

Evaluation of Time to Termination of Life Insurance Contracts with Benefits Calculations

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Abstract: *In life insurance industry, the available benefits are keys to convince people to purchase the life insurance products. However, in developing countries, the people do not purchase life insurance products because they are not attractive to them. The purpose of this paper was to evaluate time to termination of life insurance contracts with benefits calculations. The objectives were specifically to: evaluate time to stop paying premiums payments by estimating survival function using Kaplan Meier estimator, model the dependency of the survival time to the risk factors of age, gender and marital status using Cox proportional hazard regression model, calculate premiums and reserves for the life insurance contracts using equivalence principle and calculate surrender values and paid up sum assured when the policyholder stops paying premium payments. From the results, the survival probability of life insurance contract in the beginning of a contract is higher compare to survival probability of it in the last period of a contract which means that age influenced significantly the survival time. The whole life insurance contract is cheaper compare to endowment assurance and pure endowment contracts because its premiums, reserves and benefits are down compare to other types of life insurance contracts. Therefore, we recommended to use survival analysis tool because it helps to take decisions with strong facts especially for time to an event analysis and to compute adequately premiums and reserves for getting the available benefits to an event which are accurate. We have also shown the room for further studies.*

Keywords: Kaplan Meier estimator, Cox proportional hazard model, Whole life insurance, Endowment insurance, Pure endowment insurance, Surrender values, Paid-up sum assured

1. Introduction

A central challenge in selling life insurance is the reality of the event that triggers policy benefits death. Many Americans simply don't want to address the need to provide for family members after their own demise. This contributes to the findings of a 2011 survey of sales agents by the Agent's Sales Journal and the Life Foundation, where 56 percent of agents identified client procrastination as a big challenge to their business.

When you sell any type of insurance, you essentially ask people to pay money for benefits they may never use. With life insurance, especially term policies that expire, people pay monthly or annual premiums and they may never get the benefits. Even on whole life policies that function as a combined life insurance and investment plan, the rates of return are so low that it is difficult for salespeople to convince consumers that the policies are a wise investment.

Life insurance products constantly evolve, and many agents struggle with learning all facets of the policy. In the 2011 survey of sales agents by the Agent's Sales Journal and the LIFE Foundation, 2 percent of agents said they didn't understand their products well and 12 percent said the lack of customer awareness of life insurance was a hiccup. Additionally, a March 2010 article in the Wall Street Journal noted that contemporary life insurance sellers still suffer from the negative stigma resulting from the unscrupulous and misleading ways agents in the past used deceive customers to sell policies, especially whole life plans.

Souza, M.M. stated that the hindrances of life insurance for developing countries are localization and inflation problems. Localization problems are the lack of corporate structure,

management, other personnel and capitalization. The inflation hindrance is one of the major problems faced in developing countries specifically insurance area because in these countries, the prices increase abnormally. This work is an actuarial study for assessing time to termination of life insurance contracts using survival analysis and to compute the profits correctly so that both purchasers and insurers can be in the business of life insurance.

2. Research Methodology

2.1. Kaplan Meier Estimator

The Kaplan Meier estimator is also named as product estimator limit is statistic test of non-parametric used to approximate survival probabilities at time T. It was mentioned that the Kaplan Meier approximate is a step function continuously in right direction that has jumps at the event time only. It has 3 assumptions: At any duration, policyholders who are not completed the contract have the equal survival prospects as those who are still followed, the chances of survival are equal for policyholders recruited before and after in the study and the event occurs specifically on duration.

By considering that n symbolize the number of life insurance contracts stopped at duration t . It is generally either zero or one, but we allow the chance of fastened survival durations in which case may be larger than one. As N symbolize the number of policyholders at risk just at duration t ; i.e., number of policyholders in the sample whom their contracts either not ceased or uncompleted at duration t .

Estimator of Kaplan Meier is:

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$$KM(T) = \prod_{t \leq T} (1 - \frac{n}{N}) \tag{1}$$

In the expression above, it varies only at duration t where $n \geq 1$.

2.2. Cox proportional hazard model

The Cox proportional hazards regression model is the most known for survival analysis which is used to test the effect of independent variables to survival duration. In the model, the hazard rate is the measure of effect, which is the failure risk (i.e., the risk or chance of getting the event), given that the person has continued to live in a specific duration.

For the utilization of the Cox proportional hazards regression model, we possess two crucial assumptions such as: the survival durations between different people in the sample must be unique and the ratio of hazard over duration is the same.

