (4-3)\alpha\}\, d\alpha$

$= 0.5 \int_0^1 2\alpha + 1 + 4 - \alpha\}\, d\alpha = 0.5 \int_0^1 (5 + \alpha)\, d\alpha = 2.75$

Similarly we get
R(1,2,3) = 2 , R(0,1,3) = 1.25 , R(2,3,5) = 3.25 , R(3,4,6) = 4.25
R(1,2,4) = 2.25 , R(0,1,2) = 1 , R(1,2,5) = 2.5
We obtain crisp non linear programming problem is

Max $\widetilde{W}$ = 2.75 $\tilde{c}_1^2$ + 2 $\tilde{c}_2^2$
Subject to
1.25 $\tilde{c}_1$ + 3.25 $\tilde{c}_2$ ≤ 4.25
2.25 $\tilde{c}_1$ - $\tilde{c}_2$ ≤ 2.5 & $\tilde{c}_1$, $\tilde{c}_2$ ≥ 0

By applying KT condition (3.1) we get
→ 5.5 $\tilde{c}_1$ - 1.25 φ_1 - 2.25 φ_2 = 0
→ 4 $\tilde{c}_2$ - 3.25 φ_1 + φ_2 = 0

By applying KT Condition (3.2) we get
→ φ_1 [1.25 $\tilde{c}_1$ + 3.25 $\tilde{c}_2$ - 4.25] = 0
→ φ_2 [2.25 $\tilde{c}_1$ - 1.00 $\tilde{c}_2$ - 2.5]= 0

By applying Condition (3.3) we get
→ 1.25 $\tilde{c}_1$ + 3.25 $\tilde{c}_2$ - 4.25 ≤ 0
→ 2.25 $\tilde{c}_1$ - 1.00 $\tilde{c}_2$ - 2.5 ≤0
Solving the above equations we get the optimal solution is
$\tilde{c}_1$ = 0.3302 & $\tilde{c}_2$ = 1.1806

Max $\widetilde{W}$ = 3.0874
But [17] gives the optimum value is $(\frac{372}{49}, 0, 0)$
To compare the optimality , we convert that into the crisp value
Then we get ,
$R(\frac{372}{49}, 0, 0)$ = 1.8969
Therefore comparing the both the solution our solution gets optimum.

Example 2:
Consider the following fuzzy nonlinear programming problem in [17,14]

Max $\widetilde{W}$ = $\tilde{c}_1^2$ + $\tilde{c}_2^2$

Subject to
(0,1,2) $\tilde{c}_1$ + (1,2,3) $\tilde{c}_2$ ≤ (1,10,27)
(1,2,3) $\tilde{c}_1$ + (0,1,2) $\tilde{c}_2$ ≤ (2,11,28) & $\tilde{c}_1$, $\tilde{c}_2$ ≥ 0

The same way we proceed the problem get the optimum solution is
$\tilde{c}_1$ = 5.2 & $\tilde{c}_2$ = 2.6

Max $\widetilde{W}$ = 15.6

But [17] gives the optimum value is (25,0,0)

To compare the optimality , we convert that into the crisp value
Then we get ,
R(25,0,0) = 6.25
Similarly our solution gets optimum.

Example 3

Consider the problem Non linear programming problem with trapezoidal fuzzy number [8]

Max $\tilde{w}$ = (0.25, 2.25, 2, 2)$\tilde{c}_1$+ (1.25,3.25,2,2)$\tilde{c}_2$ − (0.25,2.25,2,2)$\tilde{c}_1{}^2$

Subject to

(0.75,1.25,0.5,0.5)$\tilde{c}_1$+(2.25,4.25,2,2)$\tilde{c}_2$ ≤ (2.25,4.25,2,2)

(0.75,1.25,0.5,0.5)$\tilde{c}_1$+ (0.75,1.25,0.5,0.5)$\tilde{c}_2$ ≤ (0.25,2.25,2,2)

Now convert the problem using RRM,we get

Max $\tilde{w} = 1.625\tilde{c}_1 + 2.125\,\tilde{c}_2 - 1.625\tilde{c}_1{}^2$

Subject to

$0.75\,\tilde{c}_1 + 2.625\,\tilde{c}_2 \leq 2.625$

$0.75\,\tilde{c}_1 + 0.75\,\tilde{c}_2 \leq 1.625$

By applying KT condition (3.1) we get

$1.625 - 3.25\tilde{c}_1 - \varphi_1 0.75 - \varphi_2 0.75 = 0$

$2.125 - \varphi_1\, 2.625 - \varphi_2\, 0.75 = 0$

By applying KT condition (3.2) we get

$\varphi_1\,[0.75\tilde{c}_1 + 2.625\,\tilde{c}_2 - 2.625] = 0$

$\varphi_2[0.75\,\tilde{c}_1 + 0.75\,\tilde{c}_2 - 1.625] = 0$

By applying KT condition (3.3) we get

$0.75\tilde{c}_1 + 2.625\,\tilde{c}_2 - 2.625 \leq 0$

$0.75\,\tilde{c}_1 + 0.75\,\tilde{c}_2 - 1.625 \leq 0$

Solving the above equations,we get the optimal solution is

$\tilde{c}_1 = 0.315$ &$\tilde{c}_2 = 0.910$

Max $\widetilde{W}$ = 2.285 but [8] got the solution is 1.46

3. Comparison

Example 1	Example 2	Example 3	
Max $\widetilde{W}$ = 3.0874	Max $\widetilde{W}$ = 15.6	Max $\widetilde{W}$ = 2.285	Proposed Method
Max $\widetilde{W}$=1.8969	Max $\widetilde{W}$=6.25	Max $\widetilde{W}$= 1.46	Previous Method

4. Conclusion

We used the proposed method in this research to achieve optimal solution to non-linear programming problems with linear constraints, and we confirmed the accuracy of the proposed method using numerical examples.

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