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A Deterministic Inventory Model for Non-Instantaneous Deteriorating Items with Allowed Backorders and Quadratic Time Varying Holding Cost

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Abstract: ***In this manuscript, an inventory model has been formulated in which shortages are allowed and fully backlogged. Deterioration rate is constant and demand is a function of selling price. Holding cost is a quadratic function of time. Here we have to derive optimum cycle time so as to minimize the total cost. To illustrate the model, numerical examples are used.***

Keywords: Deterioration, selling price dependent demand, backlogging

1. Introduction

During past few years many authors have studied inventory models considering different demand and deterioration rate. Misra[4] have studied inventory model for deteriorating items with constant rate of deterioration. Aggarwal [2] developed an order-level inventory model for a system with constant rate of deterioration. Further, the price sensitive nature of the demand cannot be ignored. Almost all the items are price sensitive. It is well known that lesser the price of an item, greater is the demand of that item whereas higher selling price has the reverse effects. Whitin [6] developed a deterministic inventory model by incorporating the effect of selling price on demand. Abad [1], arcelus and Srinivasan [3] followed the suit and developed inventory model considering dependence on pricing and lot size. Yadav et al [7] has developed a deterministic inventory model for deteriorating items with selling price dependent demand and variable deterioration under inflation with time dependent holding cost. Sugpriya and Jeyaraman [5] developed an EPQ model for single product subject to non-instantaneous deterioration under a production inventory policy in which holding cost varying with time is considered.

In the present paper, we have tried to develop an inventory model for non-instantaneous deteriorating items and the demand is a function of selling price. Shortages are allowed and completely backlogged, holding cost varies with quadratic function of time.

Assumptions and Notations:

1) Demand rate D(p) is price dependent and is given by $D(p) = \alpha p^2 + \beta p + \gamma;\ \alpha \geq 0, \quad \beta \neq 0, \quad \gamma \neq 0.$
2) Deterioration rate r(t) is constant i.e. r(t) = θ.
3) Inventory holding cost per unit time is $H = h_1 + h_2 t + h_3 t^2$; $h_1 > 0,\ h_2 > 0, h_3 > 0$.
4) At the beginning of every period, the initial stock level is raised to order level i.e.
   S=Initial stock level at the beginning of every inventory
5) S_1 = Maximum shortage level.
6) C_d = Deterioration cost per unit per unit time.
7) C_s= Shortage cost per unit per unit time.
8) q(t) is the Inventory level at time 't'.
9) Shortages are allowed and fully backlogged.

Initially at time t=0, the initial stock was S. Thereafter, the level of the inventory decreases mainly due to meet up demand and partially due to deterioration. Therefore the stock level reaches to zero at t=t_1. Now shortages occur and accumulate to the level S_1 at t=T.

The inventory system is governed by the differential equations described below:

$$\frac{dq(t)}{dt} + \theta q(t) = -(\alpha p^2 + \beta p + \gamma) \ ; \quad 0 \leq t \leq t_1 \qquad (1)$$

$$\frac{dq(t)}{dt} = -(\alpha p^2 + \beta p + \gamma) \ ; \quad t_1 \leq t \leq T \qquad (2)$$

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The solutions of the above differential equations after adjusting the constant of integration and applying the boundary conditions $q(t_1) = 0$ and q(0)=S are

$$q(t) = -\frac{(\alpha p^2+\beta p+\gamma)}{\theta} + \frac{(S\theta+\alpha p^2+\beta p+\gamma)}{\theta} e^{-\theta t} ; 0 \le t \le t_1 \quad (3)$$

and $q(t) = (\alpha p^2+\beta p+\gamma)(t_1 - t)$; $\quad t_1 \le t \le T \quad$ (4)

Also at $t = t_1$; $q(t_1) = 0$, $\therefore$ from (3) we have

$$t_1 = \frac{1}{\theta}\log(1 + \frac{S\theta}{\alpha p^2+\beta p+\gamma}) \quad (5)$$

And at t=T, $q(t) = -S_1$

$$S_1 = (\alpha p^2+\beta p+\gamma)\left[T - \frac{1}{\theta}\log\left(1+\frac{S\theta}{\alpha p^2+\beta p+\gamma}\right)\right] \quad (6)$$