Consider that Cox proportional hazards regression model is called a model of semi-parametric because of no assumption on the baseline hazard function shape.

Cox proportional hazards regression model is:

$$h(t) = h_0(t) \exp(\beta_1 x_1 + \dots + \beta_n x_n) \tag{2}$$

$$\frac{h(t)}{h_0(t)} = \exp(\beta_1 x_1 + \dots + \beta_n x_n) \tag{3}$$

$$h(t) = h_0(t) e^{\beta_1 x_1} e^{\beta_2 x_2} \dots e^{\beta_n x_n} \Leftrightarrow \frac{h(t)}{h_0(t)} = e^{\beta_1 x_1} e^{\beta_2 x_2} \dots e^{\beta_n x_n} \tag{4}$$

Where $\frac{h(t)}{h_0(t)}$ is the ratio of hazard, $h(t)$ is the expected hazard at time t and $h_0(t)$ is the hazard baseline.

By applying logarithm of natural both sides of equation (4), then We have:

$$\text{Log} \left(\frac{h(t)}{h_0(t)} \right) = \beta_1 x_1 + \beta_2 x_2 \dots + \beta_n x_n \tag{5}$$

As we see equation (5), it is a multiple linear regression model. The interest is practically in the relations of the independent variables (x_1, x_2, \dots, x_n) and the dependent variable or outcome (logarithm of natural of ratio of hazard). The relations are described by the coefficients of regression ($\beta_1, \beta_2, \dots, \beta_n$).

Cox showed partial likelihood as the way for approximating parameters in the model for evaluating the effect of predictors to outcome when there are no information about function of hazard baseline ($h_0(t)$).

By considering that not completing is uninformative and independent for β , the likelihood partial is then based on, for all durations of event, the status of policyholders not succeeding at each duration of event, number not succeeding and the policyholders at risk at that duration. The function is:

$$L(\beta) = \prod_{i \in T} \left(\frac{e^{\beta' x_i}}{\sum_{j \in A_i} e^{\beta' x_j}} \right) \tag{6}$$

Where T is the indices set observed time of event, x_i is the vector of covariate for the individual not succeeding at the i^{th} observed time of event, and A_i are the set of subjects at risk.

For computing the extremum, we solve the following equation:

$$\frac{\partial L(\beta)}{\partial \beta} = 0 \text{ or } \frac{\partial \ln(L(\beta))}{\partial \beta} = 0 \tag{7}$$

By examining that the extremum is maximum or minimum, we compute the second derivative and then we equalize the expression by zero:

$$\frac{\partial^2 L(\beta)}{\partial \beta^2} < 0 \text{ or } \frac{\partial^2 \ln(L(\beta))}{\partial \beta^2} < 0 \tag{8}$$

For this case above, the estimate value is the maximum.

$$\frac{\partial^2 L(\beta)}{\partial \beta^2} > 0 \text{ or } \frac{\partial^2 \ln(L(\beta))}{\partial \beta^2} > 0 \tag{9}$$

For this case above, the estimate value is the minimum.

After getting the approximate values of β , we want to check if they are meaningful to the effect of covariates on survival duration. It signifies that the natural logarithm ratio of hazard becomes larger of β_j for x_j becomes larger for one unit (all covariates are constants) and e^{β_j} shows that the ratio of hazard becomes larger of e^{β_j} for x_j becomes larger for one unit.

For $\beta = 0$: the ratio of hazard for that covariate $e^0 = 1$ which signifies that risk factor does not affect survival duration.

For $\beta < 0$: it shows that increasing x_j associated with longer survival times and lower risk (hazard) when Pvalue is less than 0.05 as we consider 95% interval of confidence.

For $\beta > 0$: it shows that increasing x_j associated with shorter survival times and increased risk (hazard) when

Pvalue is less than 0.05 as we consider 95% interval of confidence.

By referring to ratios of hazard;

For $0 < e^\beta < 1$, it shows that risk factor becomes less the chance of getting a risk when Pvalue is less than 0.05 otherwise it does not become less significantly the risk of getting an event.

For $e^\beta > 1$, it shows that factor of risk becomes larger the chance of getting risk when Pvalue is less than 0.05 otherwise it does not become larger meaningfully the risk of getting an event.

2.3. Equivalence principle

The equivalence principle allows to compute the premiums and reserves. It is stated as follow:

The mean value of loss is zero: $E(L) = 0$.

