Study on Modified Unit Step Function and Solution of Trigonometric and Logarithmic Function Using Laplace Transform

Yadav Saiganesh R.[1], Dr. R. N. Ingle[2*]

[1]Research Scholar N.E.S Science College, Nanded, Maharashtra, India

[2]Professor & Head, Department of Mathematics, Bahirji Smarak Mahavidyalaya, Basmathnagar, Hingoli, Maharashtra, India
[*]Corresponding author: *ingleraju11[at]gmail.com*

Abstract: ***Role of mathematics flawless in day to day life. Laplace transform is one of the tool used by mathematician and researcher to finding the solution to their problems. In this paper we fascinated to discuss about properties and uses of Laplace Transform in various field, also find solutions of trigonometry functions, logarithmic functions using Laplace Transform. Some of the important properties are detailed deeply in this paper with proof, with effective examples are also mentioned in this paper.***

Keywords: Modified unit step function, Trigonometric functions, Logarithmic functions, Differential equation, Algebric equation

1. Introduction

Laplace Transform is named in honour of the great French Mathematician Pierre Simon De Laplace. The best way to convert differential equation into algebraic equations is use Laplace Transformation. Laplace Transform plays major role in control system engineering to analyse the control system, Laplace Transform of different functions have to be carried out. This paper overview of Laplace transform properties with definition and its application in mathematics, physics etc. The Laplace is a particular type of integral transform. It is also primarily used in the analysis of temporarily events in the electrical circuits where frequency domain analysis is used.

A. Definition of Laplace Transform:

Let g(r) be function defined for all positive values of r,
L{ g(r) }= G(m) $=\int_0^\infty e^{-mr} \; g(r)\, dr$

B. Properties of Laplace Transform:

1) Linearity Property:
   If L{g_1(r)} = G_1 (m), L{g_2(r) } = G_2 (m) then
   L{ C_1 g_1(r) + $C_2 g_2$(r)} = C_1 L{ g_1(r)} + C_2 { g_2(r) }

2) First Shifting property:
   If L{g(r)}= G(m) the L{ $e^{ar}\; g(r)$} = G (m - a)

3) Second Shifting property:
   If L{g(r)}= G (m) = and F(r) $=\{^{G(m-a)}_{0} \quad {}^{r>a}_{r<a}$ then
   L{g(r)} = $e^{-ar}\, G(m)$

4) Laplace Transformation of derivatives:
   If L{g(r)}= G (m) then
   L{g'(r)} = m G(m)- g(0)

5) Laplace Transformation of integrals:
   If {L g(r)} = G(m) then
   L{$\int_0^\infty g(u)du$ } = $\frac{G(m)}{m}$

6) Multiplication by (r^n)
   If L{g(r)} = G(m) , then
   $L\{r^n g(r)\} = (-1)^n G^n(m)$

7) Division by r
   If L{g(r)} = G(m), then

$$\frac{g(r)}{m} = \int_0^\infty G(u)du$$

C. Modified Unit Step function:

Find L{g(r-a)}, where g (r-a) = $\{^{-1}_{0} \quad {}^{r>a}_{r<a}$

Proof:

Let,

g (r-a) = $\{^{-1}_{0} \quad {}^{r>a}_{r<a}$

L{g(r-a)} = $\int_0^\infty e^{-mr}\; g(r-a)dr$

L{g(r-a)} = $\int_0^a e^{-mr}\; g(r-a)dr + \int_a^\infty e^{-mr}\; g(r-a)dr$

L{g(r-a)} = $\int_0^a e^{-mr}.0\; dr + \int_a^\infty e^{-mr}\,(-1)dr$

L{g(r-a)} = 0 – $\left\{\frac{e^{-mr}}{-m}\right\}^{\infty}_{r=a}$

L{g(r-a)} = $-\frac{e^{-mr}}{m}$

D. Solution of Trigonometric functions using Laplace Transfrom:

1) Find Laplace Transform of (sinr), where sin r = $r - \frac{r^3}{3!} + \frac{r^5}{5!} - \frac{r^7}{7!} + \cdots$

Solution:

L(sin r) = L($r-\frac{r^3}{3!}+\frac{r^5}{5!}-\frac{r^7}{7!}+\cdots$)

L(sin r) = L(r)- $L\left(\frac{r^3}{3!}\right)+L\left(\frac{r^5}{5!}\right)-L\left(\frac{r^7}{7!}\right)+\cdots\left(where\ L(r^n)=\frac{n!}{m^{n+1}}\right)$