The total cost per cycle consists of the following cost components:

1. Deterioration cost per cycle is given by D.C.= $C_d \int_0^{t_1} \theta q(t) dt$

$$\text{D.C.} = -C_d[(\alpha p^2+\beta p+\gamma)t_1 + \frac{(S\theta+\alpha p^2+\beta p+\gamma)}{\theta}(e^{-\theta t_1} - 1)] \quad (7)$$

2. Shortage cost per cycle due to backlog is given by S.C.=$-C_s \int_{t_1}^{T} q(t)dt$

$$\text{S.C.} = \frac{C_s}{2}(\alpha p^2+\beta p+\gamma)[T - t_1]^2 \quad (8)$$

3. Inventory holding cost per cycle is given by

$$\text{IHC} = \int_0^{t_1}(h_1 + h_2 t + h_3 t^2)\, q(t)\, dt$$

$$\text{IHC} = h_1[-\frac{(\alpha p^2+\beta p+\gamma)t_1}{\theta} + \frac{(S\theta+\alpha p^2+\beta p+\gamma)}{\theta^2}(1-e^{-\theta t_1})] + h_2[-\frac{(\alpha p^2+\beta p+\gamma){t_1}^2}{2\theta} + \frac{(S\theta+\alpha p^2+\beta p+\gamma)}{\theta^3}(1 - e^{-\theta t_1} - \theta t_1 e^{-\theta t_1})] + h_3[-\frac{(\alpha p^2+\beta p+\gamma){t_1}^3}{3\theta} + \frac{(S\theta+\alpha p^2+\beta p+\gamma)}{\theta^4}\{2(1 - e^{-\theta t_1} - \theta t_1 e^{-\theta t_1}) - \theta^2 t_1^2 e^{-\theta t_1}\}] \quad (9)$$

Therefore, the total cost of inventory is given by

T.C. = [D.C. + IHC + S.C.]

i.e.

$$\text{T.C.} = -C_d\left[(\alpha p^2+\beta p+\gamma)t_1 + \frac{(S\theta+\alpha p^2+\beta p+\gamma)}{\theta}\left(-1+e^{-\theta t_1}\right)\right] + h_1[-\frac{(\alpha p^2+\beta p+\gamma)t_1}{\theta} + \frac{(S\theta+\alpha p^2+\beta p+\gamma)}{\theta^2}(1-e^{-\theta t_1})] + h_2[-\frac{(\alpha p^2+\beta p+\gamma){t_1}^2}{2\theta} + \frac{(S\theta+\alpha p^2+\beta p+\gamma)}{\theta^3}(1 - e^{-\theta t_1} - \theta t_1 e^{-\theta t_1})] + h_3[-\frac{(\alpha p^2+\beta p+\gamma){t_1}^3}{3\theta} + \frac{(S\theta+\alpha p^2+\beta p+\gamma)}{\theta^4}\{2(1 - e^{-\theta t_1} - \theta t_1 e^{-\theta t_1}) - \theta^2 t_1^2 e^{-\theta t_1}\}] + \frac{C_s}{2}(\alpha p^2+\beta p+\gamma)[T - t_1]^2 \quad (10)$$

Solution Procedure:

Now the necessary condition for the total cost per unit time to be minimum is

$$\frac{\partial TC}{\partial t_1} = 0, \text{ provided } \frac{\partial^2 TC}{\partial t_1^2} > 0$$

i.e.

$$-(\frac{\alpha p^2+\beta p+\gamma}{\theta})[C_d\theta + C_s\theta(T - t_1) + h_1 + h_2 t_1 + h_3 t_1^2] + \frac{(S\theta+\alpha p^2+\beta p+\gamma)}{\theta}[C_d\theta + h_1 + h_2 t_1 + h_3 t_1^2]\, e^{-\theta t_1}] = 0$$

