# Reliability and Validity of Generalized Grodzins Relation 

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#### Abstract

In this paper, an attempt is made to check the reliability of generalized Grodzins empirical relation on B(E2) transition probabilities. The data on available values of transition probabilities is taken for nuclei which possess large neutron excess and are close to the neutron drip line. A comparison of $\operatorname{B}(E 2)$ transition probability values calculated by using generalized Grodzins relation with the experimentally measured values for the various transitions between the angular momentum states of the ground state band is presented for some unstable dripline nuclei such as ${ }^{128,130} \mathrm{Ce},{ }^{150} \mathrm{Nd},{ }^{152,154} \mathrm{Sm},{ }^{152,154,156} \mathrm{Gd},{ }^{156,162} \mathrm{Dy},{ }^{168} \mathrm{Er},{ }^{172,174} \mathrm{Yb}$ and ${ }^{130-136} \mathrm{U}$.


Keywords: Transition Probabilities; Ground State Band; Angular Momentum; Even-Even Nuclei

## 1. Reduced Transition Probability [B(E2)]

An excited nucleus may decay to a lower state or the ground state through electromagnetic transitions. Such transitions have definite probability depending upon the nature of the initial and final states. The $B(E 2)$ values for the even-even nuclei are important in studying the nuclear structure because these have the direct signature of the intrinsic quadrupole moment and hence the intrinsic deformation of the nucleus.

The relation is given by

$$
B\left(E 2 ; 0_{1}^{+} \rightarrow 2_{1}^{+}\right)=\frac{5}{16 \pi} Q_{0}^{2}
$$

where $Q_{0}$ represents intrinsic quadrupole moment.
Further, the $\mathrm{B}(\mathrm{E} 2)$ values for the transitions between the two states $I_{i}$ and $I_{f}$ are related by

$$
B\left(E 2 ; I_{i} \rightarrow I_{f}\right)=\frac{\left(2 I_{f}+1\right) B\left(E 2 ; I_{f} \rightarrow I_{i}\right)}{\left(2 I_{i}+1\right)}
$$

Phenomenological Models play an important role in the analysis of the experimental data. They are especially useful if it becomes possible to derive on their basis relations that do not include free parameters. A well known example of such a relation is the Grodzins relation between the excitation energy of the first $2^{+}$state and the $B\left(E 2 ; 2_{1}^{+} \rightarrow 0_{1}^{+}\right)[1,2]$ and is given by

$$
E\left(2_{1}^{+}\right)=\frac{1225 \mathrm{MeV}}{\beta_{2}^{2} A^{7 / 3}}
$$

But as $\beta_{2}$ is related to $\mathrm{B}(\mathrm{E} 2) \uparrow$ by the formula [3]

$$
\begin{aligned}
& \beta_{2}=\frac{4 \pi}{3 Z R_{0}^{2}}\left[\frac{B(E 2) \uparrow}{e^{2}}\right]^{\frac{1}{2}} \\
& \quad[E(I+2)-E(I)] B(E 2 ; I+2 \rightarrow I) \frac{(2 I+5)}{(I+1)(I+2)}=\frac{5}{2} E\left(2_{1}^{+}\right) B\left(E 2 ; 2_{1}^{+} \rightarrow 0_{1}^{+}\right)
\end{aligned}
$$

This parameter free-relation connects the spin dependence of the transition energies $[E(I+2)-E(I)]$ of the members of the quasirotational ground-state band with the E2 reduced
where $R_{0}=r_{0} A^{1 / 3}$ is usually taken to be $1.2 \mathrm{~A}^{1 / 3} \mathrm{fm}$ and $\mathrm{B}(\mathrm{E} 2) \uparrow$ is in units of $\mathrm{e}^{2} \mathrm{~b}^{2}$.
Then, $\begin{array}{r}E\left(2_{1}^{+}\right)= \\ {\left[\frac{1225 M e V}{3 Z R_{0}^{2}}\right]^{1 / 2} \frac{B(E 2) \uparrow}{e^{2}}} \\ E\left(2_{1}^{+}\right) \propto \frac{1}{B(E 2) \uparrow}\end{array}$
This is old Grodzins relation. This relation shows that $\gamma$-ray E2 transition probabilities from the first $2^{+}$states of the even-even nuclei to the ground states are approximately inversely proportional to $E\left(2_{1}^{+}\right)$.