L(sin r) = $\frac{1}{m^2}-\frac{1}{m^4}+\frac{1}{m^6}-\frac{1}{m^8}+\cdots$

2) Find Laplace Transform of (cosr), where cos r = $1-\frac{r^2}{2!}+\frac{r^4}{4!}-\frac{r^6}{6!}+\cdots$

Solution:

L(cos r) = L($1-\frac{r^2}{2!}+\frac{r^4}{4!}-\frac{r^6}{6!}+\cdots$)

L(cos r) = L(1)- $L\left(\frac{r^2}{2!}\right)+L\left(\frac{r^4}{4!}\right)-L\left(\frac{r^6}{6!}\right)+\cdots\left(where\ L(r^n)=\frac{n!}{m^{n+1}}\right)$

L(cos r) = $\frac{1}{m^1}-\frac{1}{m^3}+\frac{1}{m^5}-\frac{1}{m^7}+\cdots$

3) Find Laplace Transform of (tan r), where tan r = $r+\frac{r^3}{3}+\frac{2r^5}{15}+\frac{17r^7}{315}+\cdots$

Solution:

L(tan r) = L($r+\frac{r^3}{3}+\frac{2r^5}{15}+\frac{17r^7}{315}+\cdots$)

L(tan r) = L(r) + $L\left(\frac{r^3}{3}\right)+L\left(\frac{2r^5}{15}\right)+L\left(\frac{17r^7}{315}\right)+\cdots\left(where\ L(r^n)=\frac{n!}{m^{n+1}}\right)$

L(tan r) = $\frac{1}{m^2}+\frac{2}{m^4}+\frac{2\times5!}{15m^6}+\frac{17\times7!}{315m^8}+\cdots$

E. Solution of Logarithmic functions using Laplace Transform:

1) Find Laplace Transform of log (1+r), where log(1+r) = $r-\frac{r^2}{2}+\frac{r^3}{3}-\frac{r^4}{4}+\cdots$

Solution:

L{log(1+r)}= L($r-\frac{r^2}{2}+\frac{r^3}{3}-\frac{r^4}{4}+\cdots$)

L{log(1+r)}= L(r)- $L\left(\frac{r^2}{2}\right)+L\left(\frac{r^3}{3}\right)-L\left(\frac{r^4}{4}\right)+\cdots\left(where\ L(r^n)=\frac{n!}{m^{n+1}}\right)$

L{log(1+r)} = $\frac{1}{m^2}-\frac{1}{m^3}+\frac{2}{m^4}-\frac{6}{m^5}+\cdots$

2) Find Laplace Transform of log (1-r), where log (1-r) = -$r-\frac{r^2}{2}-\frac{r^3}{3}-\frac{r^4}{4}-\cdots$

Solution:

L{log(1-r)}= L($-r-\frac{r^2}{2}-\frac{r^3}{3}-\frac{r^4}{4}-\cdots$)

L{log(1-r)}= -L(r)- $L\left(\frac{r^2}{2}\right)-L\left(\frac{r^3}{3}\right)-L\left(\frac{r^4}{4}\right)-\cdots\left(where\ L(r^n)=\frac{n!}{m^{n+1}}\right)$

L{log(1-r)} = $-\frac{1}{m^2}-\frac{1}{m^3}-\frac{2}{m^4}-\frac{6}{m^5}-\cdots$

2. Conclusion

In this paper overview of Laplace Transform and find solution of trigonometry function and logarithmic functions using Laplace Transform. Major properties of Laplace Transform and a few special functions like modified unit step function also discussed.

References

[1] J.K. Goyal and K.P. Gupta, Laplace and Fourier Transforms by Prgati Prakashan.
[2] H.K.Dass, Advanced Engineering Mathematics, S.Chand& Company Ltd.
[3] Grove A.C., An Introduction to Laplace Transforms and Z- Transforms, Prentice Hall 1991.
[4] Doetsch G., Introduction to Theory and Application of Laplace Transforms, Springer Verlag,1990.
[5] Murray Spigel, Schaum Outline of Laplace Transforms, Schaum Outline Series, Mc-Graw Hill 2012.
[6] J.W.Brown and R.V. Churchill, Complex variables and Applications, International Students edition 2009,7th edition.
[7] S.Punnusamy, Complex Analysis, Narosa Publishing house, 2nd edition.