Economic Load and Emission Dispatch: Strategic Game Approach

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Abstract: ***The economic load and emission dispatch (ELED) of thermal power generation is a multi-objective mathematical optimization problem consisting of two objective functions. The first objective is to minimize of cost of fuel of generators of a power system, while another objective is to minimize emission cost. Both objectives functions are subjected to constraints such as power balance, transmission loss, generation limits constraints. In this paper, integrate strategic game theory based optimal response program into ELED problem. The game theory based response program evaluates optimal values of incentive for load and emission dispatch. The ELED and game theoretical algorithm presented through this work minimizes fuel cost and emission cost. It simultaneously determines optimal incentive values results for maximum relief on the power system. The developed program is tested on IEEE 30 bus system and obtained results are compared with Lagranges optimization algorithm (Iterative Method), indicates practical benefits of the proposed algorithm.***

Keywords: Economic Dispatch, Game Theory, Lagrange's Iterative Technique, Nash Equilibrium, Optimization Strategy.

Nomenclature

P_{gi}	Power generated in MW by an i^{th} generator.
P_{Di}	Power demand in MW by an i^{th} generator.
P_{Li}	Power losses in MW by an i^{th} generator.
n	No. of players in the game.
F_c, E_w	Fuel cost and emission cost respectively \$/Hr.
s_n	Strategies of n players.
$\overline{x}$	Optimizer of an algorithm.
c_n	Generator cost in \$/Hr.
$R_{>0}$	Set of non-negative real numbers.
p_n	Payoff for an n^{th} player.

1. Introduction

In electrical power systems, the generation largely depends on thermal power plants. The fossil fuel is more conveniently used which causes emission of a large amount of pollutantgas. This inspires researchers to work in minimizing this environmental effect. After all crude fuel remains a widely used an element for generating electricity. Therefore, the significant problems of using crude fuels in electricity generation systems are to find out optimal strategy to minimizethe fuel cost and emission of harmful gases as well [1]. Economic load dispatch (ELD) deals with minimization of fuelcost with consideration of optimal power generation of each generating unit of the generation system on the other hand emission dispatch problem includes minimization of emission of harmful gases and unsafe particles in an environment [2].

The desired motto of optimization problems like minimization of cost of generation, minimization of total power loss in the system. Determining the active power (Real) outputs of the generators such that the total cost of generation in the power system to be minimized is simply termed as a problem of economic load dispatch (ELD). Most of the generation systems are of three categories: hydro, nuclear, and thermal (using input fuel like coal, crude oil, natural gas). Nuclear plants generally operate at a level of constant power output. Operating costs of the hydropower plant do not fluctuate much with the output. The cost of operation of thermal plants, nevertheless, change notably with power output. Overall, for any specific load economic dispatch (ED) finds power output of each generating plant which can minimize the cost of fuel need to meet the total demand of system [3], [9].

An economic emission dispatch problem (EED) is put forward to look into emission control. The recent consumption of energy heavily depends on crude or fossil fuels which are count for 75-90% of total energy consumed. Thermal power generation units are accountable for large atmospheric pollution due to too high concentrations of pollutants like NO_x, CO, CO_2, and SO_2 are contaminated as discussed in [2]. Due to environmental constraints, allocation of generation is governed not only by the cost of generation but also allow- able maximum emission level. EED, follows the minimum emission operating level of an electrical power system, an optimization problem. From [4], EED is a clear approach to find out the optimal values of generated power output for thermal units of the system by minimizing generation cost and level of emission simultaneously.

The strategic approach from [5] is applied to the IEEE 30 bus system along with Nash equilibrium (NE) to analyze ELD and EED of active power generation of thermal plants, in this work. Nash equilibrium points for this strategy is put forth to find out optimal operating points of generators. The results from the game theoretic approach are compared with lagrange's iterative technique for interpreting the benefits of strategic response algorithm. Economic dispatch of the power system is a most common topic of study [6]. The research work like [7], [8] deals with the computation of NE by using the iterative technique.

In this paper, first contribution leads develop strategic Nash algorithm for economic load and emission dispatch. Particularly finding an optimum response of the system for a given problem statement. Section II describes formulation problem with respect to power system. Some concepts are illustrated in Section III for understanding pre-requisites game theory. Then discussed a framework for strategic game and proposean algorithm in Section IV. Later test this algorithm on IEEE 30 bus standard test system as a case study in Section V. Results from the strategic game approach is compared with Lagrange's iterative method. Based on results here reach to a conclusion in Section VI.