## 2. Generalised Grodzins Relation

Jolas et al. [4], have generalized the Grodzins relation from $2_{1}^{+}$state to all the members of the ground state band for nuclei which are well deformed or at least deformed and have a quasirotational ground band. Including the rotationvibration interaction, and using the sum rule approach, they derived expressions for $B(E 2, I+2 \rightarrow I) \times E_{\gamma}[I+2 \rightarrow I]$, related to $\mathrm{B}\left(\mathrm{E} 2,2_{1}^{+} \rightarrow 0_{1}^{+}\right) \times \mathrm{E}\left(2_{1}^{+}\right)$. The sum rules were derived based on Bohr-Hamiltonian. They also used the assumption that the rotational contribution to the transition energies between the $\beta, \gamma$ bands and the ground band is small compared to vibrational contribution. For the axially symmetric well deformed nuclei, for which the contribution of the excited bands to the ground state band is small, authors of Ref. [4], have generalized Grodzins relation and obtain the form as
transition probabilities $\mathrm{B}\left[E 2 ;(I+2)_{1} \rightarrow I_{1}\right]$. In this paper, we checked the reliability and validity of this relation for some even-even nuclei of lanthanide series and four uranium isotopes of actinide series.

## 3. Results

Table1: Experimental ratios $E_{4}^{+} / E_{2}^{+}$

| ${ }^{128} \mathrm{Ce}$ | ${ }^{130} \mathrm{Ce}$ | ${ }^{150} \mathrm{Nd}$ | ${ }^{152} \mathrm{Sm}$ | ${ }^{154} \mathrm{Sm}$ | ${ }^{152} \mathrm{Gd}$ | ${ }^{154} \mathrm{Gd}$ | ${ }^{156} \mathrm{Gd}$ | ${ }^{156} \mathrm{Dy}$ | ${ }^{162} \mathrm{Dy}$ | ${ }^{168} \mathrm{Er}$ | ${ }^{172} \mathrm{Yb}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2.92 | 2.79 | 2.93 | 3.00 | 3.32 | 2.19 | 3.01 | 3.23 | 2.93 | 3.29 | 3.31 | 3.31 |

Table 2: Comparison of experimental and calculated $\mathrm{B}[\mathrm{E} 2$; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{128} \mathrm{Ce} . \mathrm{B}(\mathrm{E} 2)$ values are in units of $e^{2} b^{2}$

| Spin (I) | Transition <br> $\left(I_{f} \rightarrow I_{i}\right)$ | Experimental <br> B(E2) values [5] | Calculated <br> B(E2) values |
| :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | $0.43(4)$ | 0.430 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ | $0.68(13)$ | 0.743 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ | $0.56(5)$ | 0.931 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ | $0.96(14)$ | 1.105 |
| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ | $0.13(2)$ | 1.341 |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ |  | 2.039 |
| $12^{+}$ | $14^{+} \rightarrow 12^{+}$ |  | 2.489 |
| $14^{+}$ | $16^{+} \rightarrow 14^{+}$ |  | 2.345 |