Numerical Example: To illustrate the results obtained for the suggested model, the parameters of the inventory system are: θ=0.30, p=5, $\alpha = 4$, $\beta = 1$, $\gamma = 2$, $h_1 = 4$, $h_2 = 0.2$, $h_3 = 0.05$, $C_d = 1.50$, $C_s = 1$, $T = 4$ in appropriate units, we obtain the optimal values of 'S', 't_1' and TC as $S^* = 34.6873$, $t_1^* = 0.3093$,, $TC^* = 752.3576$

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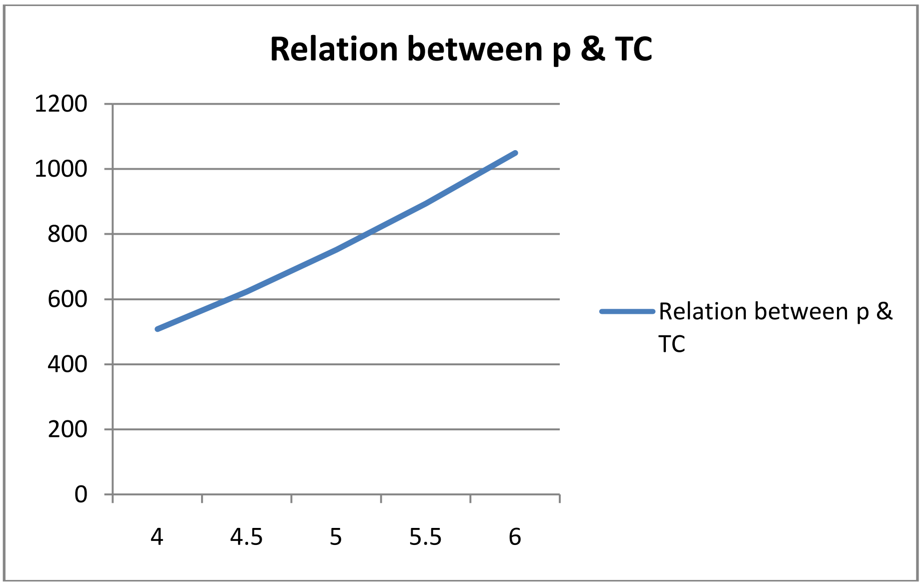
Table 1: Relation between ‘p’ and ‘TC’

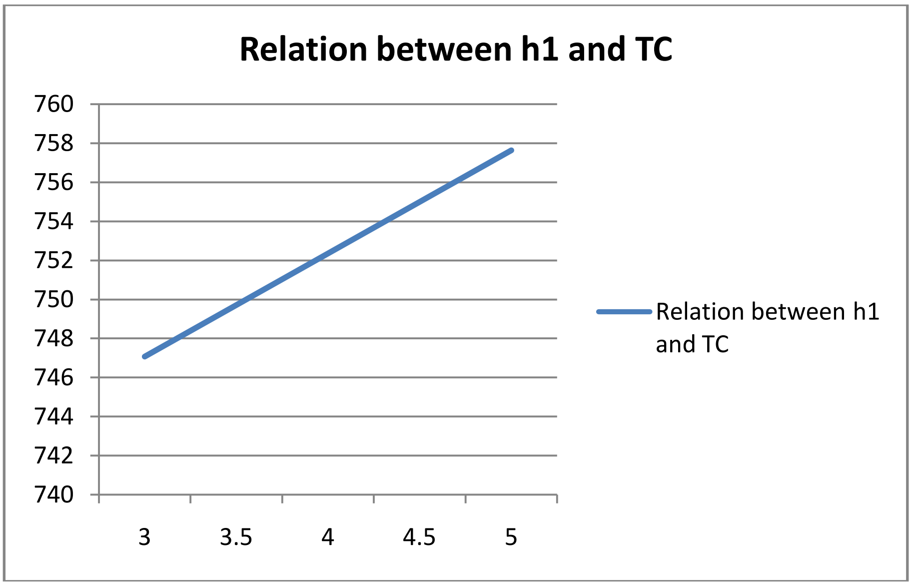
p	TC
4	508.0765
4.5	623.6149
5	752.3576
5.5	894.3048
6	1049.456



When the value of ‘p’ increases, there is an increase in the total cost of the system.