2. Mathematical Modeling of ED Problem

An economic dispatch problem generally mentions to fueling cost minimization and hazardous gases emission as well as particulates at the same time satisfying total load demand including equality and inequality constraints. This section describes the mathematical problem formulation of EED problem with consideration of equality constraints i.e. power balance constraints and inequality constraints i.e. generation limit constraints [1], [2], [6].

a) Objective function

The objective function of the ED is also called a cost function. The cost function includes the cost of fuel, initial as well as no load operating costs of power generation units.

ELD problem usually described as a quadratic function of cost and represented as,

$$F_c\ (Pgi) = \sum_{i=1}^{n} a_i P_{gi^2} + b_i P_{gi} + c_i \qquad (1)$$

where F_c is total fuel cost or generation cost in (Rs/h) and a_i, b_i, and c_i are fuel cost coefficients of the i^{th} generation unit. In addition to that P_{gi} stands for active power generation of i^{th} generation unit in MW and n refers the total number of generation units.

EED is also formulated the same as a quadratic function and represented as,

$$E_w\ (Pgi) = \sum_{i=1}^{n} d_i P_{gi^2} + e_i P_{gi} + f_i \qquad (2)$$

where E_w is emission measured in (kg/h) and d_i, e_i, and $f_i$are emission coefficients of the i^{th} generation unit. Both the objectives of ED problem are incommensurable and contradictory in nature.

b) Constraints

The objective functions equations are subjected to below network constraints. According to power balance constraints for each time interval in system network, total generated power equals to total load demand and total power losses usually referred to as equality constraints represented as,

$$\sum_{i=1}^{n} P_{gi} = P_{Di} + P_{Li} \qquad (3)$$

For stable power generating operation, the total power of the generation unit shouldn't violate the minimum and maximum limits which referred to as unit capacity or generation limit or inequality constraints represented as,

$$P_{gi,\,\min} \le P_{gi} \le P_{gi,\max} \qquad (4)$$

A practical operation of generation plants is restricted to operate at specific operating limits due to ramp rate. Hence they have to operate in certain operation zones. For each generating unit, the power output with respect to the ramprate restrictions represented as,

$$P_{git} - P_{gi\,(t-1)} \le UR_i$$

$$P_{gi\,(t-1)} - P_{git} \le LR_i \qquad (5)$$

Above discussed constraints are only about practical operationbut there are other constraints like demand response cost, load shedding cost etc. incurred can be included in further studies.

3. Game Theory for Multi-Player Transactions

Game theory is a subject which used to find out optimal option. In economics, it is used to analyze different strategies of players. Nowadays, the application of game theory to various branches of science is common practice. The fundamental understanding of game theory was to apply the logic involved in games to real life mechanisms.

In the domain of game theory, the action of each decision maker or player depends on other decision maker or player. Game theory utilizes the fundamentals of economics and mathematical tools to answer decision making actions. A game illustrates strategic dealing that consists of the constraints on actions that the players can perform and the player's interests but it does not describe the actions taken by players. A solution to this is a systematic interpretation of outcomes that can appear in a game theory family [16]. The game theory proposes logical solutions for each class of games and exploretheir properties.

One of the most fundamental concepts of game theory is Nash equilibrium and it is an extensive method of outcome prediction of strategies taken by players. At first, A. Cournot used game theory and Nash equilibrium. By the theory of Cournot, enterprises decide their production quantities to enhance their own profit which is interdependent on other players. According to J. Nash, for finite games, there always exists an equilibrium point. Stable outcome depending on the payoffs gest by players at the end of the game [3]. All participated players select strategies which are optimum to them with respect to other player's choices at an equilibrium point. According to Nash equilibrium, each player in the gameresponds to the next players with the best optimal decision.

