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Factorisation of a Polynomial by Power Completion: Using the Binomial Theorem

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Abstract: ***Factorisation by power completion has been done for quadratic expressions only over the years. The process has not been used for higher powers as most studies showed that it requires multiple substitutions and is not that necessary. However, it is a necessary process in identifying perfect powers, their multiples and determining the pertext power closest to a given polynomial expression.***

Keywords: cube Multiples, polynomial powers, Binomial theorem, polynomials, Factorisation, power completion

1. Background

The binomial theorem together with the general polynomial notation can be used to create a formula which performs factorisation of polynomials by power completion that is, completing the square, cube and higher powers. This article will provide the basic formula and the derivation process.

2. The Derivation Process

Given a polynomial p(x) of order n,
$p(x) = ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} + .. + \mathrm{k}$ (K.A Stroud 2001 pp197)
Where a, b , c ,d ,..&k are constants .

If p(x) is a multiple of a perfect power of n, then by modifying the binomial theorem to become

$f(x) = a(x+r)^n = a\left(\mathrm{x}^n + nx^{n-1}r + \frac{n(n-1)x^{n-2}r^2}{2!} + \right.$ *nn−1n−2xn−3r33!+..+rn* (modified the series from K.A Stroud 2001 pp269)

Since p(x)=f(x), corresponding terms in both expressions should be equal

$$ax^n = ax^n$$
$$b = anr$$
$$i.e\ r = \frac{b}{an}$$
$$c = \frac{an(n-1)r^2}{2!}$$

Substituting for r gives

$$\frac{c}{a} = \frac{n(n-1)b^2}{2!\,a^2n^2}$$

$$d = \frac{an(n-1)(n-2)r^3}{3!}$$

Substituting for r gives

$$\frac{d}{a} = \frac{n(n-1)(n-2)b^3}{3!\,a^3n^3}$$
$$k = r^n$$

Substituting for r gives

$$\frac{k}{a} = \frac{b^n}{a^nn^n}$$

Factoring a in p(x) gives

$$p(x) = a\left(x^n + \frac{b}{a}x^{n-1} + \frac{c}{a}x^{n-2} + \frac{d}{a}x^{n-3} + .. \frac{k}{a}\right)$$

From p(x)=f(x), p(x) can be written as

$$p(x) = a(x+r)^n$$

Substituting for r gives

$$p(x) = a\left(x + \frac{b}{an}\right)^n = f(x)$$

The character b/an is a characteristic of the perfect power closest to a given polynomial, hence the second term of a polynomial is always the same as that of the closest perfect power.

However, if p(x) is not a multiple of a perfect power, then there is always at least one non zero difference between the terms in p(x) and those in f(x) starting from the third term

$$i.e\ p(x) - f(x) = a\left(\left(\frac{c}{a} - \frac{n(n-1)b^2}{2!\,a^2n^2}\right)x^{n-2} + \left(\frac{d}{a} - \frac{n(n-1)(n-2)b^3}{3!\,a^3n^3}\right)x^{n-3} + .. + \left(\frac{k}{a} - \frac{b^n}{a^nn^n}\right)\right) = d\_$$

Making p(x) the subject of the formula gives

$$p(x) = f(x) + d\_$$

Substituting for f(x) and d_ gives

$$p(x) = a\left(\left(x + \frac{b}{na}\right)^n + \left(\frac{c}{a} - \frac{n(n-1)b^2}{2!\,n^2a^2}\right)x^{n-2} + \left(\frac{d}{a} - \frac{n(n-1)(n-2)b^3}{3!\,n^3a^3}\right)x^{n-3} + .. + \left(\frac{k}{a} - \frac{b^n}{n^na^n}\right)\right)$$

Which gives the general method for factorising a polynomial by completing a power.

Completing the nth power, take the first n terms of the expression, that is, for completing the square take the first two terms, the first three for completing the cube and soon.

This method can be used to verify on whether a given polynomial is a perfect power, a multiple of a perfect power or not a perfect power.

References

[1] K. A Stroud (2001)Engineering Mathematics, 5th Edition pp197&269, Industrial Press INC New York