Table 2: Relation between ‘h_1’ and ‘TC’

h1	TC
3	747.075
3.5	749.7163
4	752.3576
4.5	754.9989
5	757.6402



When the value of ‘h_1’ increases, there is an increase in the total cost of the system.

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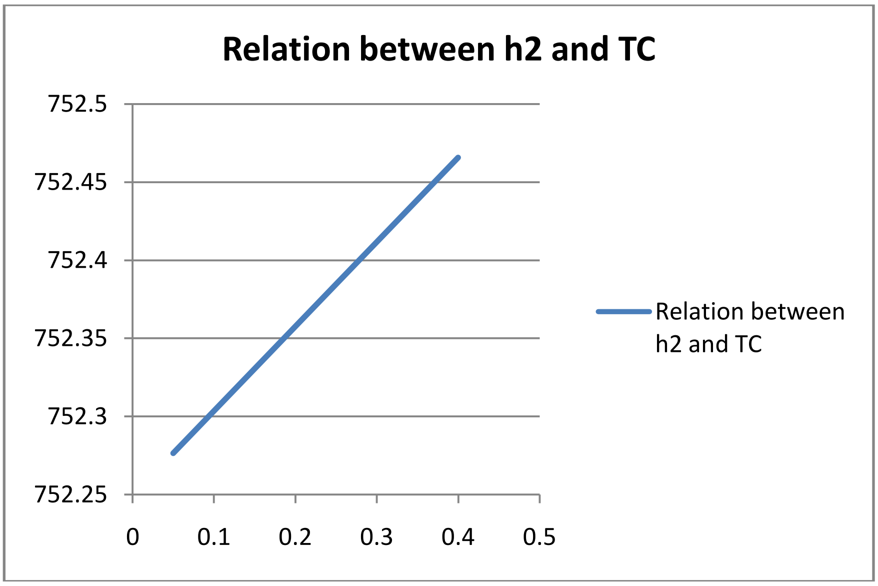
Table 3: Relation between 'h_2' and 'TC'

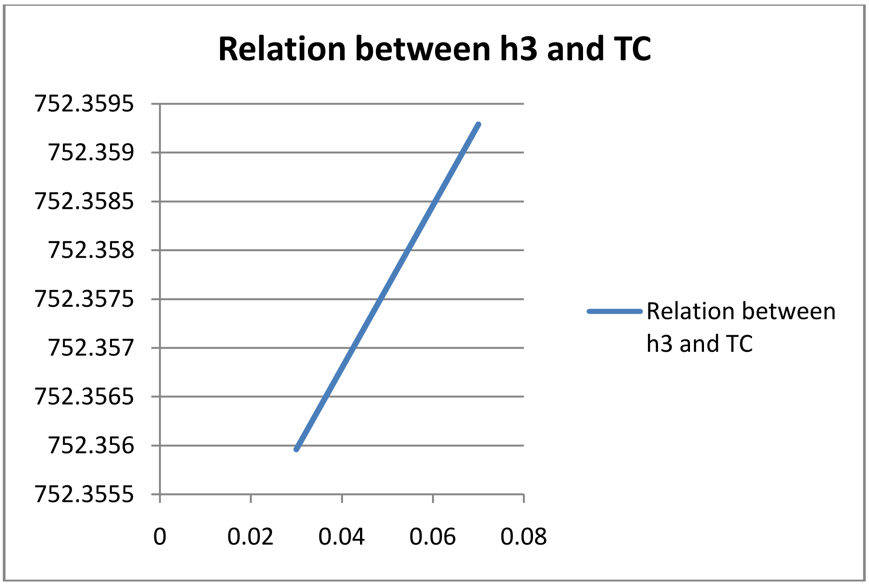
h2	TC
0.05	752.2765
0.1	752.3036
0.2	752.3576
0.3	752.4117
0.4	752.4657



When the value of 'h_2' increases, there is an increase in the total cost of the system.

Table 4: Relation between 'h_3' and 'TC'

h3	TC
0.03	752.356
0.04	752.3568
0.05	752.3576
0.06	752.3585
0.07	752.3593



When the value of 'h_3', increases, there is an increase in the total cost of the system.