Table 3: Comparison of experimental and calculated $\mathrm{B}[\mathrm{E} 2$; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{130} \mathrm{Ce} . \mathrm{B}(\mathrm{E} 2)$ values are in units of $e^{2} b^{2}$

| Spin <br> (I) | Transition <br> $\left(I_{+} \rightarrow I_{i}\right)$ | Experimental <br> B(E2) values [6,7] | Calculated <br> B(E2) values |
| :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | $0.418(35)$ | 0.418 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ | $0.656_{47}^{90}$ | 0.771 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ | $0.743_{551}^{646}$ | 0.993 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ | $0.160_{264}^{484}$ | 1.194 |
| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ | $0.540{ }_{107}^{176}$ | 1.498 |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ |  | 2.775 |
| $12^{+}$ | $14^{+} \rightarrow 12^{+}$ |  | 3.027 |
| $14^{+}$ | $16^{+} \rightarrow 14^{+}$ |  | 2.774 |

Table 4: Comparison of experimental and calculated $\mathrm{B}[\mathrm{E} 2$; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{150} \mathrm{Nd}$. $\mathrm{B}(\mathrm{E} 2)$ values are in units of $\mathrm{e}^{2} \mathrm{~b}^{2}$

| Spin (I) | Transition <br> $\left(I_{f} \rightarrow I_{i}\right)$ | Experimental <br> B(E2) values [8] | Calculated <br> B(E2) values |
| :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | $0.544(9)$ | 0.544 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ | $0.862(9)$ | 0.939 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ | $0.994(9)$ | 1.199 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ |  | 1.423 |

Table 5: Comparison of experimental and calculated B[E2; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{152} \mathrm{Sm}$. B(E2) values are in units of $e^{2} b^{2}$

| Spin (I) | Transition <br> $\left(I_{f} \rightarrow I_{i}\right)$ | Experimental <br> B(E2) values [9] | Calculated <br> B(E2) values |
| :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | 0.6912 | 0.6912 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ | 1.0032 | 1.011 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ | 1.176 | 1.313 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ | 1.368 | 1.525 |
| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ | 1.536 | 1.713 |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ |  | 1.892 |
| $12^{+}$ | $14^{+} \rightarrow 12^{+}$ |  | 2.069 |
| $14^{+}$ | $16^{+} \rightarrow 14^{+}$ |  |  |

Table 6: Comparison of experimental and calculated B[E2; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{154} \mathrm{Sm}$. B(E2) values are in units of $\mathrm{e}^{2} \mathrm{~b}^{2}$

| Spin (I) | Transition <br> $\left(I_{f} \rightarrow I_{i}\right)$ | Experimental <br> B(E2) values [10] | Calculated <br> B(E2) values |
| :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | 0.856 | 0.856 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ | 1.20 | 1.224 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ | 1.42 | 1.345 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ | 1.56 | 1.409 |
| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ | 1.54 | 1.449 |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ | 1.38 | 1.475 |
| $12^{+}$ | $14^{+} \rightarrow 12^{+}$ |  | 1.492 |

Table 7: Comparison of experimental and calculated B[E2; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{152} \mathrm{Gd}$. B(E2) values are in units of $\mathrm{e}^{2} b^{2}$

| Spin (I) | Transition <br> $\left(I_{f} \rightarrow I_{i}\right)$ | Experimental <br> B(E2) values [11] | Calculated <br> B(E2) values |
| :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | $0.33(2)$ | 0.33 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ | $0.64(4)$ | 0.92 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ | $0.95(19)$ | 1.38 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ |  | 1.80 |
| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ |  | 2.19 |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ |  | 2.57 |
| $12^{+}$ | $14^{+} \rightarrow 12^{+}$ |  | 2.89 |
| $14^{+}$ | $16^{+} \rightarrow 14^{+}$ |  | 3.20 |

Table 8: Comparison of experimental and calculated $\mathrm{B}[\mathrm{E} 2$; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{154} \mathrm{Gd}$. B(E2) values are in units of $\mathrm{e}^{2} \mathrm{~b}^{2}$

| Spin <br> (I) | Transition <br> $\left(I_{f} \rightarrow I_{i}\right)$ | Experimental <br> $\mathrm{B}(\mathrm{E} 2)$ values [12] $]$ | Calculated <br> $\mathrm{B}(\mathrm{E} 2)$ values |
| :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | $0.77(2)$ | 0.77 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ | $1.20(28)$ | 1.27 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ | $1.30(3)$ | 1.57 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ | $1.53(8)$ | 1.82 |
| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ |  | 2.05 |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ |  | 2.28 |
| $12^{+}$ | $14^{+} \rightarrow 12^{+}$ |  | 2.50 |
| $14^{+}$ | $16^{+} \rightarrow 14^{+}$ |  | 2.74 |