L is the value of loss (Benefits present value - Premiums present value) by assuming that that there are no expenses amount.

2.4. Premiums and reserves

By Considering that T be future life time to end of a contract which is positive and random variable continuous and considering that Y is the age limit and X is the life age then we have: $0 < T < Y - X - 1$ and knowing that the 1967-70 mortality table will be used in computations.

Let $K(x) = k = [T]$

Where [.] symbolizes "largest integer" then K is a discrete random variable such that: $k=0, 1, 2, 3, \dots, [Y-X-1]$.

Whole life insurance:

A whole life insurance contract is an agreement that gives amount of money agreed at the year termination of ceasing premium payments.

Premiums are computed as follows:

$$P_x = \frac{SA_x}{a_x}$$

Reserves are computed as follows:

$${}_kV_x = SA_{x+k} - P\ddot{a}_{x+k}$$

Where S is the sum assured

Pure endowment assurance:

A pure endowment contract is an agreement that gives amount of money agreed at the termination of m years if the life is not dead.

Premiums are computed as follows:

$$\begin{cases} P_{x:n} = \frac{SA_{x:n}}{\ddot{a}_{x:n}}, k \geq n \\ 0, k < n \end{cases}$$

Reserves are computed as follows:

$$\begin{cases} {}_kV_{x:n} = SA_{x+k:n-k} - P_{x:n}\ddot{a}_{x+k:n-k}, k \geq n \\ 0, k < n \end{cases}$$

Where n is the contract period.

Endowment assurance:

An endowment insurance contract is an agreement that gives amount of money agreed at the termination of m years if the person still lives or at the end of the year of ceasing premium payments when cessation happens specifically during a period.

Premiums are computed as follows:

$$\begin{cases} P_{x:n} = \frac{SA_{x:n}}{\ddot{a}_{x:n}}, k \geq n \\ P_{x:n} = \frac{SA_{x:n}}{\ddot{a}_{x:n}}, k < n \end{cases} \Leftrightarrow P_{x:n} = P_{x:n} + P_{x:n}$$

Reserves are computed as follows:

$$\begin{cases} {}_kV_{x:n} = SA_{x+k:n-k} - P_{x:n}\ddot{a}_{x+k:n-k}, k \geq n \\ {}_kV_{x:n} = SA_{x+k:n-k} - P_{x:n}\ddot{a}_{x+k:n-k}, k < n \end{cases}$$

2.5. Available benefits

The available benefits when the person ceases the payments of premium are: surrender value and paid-up sum assured.

Surrender value is the amount of money given in most policies of life insurance (except assurances of term) to be payable to the person if the payment of premiums is not continued.

Paid-up sum assured (PUSA) is the reduced benefits given to the person who would like to cease payments of premium as usual and he/she claims for keeping the contract in force.

Surrender value is computed as follows:

$$(SV)_t = s_{t-1}V_{x+1}$$

Where

s- the rate of surrender that must belong to [0,1]

t - the duration

x-the age at the beginning of the contract

${}_{t-1}V_{x+1}$ - Reserve

- Paid-up sum assured (PUSA) is computed as follows:

For whole life: ${}_tW_x = \frac{{}_tV_x}{A_{x+t}}$

For pure endowment: ${}_tW_{x:n} = \frac{{}_tV_{x:n}}{A_{x+t:n-t}}$

For endowment assurance: ${}_tW_{x:n} = \frac{{}_tV_{x:n}}{A_{x+t:n-t}}$

3. Results, conclusions and recommendations

In our work, we had two separate pieces of computations: survival computations and actuarial computations.

In survival part, we considered future life time of life insurance contracts within a period contract from different individuals as survival time, stop paying premiums payments as an event of interest and finally, we considered age, gender and marital status of individuals who were in the contract as risk factors to survival time.

In actuarial part, we considered the following variables such as sum assured given to the individual when the event occurred, interest rate, the assured person age, the age of limit, the contract period and the table of mortality of 1967-70.

There were criteria made such as: The period of the contract was 20 years, the age of individual that allowed to be in the contract was 40 years old, the individuals who were between 40 years and 50 years given a chance to take a contract by the condition that the limiting age would be 60 years old and the dataset is the same to all types of insurance contracts considered in our study.