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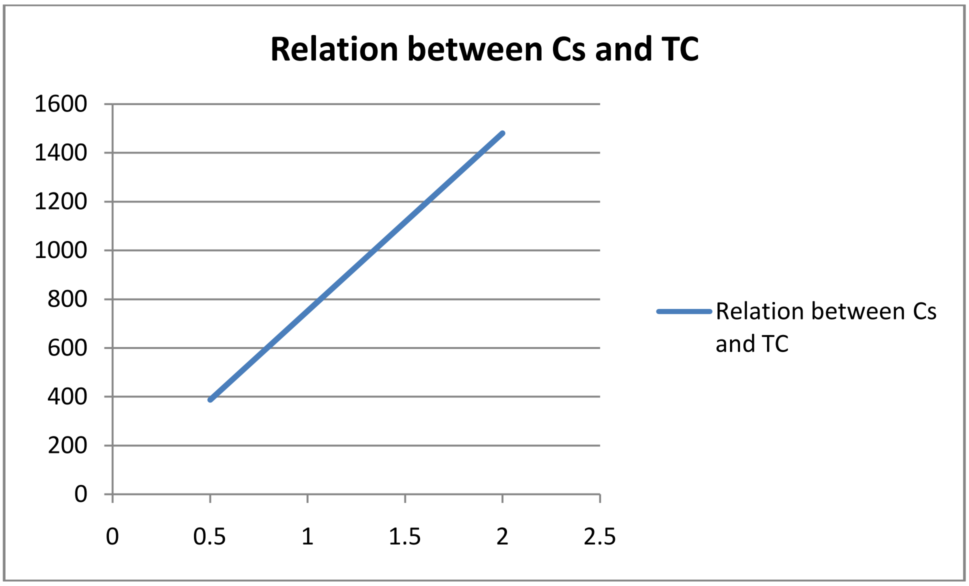
Table 5: Relation between 'C_s' and 'TC'

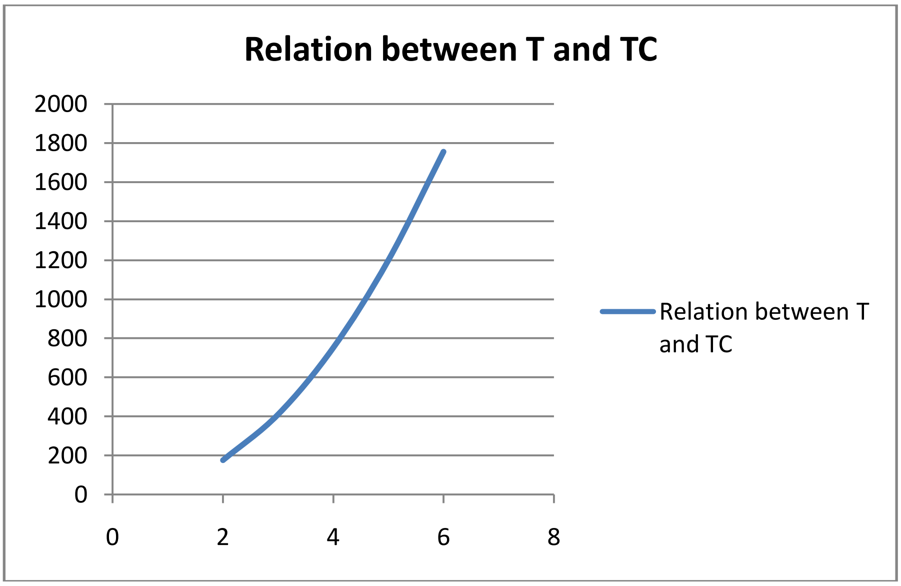
C_s	TC
0.5	387.9887
0.75	570.1732
1	752.3576
1.5	1116.727
2	1481.095



When the value of 'C_s' increases, there is an increase in the total cost of the system.

Table 6: Relation between 'T' and 'TC'

T	TC
2	176.5478
3	410.9527
4	752.3576
5	1200.763
6	1756.167



When the value of 'T' increases, there is an increase in the total cost of the system.