Table 9: Comparison of experimental and calculated $\mathrm{B}[\mathrm{E} 2 ; \mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{156} \mathrm{Gd}$. $B(E 2)$ values are in units of $e^{2} b^{2}$

| Spin (I) | Transition <br> $\left(I_{f} \rightarrow I_{i}\right)$ | Experimental <br> B(E2) values [13] | Calculated <br> B(E2) values |
| :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | $0.91(11)$ | 0.91 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ | $1.30(5)$ | 1.33 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ |  | 1.56 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ |  | 1.73 |
| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ |  | 1.90 |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ |  | 2.08 |
| $12^{+}$ | $14^{+} \rightarrow 12^{+}$ |  | 2.28 |
| $14^{+}$ | $16^{+} \rightarrow 14^{+}$ |  | 2.49 |

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Table 10: Comparison of experimental and calculated $\mathrm{B}[\mathrm{E} 2$; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{156} \mathrm{Dy}$. $\mathrm{B}(\mathrm{E} 2)$ values are in units of $\mathrm{e}^{2} \mathrm{~b}^{2}$

| Spin (I) | Transition <br> $\left(I_{f} \rightarrow I_{i}\right)$ | Experimental <br> B(E2) values [14] | Calculated <br> B(E2) values |
| :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | $0.834_{-103}^{+137}$ | 0.834 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ | $1.220(13)$ | 1.426 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ | $1.323(22)$ | 1.801 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ | $1.378_{-40}^{+43}$ | 2.114 |
| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ | $1.390_{-95}^{+97}$ | 2.400 |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ | $1.400_{-88}^{+100}$ | 2.693 |
| $12^{+}$ | $14^{+} \rightarrow 12^{+}$ | $1.463_{-258}^{+400}$ | 2.977 |

Table 11: Comparison of experimental and calculated $\mathrm{B}[\mathrm{E} 2$; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{162} \mathrm{Dy}$. $\mathrm{B}(\mathrm{E} 2)$ values are in units of $e^{2} b^{2}$

| Spin <br> (I) | Transition <br> $\left(I_{+} \rightarrow I_{i}\right)$ | Experimental <br> B(E2) values [15] | Calculated <br> B(E2) values |
| :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | $1.05 \pm 0.01$ | 1.05 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ | $1.50 \pm 0.07$ | 1.51 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ | $1.58 \pm 0.08$ | 1.71 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ | $1.66 \pm 0.09$ | 1.85 |
| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ | $2.17 \pm 0.22$ | 1.98 |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ | $1.74 \pm 0.22$ | 2.09 |

Table 12: Comparison of experimental and calculated $B[E 2$; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{168} \mathrm{Er}$. $\mathrm{B}(\mathrm{E} 2)$ values are in units of $\mathrm{e}^{2} b^{2}$

| Spin <br> $(\mathrm{I})$ | Transition <br> $\left(I_{f} \rightarrow I_{i}\right)$ | Experimental <br> $\mathrm{B}(\mathrm{E} 2)$ values [16] | Calculated <br> $\mathrm{B}(\mathrm{E} 2)$ values |
| :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | $1.14(6)$ | 1.14 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ | $1.75(12)$ | 1.64 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ | $2.42(30)$ | 1.84 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ | $1.93(20)$ | 1.96 |
| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ | $1.66(21)$ | 2.07 |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ |  | 2.17 |
| $12^{+}$ | $14^{+} \rightarrow 12^{+}$ |  | 2.27 |
| $14^{+}$ | $16^{+} \rightarrow 14^{+}$ |  | 2.40 |