Formation of games consists of basic three elements i.e. players, strategies, and payoffs. For example, consider a set of players *N*= 1, 2, 3,..., *n* which is a finite set and implicated by *k*. Let set of pure strategies to each player to determine their viable strategies be $s = (s_1, s_2, \ldots, s_n) \in A$ and payoff functions representing preferences of each player i.e. player receive after game ends are $p = (p_1, p_2, \ldots, p_n)$. The best optimal response in the game is given by,

$$S_k^* \in OR\,(S_{-k}) \tag{6}$$

$\forall\ s_k \in A_i,\ p_i(s_k, s_{-k}) \geq p_i(s_k, s_{-k})$. The existence of equilibrium is important as an equilibrium point can be mixed or pure strategy. When there exist pure equillibrium point that shows a maximum profit. On the other hand, when there exist mixed strategy equilibrium point players can choose enhanced profit with probability. In short, pure strategy Nash equilibriumpoint in which player unable to get higher payoff deviating its profile alone [9]

4. Strategic Game Approach for Economic Load and Emission Dispatch Problem

In [11] and other solution strategies for the distributed algorithm developed for economic dispatch in [12], [13] and [14], the generators find the optimizer of ED problem with cooperation. In cooperative game framework, generators are ready to share such as cost function, gradient, etc. Corresponding to other generators in network. Here strategic game framework, generator units focused to maximize their individual profit without sharing information to other participants even to System Operator (SO). Above two frameworks can accommodated in the architecture of future grid (Smart Grid) [1]. Recent research work [1], [20] and [15] provided different cases for strategic framework based economic dispatch but not given information about the equilibrium of the resultant game.

The aim of this problem to pursue power generated $x \in \mathbb{R}^N \geq 0$ to minimize the total cost induced by each generator to meet demand y.

minimize *x*;

$$\sum_{n=1}^{N} f_n(x_n) \tag{7}$$

subject to $\sum_{n=1}^{N} x_n = y_t$. Here note that $\bar{x}$ be an optimizer which is unique cost function and establish efficient Nash Equilibrium (NE) of game discussed in Section III. Here at first, find existence of efficient NE for the economic dispatch game. Let $\bar{x} = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) \in \mathbb{R}^N \geq 0$ be the solution of ED problem in cooperative game framework in (7).

In problem discussed in (7), all cost functions are known to every generator. In future grid, when generators are strategic in action the power generation has done by through optimization which in turns to game theoretic formulation of ED problem. Particularly for n generator, cost per unit power will be $C_n \in \mathbb{R} > 0$. Here it represent all costs except n th generator cost as $c_{-n} = (c_1, \ldots, c_{n-1}, c_{n+1}, \ldots, c_N)$. Represented all costs are $c = (c_1, \ldots, c_N) \in \mathbb{R}^N \geq 0$

For strategic ED framework problem,
minimize *x*;

$$\sum_{n=1}^{N} c_n(x_n) \tag{8}$$

subject to $\sum_{n=1}^{N} x_n = y$. The function discussed in (7) is linear as that of nonlinear and complex in (8). For problem (8) the generating unit with minimum cost will be able to provide y units of power. The aim of each generator is to generate units $c_n \geq 0$ which maximizes payoff function $u_n: \mathbb{R}^2 \geq 0 \rightarrow \mathbb{R}$

$$u_n(c_n, x_n \text{ or } (c_n, c_{-1})) = c_n\, x_n \text{ or } (c_n, c_{-1}) - f_n(x_n \text{ or } (c_n, c_{-1})) \tag{9}$$

Where $x_n \text{ or } (c_n, c_{-1})$is optimizer and for n generator $x_n \text{ or } (c_n, c_{-n})$. For sake of space, denote this as *x* or (c_n, c_{-1}) and *x* or (c) as optimizer of (8). Now define pure nash equilibruim for discussed ED problem. The pure nash equilibrium of ED problem is generation profile of the group $\bar{c} \in \mathbb{R}^N \geq 0$ to which there is optimizer *x* or $(\bar{c})$ of function (8) that subject to following, for each *n* generator $n \in \{1, \ldots, N\}$ and each cost $c_n \in \mathbb{R}_{\geq 0}$ for $c_n \neq c_n$ and every optimizer *x* OR $(c_n, \bar{c}_{-1})$ of (8) generation profile $(c_n, \bar{c}_{-1})$ will write,

$$u_n(c_n, x_n \text{ or } (c_n, \bar{c}_{-1})) \leq u_n(\bar{c}_n, x_n \text{ or } (\bar{c})) \tag{10}$$

Nash equilibrium for which the optimizer $\bar{x}$ of (7) is optimizer of (8) of the given cost c^* as well is an Efficient NE [10] and thus,

$$\bar{x}_n = argmax_{x \geq 0}\, \bar{c}_n x - f_n(x) \tag{11}$$

for all *n* generators. At the point of efficient NE, the production of generating units that they are able to provide to maximize their profit reflects optimal generation for ED problem. For the sake of space, the proofs are not given in detail but you can refer [16], [17], [18], [19]. The simplified algorithm for the strategic framework is as shown in Fig. 2. Later, use this algorithm for our illustrative case study in Section V.