3.1 Results

3.1.1. Survival part

3.1.1.1 Survival probabilities summary

Time (years)	n.risk	n.event	Survival .Pr	Std.err	Lower 95% CI	Upper 95% CI
10	89	2	0.978	0.0157	0.947	1
11	86	2	0.955	0.0221	0.912	0.999
12	84	10	0.841	0.0389	0.768	0.921
13	71	4	0.794	0.0434	0.713	0.883
14	57	3	0.752	0.0473	0.665	0.851
15	47	6	0.656	0.0552	0.556	0.774
18	10	2	0.525	0.0940	0.369	0.745
19	8	1	0.459	0.1026	0.296	0.712

The above results explain the approximation of survival probabilities, number of events, number of people who were exposed to risk, standard error and the interval of confidence of approximated probabilities of survival.

It clarifies that from 0 up to 9 years, the chance of survival was 1 and from 10 and above years the chance of survival was tending to 0.

Briefly, the life insurance contract was taken by 89 people. 30 people ceased payments of premium, 52 people lost and 7

people terminated the contract. At 12th, people ceased payments of premium on big number (10 people).

3.1.1.2. Cox Proportional hazards model outputs

Variables	Coefficients	Exp (coefficients)	Se (coefficients)	Z score	Prob (>z)
Age	0.4111	1.5084	0.0923	4.453	8.45e ⁻⁶
Gender	-0.6746	0.5094	0.3932	-1.716	0.0862
Marital status	0.1119	1.1183	0.2686	0.416	0.6771

This table above clarifies the coefficients of natural logarithm of hazard ratio with risk factors, exponential of those coefficients, error of standard of coefficients, Zscore and the probability of greater than Z score. By referring to exp(coefficient) values and pr(>z), as ages become larger, the risk of ceasing payments of premium becomes larger by 50.84% computed from [(1.5084-1.00)*100]. This variable age is meaningful to affect the survival duration because p=8.45e-6<0.05, which signifies that variable age caused the risk to cease payments of premium. The variable gender is not significant to cause survival duration changes because p=0.0862>0.05 which means that variable gender did not affect the risk to cease payments of premium. The predictor marital status is not significant to influence survival time because p=0.6771>0.05 That signifies that variable marital status did not affect the risk of ceasing payments of premium.

The model is:

$$\text{Log} \left(\frac{\hat{\beta}(t)}{\beta_0(t)} \right) = 0.411 * \text{Age} - 0.6746 * \text{Gender} + 0.119 * \text{Marital Status}$$

Briefly, variable age influenced the survival duration of ceasing payments of premium but variable gender and variable marital status did not affect it.

3.1.2. Actuarial part

3.1.2.1. Summary for net Premiums calculations

Life insurance contract	Formulas of net premiums	Net premiums
Whole life insurance	$P_x = \frac{SA_x}{\ddot{a}_x}$	$P_x = \text{£}5,300$
Endowment assurance	$P_x = \frac{SA_{x:n}}{\ddot{a}_{x:n}}$	$P_x = \text{£}13,800$
Pure endowment	$P_x = \frac{SA_{x:n}}{\ddot{a}_{x:n}}$	$P_x = \text{£}11,761.77$

3.1.2.2. Summary for reserves calculations

Time (years)	Whole life insurance	Endowment insurance	Pure endowment
0	0	0	0
1	£5,116.6	£14,133	£12,483.24
2	£10,247.5	£28,839.2	£25,736.49
3	£15,595.2	£44,366.4	£39,824.38
4	£21,160	£60,744.6	£54,793.34
5	£26,941.6	£78,027.6	£70,706.32

6	£32,940	£96,281.8	£87,655.23
7	£39,165.5	£115,533.4	£105,691.03
8	£45,608.1	£135,868.8	£124,918.95
9	£52,267.5	£157,365.6	£145,440.56
10	£59,144	£178,626.8	£167,343.06
11	£66,233	£204,074.8	£190,754.95
12	£73,352.4	£229,495	£215,837.07
13	£81,039.6	£256,432	£242,723.11
14	£88,738.3	£284,988.4	£271,583.64
15	£96,633.8	£315,328.2	£302,638.89
16	£104,710.8	£347,584	£336,098.99
17	£112,970	£381,949.8	£372,233.93
18	£121,389.6	£418,648.4	£411,359.32
19	£129,971.4	£457,900	£453,807.23
20	0	0	0

Briefly, from 0 up to 9th year, the insurer didn't provide any benefits. At 10th year, the insurer provided benefits to 2 policy holders. At 11th year, the insurer provided benefits to 2 policy holders. At 12th year, the insurer provided benefits to 10 policyholders. At 13th year, the insurer provided benefits to 4 policyholders. At 14th year, the insurer provided benefits to 3 policyholders. At 15th year, the insurer provided benefits to 6 policyholders. From 16th up to 17th year, the insurer didn't provide any benefits. At 18th year, the insurer provided benefits to 2 policyholders. At 19th year, the insurer provided benefits to 1 policyholder. At 20th year, no benefits provided.