Table 13: Comparison of experimental and calculated $B[E 2$; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{172} \mathrm{Yb}$. B(E2) values are in units of $e^{2} b^{2}$

| Spin <br> (I) | Transition <br> $\left(I_{f} \rightarrow I_{i}\right)$ | Experimental <br> B(E2) values [16] | Calculated <br> B(E2) values |
| :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | $1.20(2)$ | 1.20 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ | $1.71(20)$ | 1.73 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ | $1.82(3)$ | 1.95 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ | $2.27(4)$ | 2.10 |
| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ | $2.13(23)$ | 2.21 |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ |  | 2.32 |
| $12^{+}$ | $14^{+} \rightarrow 12^{+}$ |  | 2.43 |
| $14^{+}$ | $16^{+} \rightarrow 14^{+}$ |  | 2.53 |

Table 14: Comparison of experimental and calculated $B[E 2$; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{174} \mathrm{Yb} . \mathrm{B}(\mathrm{E} 2)$ values are in units of $\mathrm{e}^{2} \mathrm{~b}^{2}$

| Spin <br> (I) | Transition <br> $\left(I_{+} \rightarrow I_{i}\right)$ | Experimental <br> B(E2) values [17] | Calculated <br> B(E2) values |
| :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | $1.16(7)$ | 1.16 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ | $1.62(9)$ | 1.67 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ | $2.13(50)$ | 1.87 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ | $2.24(21)$ | 2.01 |


| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ | $1.88(22)$ | 2.12 |
| :---: | :---: | :---: | :---: |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ | $2.13(23)$ | 2.23 |
| $12^{+}$ | $14^{+} \rightarrow 12^{+}$ | 1.85 | 2.33 |
| $14^{+}$ | $16^{+} \rightarrow 14^{+}$ |  | 2.45 |
| $16^{+}$ | $18^{+} \rightarrow 16^{+}$ |  | 2.55 |
| $8^{+}$ | $20^{+} \rightarrow 18^{+}$ |  | 2.66 |

Table 15: Comparison of experimental and calculated $\mathrm{B}[\mathrm{E} 2$; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{130} \mathrm{U}$.
Experimental $B(E 2)$ values for ${ }^{130-136} U$ are in units of $e^{2} b^{2}$ and taken from Refs. [18-21]

| Spin <br> $(\mathrm{I})$ | Transition <br> $\left(I_{f} \rightarrow I_{i}\right)$ | Experimental <br> B(E2) values | Calculated <br> B(E2) values | Calculated B(E2) <br> values [PSM] [22] |
| :---: | :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | $1.92(23)$ | 1.92 | 1.98 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ |  | 2.79 | 2.84 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ |  | 3.22 | 3.15 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ |  | 3.52 | 3.33 |
| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ |  | 3.81 | 3.46 |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ |  | 4.09 | 3.57 |
| $12^{+}$ | $14^{+} \rightarrow 12^{+}$ |  | 4.35 | 3.66 |
| $14^{+}$ | $16^{+} \rightarrow 14^{+}$ |  | 4.62 | 3.74 |
| $16^{+}$ | $18^{+} \rightarrow 16^{+}$ |  | 4.92 | 3.78 |
| $18^{+}$ | $22^{+} \rightarrow 18^{+}$ |  | 5.19 | 0.052 |
| $20^{+}$ | $22^{+} \rightarrow 20^{+}$ |  | 5.47 |  |

Table 16: Comparison of experimental and calculated $\mathrm{B}[\mathrm{E} 2$; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{132} \mathrm{U}$.