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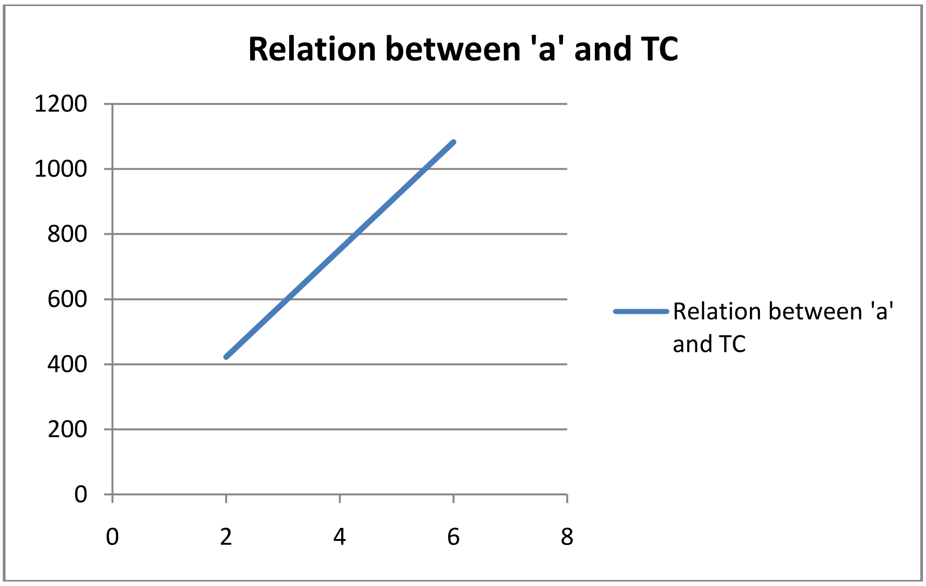
Table 7: Relation between 'a' and 'TC'

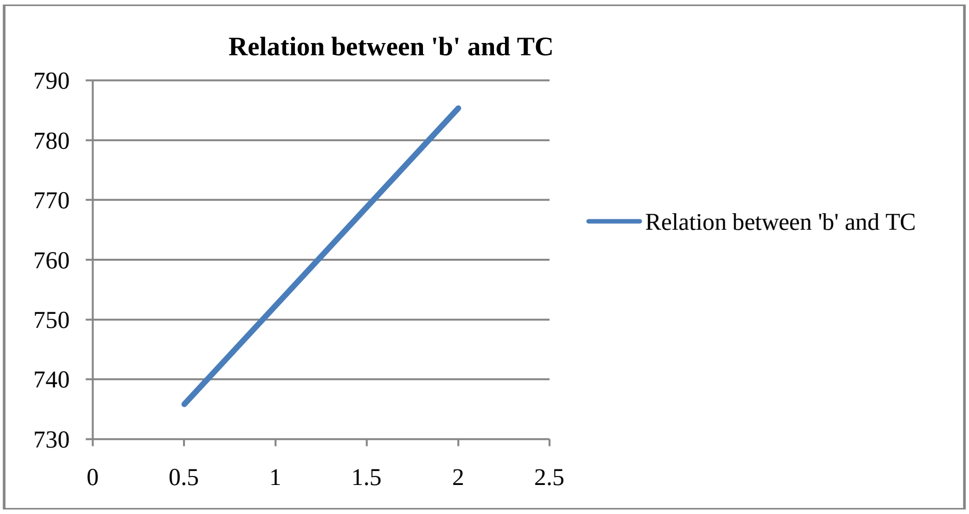
a	TC
2	422.248
3	587.3028
4	752.3576
5	917.4125
6	1082.467



When the value of 'a' increases, there is an increase in the total cost of the system.

Table 8: Relation between 'b' and 'TC'

b	TC
0.5	735.8521
0.75	744.1049
1	752.3576
1.5	768.8631
2	785.3686



When the value of 'b' increases, there is an increase in the total cost of the system.

2. Conclusion

Here, we have developed an inventory model in which the deterioration rate is constant. Demand rate is quadratic function of selling price and holding cost varies with time. We have shown the effect on total cost by changing various parameters taken into consideration.

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