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Study on Differentiating Factors

Ankur Haldar

Student of BSc Physics Hons. The Heritage College, University of Calcutta, India
Email address: ankurivuhaldar[at]gmail.com

Abstract: ***In my previous paper "Modification of Feynman Technique of Differentiation", published in Volume 11 Issue 11, November 2022 of International Journal of Science and Research (IJSR) with paper ID SR221103004432, The Feynman Differentiation Technique was developed and an idea of differentiating factors was introduced. This paper is a more detailed and generalized study of differentiating factors and differentiation.***

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1. Introduction

The Modified Feynman Differentiation Technique gives us a simple way to differentiate a special type of functions that consist the product of multiple functions. Given below is the method :

If there is a differentiable function $f(x) = \prod_{i=1}^{n} [a_i(x)]^{b_i(x)}$, where a and b are differentiable functions of x, then $\frac{df}{dx}$ will be,

$$\frac{df}{dx} = f(x) \sum_{i=1}^{n} \left(\frac{b_i}{a_i}\frac{da_i}{dx} + \frac{db_i}{dx}\log_e[a_i]\right) \quad (1)$$

In Eq.(1) we can see that $f(x)$ is differentiated by multiplying $\sum_{i=1}^{n} \left(\frac{b_i}{a_i}\frac{da_i}{dx} + \frac{db_i}{dx}\log_e[a_i]\right)$ with it.This can be called **Differentiating Factor ($\boldsymbol{\phi(x)}$)** of $f(x)$. So, we can say,

$$\phi(x) = \frac{1}{f(x)}\frac{df}{dx} = \sum_{i=1}^{n} \left(\frac{b_i}{a_i}\frac{da_i}{dx} + \frac{db_i}{dx}\log_e[a_i]\right) \quad (2)$$

This gives the rise of the idea of differentiating factor of a function.

2. Definition

For every differentiable function $f(x)$, there exists a differentiating factor $\phi_f(x)$ such that :

$$\frac{df(x)}{dx} = f(x)\phi_f(x) \quad (3)$$

It can also be said that the differentiating factor of any differentiable function $f(x)$ is the ratio of $\frac{df(x)}{dx}$ and $f(x)$

$$\phi_f(x) = \frac{1}{f(x)}\frac{df(x)}{dx} \quad (4)$$

Now, by solving Eq(3) by separating the variables, we get:

$$f(x) = Ce^{\int \phi_f(x)dx} \quad (5)$$

Where, C is a constant.

From Eq(5), we can conclude that for every continuous $\phi_f(x)$, there exists a function $f(x)$ or in other words, every continuous function is a differentiating factor of some other function.

3. Basic Rules of Differentiation using Differentiating Factors

1) If $f(x)$ is a sum of functions :

$f(x) = \sum_i a_i(x)$ [Where, $a_i(x)$ is differentiable function of x]

Then, $\phi_f(x) = \frac{\sum_i a_i(x)\phi_{a_i}(x)}{\sum_i a_i(x)}$

Therefore, $f'(x) = \sum_i a_i(x)\phi_{a_i}(x)$

2) If $f(x)$ is a product of functions :

$f(x) = \prod_i a_i(x)$ [Where, $a_i(x)$ is a differentiable function of x]

Then, $\phi_f(x) = \sum_i \phi_{a_i}(x)$

Therefore, $f'(x) = [\prod_i a_i(x)][\sum_i \phi_{a_i}(x)]$

Now, if $a_i(x) = b_i(x)^{c_i(x)}$

Then, $\phi_{a_i}(x) = c_i(x)[\phi_{b_i}(x) + \phi_{c_i}(x)\log_e(b_i(x)]$ [i.e. The modified feynman differentiation technique]

4. Differentiating Factors of Some Basic Functions

[Here, a is constant, ϕ_f is the differentiating factor of $f(x)$]

1) Algebraic:

$$f(x) = a; \phi_f = 0$$

$$f(x) = x^n; \phi_f = \frac{n}{x}$$

$$f(x) = ax^n; \phi_f = \frac{n}{x}$$

$$f(x) = a^x; \phi_f = \log_e a$$

$$f(x) = x^x; \phi_f = 1 + \log_e x$$

2) Trigonometric:

$$f(x) = \sin x; \phi_f = \cot x$$

$$f(x) = \cos x; \phi_f = -\tan x$$

$$f(x) = \tan x; \phi_f = \tan x + \cot x$$

$$f(x) = \cot x; \phi_f = -(\tan x + \cot x)$$

$$f(x) = \sec x; \phi_f = \tan x$$

$$f(x) = x; \phi_f = -\cot x$$

3) Logarithmic and Exponential:

$$f(x) = \log_e x; \phi_f = \frac{1}{x \log_e x}$$

$$f(x) = \log_a x; \phi_f = \frac{1}{x \log_e x}$$

$$f(x) = e^x; \phi_f = 1$$

4) Double Derivative

Let, $f(x)$ be a differentiable function, $f'(x)$ be the first derivative of $f(x)$ and $f''(x)$ be the second derivative of $f(x)$.

We know,

$$f'(x) = f(x)\phi_f \tag{6}$$

$$f''(x) = f'(x)\phi_{f'} = f(x)\phi_f\phi_{f'} \tag{7}$$

We can conclude that $\phi_{f'}$ is the differentiating factor of $f'(x)$ i.e. $f(x)\phi_f$. Using basic rules of section 3, we can calculate that,

$$\phi_{f'} = \phi_f + \phi_{\phi_f} \tag{8}$$

ϕ_{ϕ_f} is the differentiating factor of ϕ_f So, we can conclude that,

$$f''(x) = f(x)\phi_f(\phi_f + \phi_{\phi_f}) \tag{9}$$

5) Presence of a Similar Integrating Factor

Let $f(x)$ and $g(x)$ be two functions of x such that :

$$\int f(x)dx = g(x) \tag{10}$$

If a similar integrating factor exists then :

$$\int f(x)dx = g(x) = f(x)I_f(x) \tag{11}$$

We can also say that $f(x)$ is the first derivative of $g(x)$. Hence :

$$f(x) = g(x)\phi_g(x) \tag{12}$$

From Eq(11) and Eq(12), we get:

$$\phi_g(x)I_f(x) = 1 \tag{13}$$

Also, by differentiating Eq(12) with respect to x, we get :

$$f(x)\phi_f(x) = g(x)\phi_g(x)(\phi_g(x) + \phi_{\phi_g}(x)) \tag{14}$$

As, $f(x) = g(x)\phi_g(x)$ (According to Eq(12)), Eq(14) becomes :

$$\phi_f(x) = \phi_g(x) + \phi_{\phi_g}(x) \tag{15}$$

This equation (Can be called Basic Integral Rule) holds true for any function $f(x)$ and its integral $g(x)$ with respect to x.

5. Summary

Differentiating factor is the ratio of a differentiable function and its first derivative. For every differentiable function there exists a differentiating factor and every differentiable function is differentiating factor of some other function. Using the concept of differentiating factors, first and second derivatives of various functions can be done easily.

The same concept can be used to get the idea of a similar integrating factor. Eq(13) and Eq(15) gives some idea of this integrating factor. Though, finding a generalized way to find integrating factor of any function has not been found yet.

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