| Spin <br> (I) | Transition <br> $\left(I_{+} \rightarrow I_{i}\right)$ | Experimental <br> B(E2) values | Calculated <br> B(E2) values | Calculated B(E2) <br> values [PSM][22] |
| :---: | :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | $2.11(18)$ | 2.11 | 2.00 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ |  | 3.08 | 2.87 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ |  | 3.48 | 3.18 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ |  | 3.77 | 3.36 |
| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ |  | 4.06 | 3.48 |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ |  | 4.33 | 3.58 |
| $12^{+}$ | $14^{+} \rightarrow 12^{+}$ |  | 4.59 | 3.66 |
| $14^{+}$ | $16^{+} \rightarrow 14^{+}$ |  | 4.87 | 3.71 |
| $16^{+}$ | $18^{+} \rightarrow 16^{+}$ |  | 5.14 | 3.60 |
| $18^{+}$ | $20^{+} \rightarrow 18^{+}$ |  | 5.41 | 2.58 |

Table 17: Comparison of experimental and calculated $\mathrm{B}[\mathrm{E} 2$; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{134} \mathrm{U}$.

| Spin <br> (I) | ransition <br> $\left(I_{f} \rightarrow I_{i}\right)$ | experimental <br> B(E2) values | Calculated <br> B(E2) values | Calculated B(E2) <br> values [PSM] [22] |
| :---: | :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | $2.09(08)$ | 2.09 | 2.20 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ |  | 3.02 | 3.15 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ |  | 3.43 | 3.48 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ |  | 3.71 | 3.66 |
| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ |  | 3.97 | 3.77 |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ |  | 4.24 | 3.85 |
| $12^{+}$ | $14^{+} \rightarrow 12^{+}$ |  | 4.49 | 3.90 |
| $14^{+}$ | $16^{+} \rightarrow 14^{+}$ |  | 4.75 | 3.92 |
| $16^{+}$ | $18^{+} \rightarrow 16^{+}$ |  | 4.99 | 3.90 |
| $18^{+}$ | $20^{+} \rightarrow 18^{+}$ |  | 5.23 | 3.91 |

Table 18: Comparison of experimental and calculated B[E2; $\mathrm{I}+2 \rightarrow \mathrm{I}]$ for the ground state band transitions in ${ }^{136} \mathrm{U}$.

| ppin <br> (I) | Transition <br> $\left(I_{f} \rightarrow I_{i}\right)$ | Experimental <br> $\mathrm{B}(\mathrm{E} 2)$ values | Calculated <br> $\mathrm{B}(\mathrm{E} 2)$ values | Calculated B(E2) <br> values [PSM] [22] |
| :---: | :---: | :---: | :---: | :---: |
| $0^{+}$ | $2^{+} \rightarrow 0^{+}$ | $2.25(08)$ | 2.25 | 2.28 |
| $2^{+}$ | $4^{+} \rightarrow 2^{+}$ | $3.21(20)$ | 3.24 | 3.26 |
| $4^{+}$ | $6^{+} \rightarrow 4^{+}$ | $3.46(19)$ | 3.65 | 3.61 |
| $6^{+}$ | $8^{+} \rightarrow 6^{+}$ | $3.50(35)$ | 3.91 | 3.79 |
| $8^{+}$ | $10^{+} \rightarrow 8^{+}$ | $3.23(35)$ | 4.17 | 3.91 |
| $10^{+}$ | $12^{+} \rightarrow 10^{+}$ | $3.68(62)$ | 4.41 | 4.01 |
| $12^{+}$ | $14^{+} \rightarrow 12^{+}$ | $4.04(4)$ | 4.65 | 4.08 |


| $14^{+}$ | $16^{+} \rightarrow 14^{+}$ | $3.41(35)$ | 4.92 | 4.13 |
| :---: | :---: | :---: | :---: | :---: |
| $16^{+}$ | $18^{+} \rightarrow 16^{+}$ | $4.402(04)$ | 5.19 | 4.12 |
| $18^{+}$ | $20^{+} \rightarrow 18^{+}$ | $4.58(07)$ | 5.48 | 3.85 |
| $20^{+}$ | $22^{+} \rightarrow 20^{+}$ |  | 5.76 |  |
| $22^{+}$ | $24^{+} \rightarrow 22^{+}$ |  | 6.08 |  |
| $24^{+}$ | $26^{+} \rightarrow 24^{+}$ |  | 6.34 |  |
| $26^{+}$ | $28^{+} \rightarrow 26^{+}$ |  | 6.57 |  |
| $28^{+}$ | $30^{+} \rightarrow 28^{+}$ |  | 6.81 |  |