5. Illustrative Example: IEEE 30 Bus System

a) System Information

In this Section, will study an example of standard IEEE 30 BUS test system in order to check the fitness of the proposed algorithm as well as to study strategies of all players involved. Fig. 1 shows a single line diagram of the IEEE 30 bus, 6 generators test system. Assume all the generators to be thermal. In the test system shown in Fig. 1, all 6 generating units acts as players, the produced amount of each generating units are strategies and payoffs are established at each point.

b) Simulation Results

In this Section, interpretation of the results concerning the proposed algorithm for IEEE 30 bus system is present. For the mathematical model, certain assumptions are made. The aim is to obtain optimal values for economic load and emission dispatch for IEEE 30 Bus test system.

The fuel and emission cost coefficients are taken in the account from a Table I and Table II respectively. As soon as the game begins, all the possibilities of power generation were determined by generator system parameters shown in Table I and Table II. After the determination of possibilities of power generation, minimum and maximum values of powerdelivered are to be considered and step size for power output linear as that of nonlinear and complex in (8). For problem (8) interval has chosen. Consider generator connected to bus 1

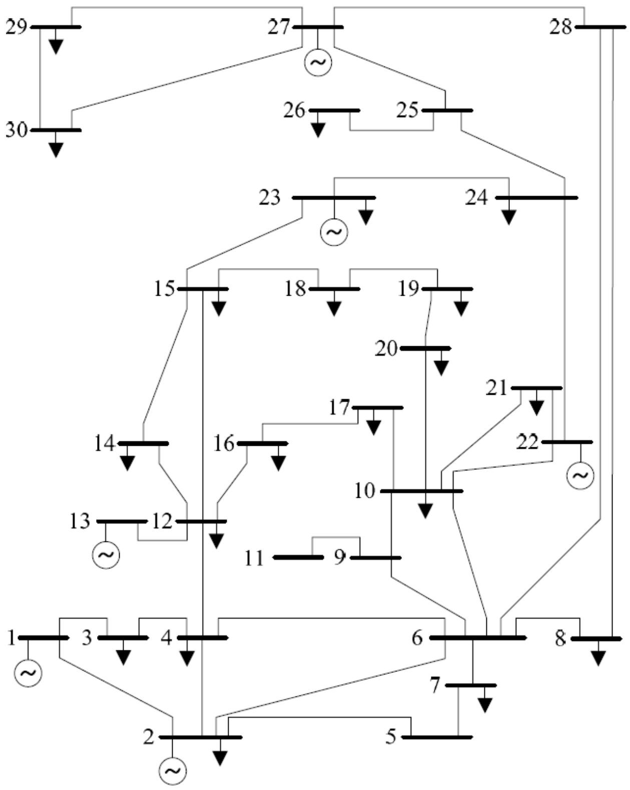

Figure 1: SLD of IEEE 30 bus, 6 generators test system [21].

as an example, possible power generation varies between 50 MW and 200 MW with a step size of 0.001MW. For satisfyingthe total power demand, the optimum value of the output of each generator is determined. These obtained optimum values are Nash equilibrium points of the considered test system. Thesystem optimum response point is determined by the minimumvalue of the resultant total cost. The generator output at Nash equilibrium points give the operational values which define the optimum strategy of generators.

Table I: Generator Cost Coefficients

Generator No	$P_{gi}Min$	$P_{gi}Max$	a_i	b_i	c_i
1	50	200	0.00375	2.00	0
2	20	80	0.01750	1.75	0
3	15	50	0.06250	1.00	0
4	10	35	0.00834	3.25	0
5	10	30	0.02500	3.00	0
6	12	40	0.02500	3.00	0

Table II: Generator Emission Coefficients

Generator No	$P_{gi}Min$	$P_{gi}Max$	α_i	β_i	γ_i
1	50	200	0.0126	-1.1000	22.983
2	20	80	0.0200	-0.1000	25.313
3	15	50	0.0270	-0.0100	25.505
4	10	35	0.0291	-0.0050	24.900
5	10	30	0.0290	-0.0040	24.700
6	12	40	0.0271	-0.0055	25.300