3.2 Conclusions

The survival probability in beginning period of the contract is bigger to the survival probability in end period of the contract which signifies that the chance of ceasing premium payments in beginning duration is less to the chance of stopping the contract in terminating duration.

Variable age affects meaningfully the survival duration. This signifies that as variable age becomes larger, the survival duration for making premium payments becomes less. But variable gender and variable marital status do not influence meaningfully survival duration of payments of premium.

The net premiums and reserves per year for assurance of endowment are bigger to net premiums and reserves for pure endowment contract and for whole insurance of life contract.

The surrender value benefits are costing little money and simply to compute compared to PUSA profits. Both surrender values and paid up sum assured for whole life insurance contract are smaller to endowment assurance contract and pure endowment contract. The surrender values for endowment assurance are bigger than to those for Pure endowment but the PUSA for endowment assurance are lower to the PUSA for pure endowment contract.

3.1.2.3. Summary for Surrender values calculations.

Time (years)	Whole life insurance	Endowment insurance	Pure endowment
0	0	0	0
1	£511.66	£1,413.3	£1,248.32
2	£1,024.75	£2,883.92	£2,573.65
3	£1,559.52	£4,436.64	£3,982.44
4	£2,116	£6,074.46	£5,479.33
5	£2,694.16	£7,802.76	£7,070.63
6	£3,294	£9,628.18	£8,765.52
7	£3,916.55	£11,553.34	£10,569.10
8	£4,560.81	£13,586.88	£12,491.89
9	£5,226.75	£15,736.56	£14,544.06
10	£5,914.4	£17,862.68	£16,734.31
11	£6,623.3	£20,407.48	£19,075.495
12	£7,335.24	£22,949.5	£21,583.71
13	£8,103.96	£25,643.2	£24,272.31
14	£8,873.83	£28,498.84	£27,158.36
15	£9,663.38	£31,532.82	£30,263.89
16	£10,471.08	£34,758.4	£33,609.90
17	£11,297	£38,194.98	£37,223.39
18	£12,138.96	£41,864.84	£41,135.93
19	£12,997.14	£45,790	£45,380.72
20	0	0	0

As the survival probability of paying premium payments decreased, probability of providing benefits from insurer increased.

3.3 Recommendations

The work recommends: to approximate the survival probabilities of anything that happens by Kaplan Meier estimator and to assess the effect of variables on the survival duration by the Cox hazards proportional regression model.

The work recommends the companies for computing correctly the available profits when an event occurred. These available profits are the ones that persuade the clients to be interested in the life insurance product.

The work recommends the companies for pricing and reserving correctly because they affect the calculation of available profits. This signifies that every company of insurance of life needs an actuary or at least an actuarial function.

3.1.2.4. Summary for Paid up sum assured calculations

Time (years)	Whole life insurance	Endowment insurance	Pure endowment
0	0	0	0
1	£30,758.4	£40,756.12	£42,116.19
2	£58,560.5	£78,677.4	£81,911.17
3	£84,756.5	£114,517.6	£119,163.31
4	£109,427.5	£148,360.2	£154,447.44
5	£132,645	£180,331.4	£187,450.48
6	£154,488.33	£210,580.9	£218,646.12
7	£174,853.8	£239,145.1	£247,985.05
8	£194,441	£266,174.6	£275,576.77
9	£212,660	£291,174.4	£301,556.21
10	£229,810	£313,457.3	£325,950.68
11	£245,945	£338,910.2	£348,793.11
12	£261,123.6	£360,659.7	£370,344.99
13	£275,420	£381,294.5	£390,544.02
14	£288,852	£400,867.03	£409,443.15
15	£301,515.7	£419,481.2	£427,214.69
16	£313,392.8	£437,168.6	£443,812.21
17	£324,598.5	£454,004.9	£459,321.24
18	£335,126.7	£470,006.4	£473,861.67
19	£345,053.8	£485,372.1	£487,388.28
20	0	0	0

Room for further research

It is likely to consider the amount of expenses in the computations of prices and reserves. It will direct to the best amount of prices and reserves with available profits when event occurred and the insurer will be much comfortable.

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