## 4. Discussion

From table 2, one notes that in ${ }^{128} \mathrm{Ce}$ the calculated B(E2) values for the low lying transitions up to $4^{+}$state are in satisfactory agreement with the experimental values. However, for higher transitions the Grodzins relation gives larger values than the observed ones. Similar observations are made for ${ }^{130} \mathrm{Ce}$. Here again the calculated $\mathrm{B}(\mathrm{E} 2)$ values for higher transitions are much larger compared to the experimentally observed values. For example, the experimentally measured value for $10^{+} \rightarrow 8^{+}$transition is 0.54 , whereas the calculated value based on the Grodzins relation is 1.498 . For neutron-rich nuclei like ${ }^{150} \mathrm{Nd}$, the calculated B (E2) values show a reasonably good agreement with the experimental values for transitions up to $6^{+}$. However, as can be seen from tables 5 and 6 , the overall agreement between the calculated and the experimental $B(E 2)$ values for the transitions upto $10^{+}$are satisfactorily reproduced. Here, it may be noted that for these two nuclei, the ratios $\mathrm{E}_{4} / \mathrm{E}_{2}$ values are greater or equal to 3.0. From what has been said earlier, it seems that generalized Grodzins relation reproduced satisfactorily agreement for $B$ (E2) transition probability values in nuclei which are nearly rigid rotators and have $E_{4} / E_{2}$ in excess of 3.0. When $E_{4} / E_{2}$ value falls, then the agreement between $B$ (E2) values predicted by generalized Grodzins relation and experimental $B(E 2)$ becomes poor, especially for the higher transitions. This fact is clear from the data presented in table 7 for ${ }^{152} \mathrm{Gd}$. For this nucleus $\mathrm{E}_{4} / \mathrm{E}_{2}=2.19$ and $\mathrm{B}(\mathrm{E} 2)$ values agree up to $2^{+}$ and after that there is disagreement between calculated and experimentally measured $B(E 2)$ values. Above discussion is further substantiated by the data presented on ${ }^{154,156} \mathrm{Gd}$ isotopes, ${ }^{156,162} \mathrm{Dy},{ }^{168} \mathrm{Er},{ }^{172,174} \mathrm{Yb}$ and ${ }^{130-136} \mathrm{U}$. In ${ }^{130-136} \mathrm{U}$ isotopes, the available experimental data on $\mathrm{B}(\mathrm{E} 2)$ values are not available except for first transition. In these isotopes calculated values are compared with another set theoretical $B(E 2)$ values and the results are in reasonable agreement with each other. One major limitation of this generalized Grodzins relation is that, it is unable to reproduce the experimental data where there is sudden change (decrease) in measured $B$ (E2) values on account of structure change with the increase in angular momentum, and this is evident from the tables 6 and 11-14.
be greater than 3.0. Nuclei in which this ratio is between 2 and 3 do not give as good agreement with Grodzins relation for the higher transitions.

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## 5. Conclusion

From the discussion and comparison of the calculated intrastate $\mathrm{B}(\mathrm{E} 2)$ transition probabilities, it can be concluded that generalized Grodzins relation produced satisfactory agreement with experiments for low lying transitions. For higher transitions the agreement between the calculated values and experimentally measured values for $\mathrm{B}(\mathrm{E} 2)$ is good in those nuclei, which are axially deformed and behave like rigid rotators. For such nuclei, the ratio of $E_{4} / E_{2}$ should