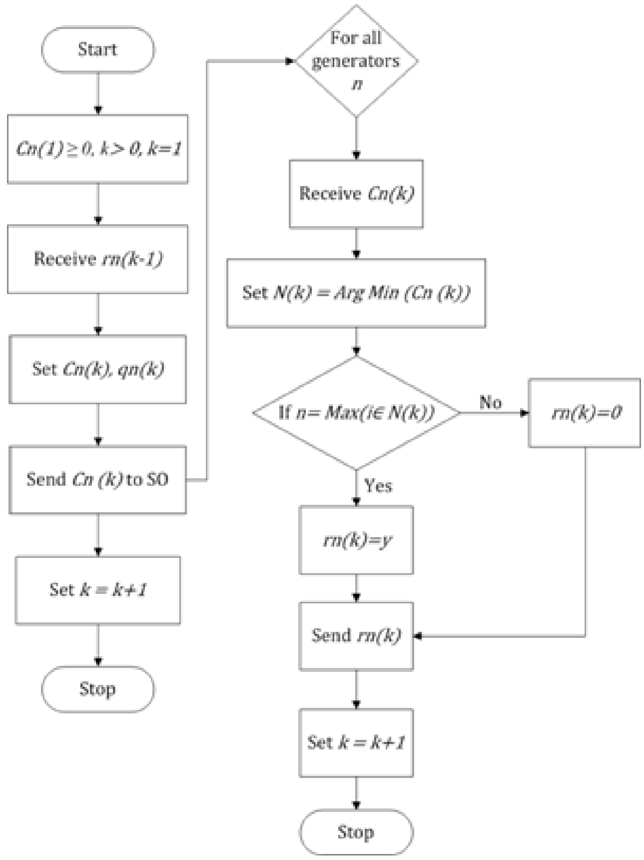

Figure 2: Proposed algorithm for the strategic game framework.

Table III: Generation units in MW for ELD with losses for a power demand of 300MW

Generator No	Lagrange Iterative Method	Game Theory Method
1	176.98	171.83
2	49.27	47.38
3	20.99	19.87
4	23.87	25.11
5	12.55	13.42
6	12	14.18

Table IV: Generation units in MW for EED with losses for a power demand of 300MW

Generator No	Lagrange Iterative Method	Game Theory Method
1	117.92	114.49
2	57.61	56.39
3	42.86	41.82
4	23.08	23.31
5	28.41	28.12
6	27.66	28.03

At first, calculated generator output P_gi units for considered test system with the Lagrange method. Later, calculated with the strategic game approach. In Table III, the values of P_gi are calculated by both methods for ELD and in Table IV, for

EED.

Table V: Result of ELD for IEEE 30 Bus Test System for a power demand of 300 MW

Parameter	Lagrange Iterative Method	Game Theory Method
Total Cost ($/hr)	802.63	789.21
Losses in MW	9.623	8.335

Table VI: Result of EED for IEEE 30 Bus Test System for a power demand of 300 MW

Parameter	Lagrange Iterative Method	Game Theory Method
Total emission (lb/hr)	11.930	11.701
Losses in MW	9.553	8.153

The losses, fuel cost and emission cost in $/hour as shown in Table V and VI. From all results, the best results are considered in below Table V and VI. In Table V and VI, the fuel cost and emission cost obtained for game theory method are more economical compared with Lagrange's method.

6. Conclusion

This paper shows a strategic game theory based optimal response for power IEEE 30 bus test system. The two main objectives in ED optimization problem are to minimize fuel cost and emission cost. Results obtained in this paper presents a cost-effective method for dispatch problem in power system network. Here assumed that the generators are aware of cost functions and to follow the optimal schedule by SO.

In this work, two methods, Lagrange's method, and strategic game theory method are performed are suitable for dispatch problem in ON-Line system condition in a power network. In a given case study of ELD and EED problems using the two techniques is determined with respect to constraints discussed with losses. The presented game theoretic algorithm, the strategic framework gives the best response of total cost compared with Lagrange's method.

On concluding remark, the results found in the case study of IEEE 30 bus test system represents a strategic game approach to an optimization problem like ED. For future work, the game-theoretic framework can be modified for combined economic load and emission dispatch (CELED). Also, the proposed algorithm can be analyzed for other optimization techniques and standard IEEE test systems.

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