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Two Fluid Anisotropic Model with Hybrid Scale Factor

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Abstract: ***We have presented two fluid Bianchi Type III Cosmological Model coupled with two noninteracting fluids, namely, bulk viscous and dark energy fluid. In order to obtain deterministic solution of the field equations we determine the Bianchi-III space-time by considering the mixture of exponential and hyperbolic scale factor to find the physical parameters and metric potentials defined in the space-time. Furthermore, we have discussed the physical behaviours of the models. Consequently, it is believed that the models obtained are physically acceptable models of the universe.***

Keywords: Bianchi type III, Bulk viscous fluid, dark energy, Hybrid Scale Factor

1. Introduction

In an attempt to produce Dark Energy (DE), in 1980 energy components might occur and their role as possible DE candidates was raised by Caldwell at the end [1]. In theory, despite the observational constraints, extensions of general relativity are the prime candidate class of theories consistent with PDL crossing [2]. On the other hand, while the current cosmological data from SNIa (Supernova Legacy Survey, Gold sample of Hubble Space Telescope) [3, 4], CMB (WMAP, BOOMERANG) [5, 6] and large scale structure (SDSS) [7] data rule out the $w \ll -1$, they mildly favour dynamically evolving DE crossing the PDL (see [8, 9], [10, 11] for theoretical and observational status of crossing the PDL).

The theory of DE is a widely accepted theory to describe the observed acceleration expansion of the Universe. But a complete understanding on the nature of dark energy is still remain a challenge to the researchers. Recent Planck results estimate a maximum contribution of 68% for DE in the cosmic mass energy budget [12]. Observations have confirmed that the cosmic speed up is a late time phenomena and has occurred at a redshift of the order $z_t \sim 0.7$. This indicates that, the Universe has undergone a transition from a decelerated phase of expansion to an accelerating phase in the recent past. This cosmic transit phenomenon speculates an evolving deceleration parameter with a signature flipping. The rate at which the transition occurs usually determines the transit redshift z_t.

In fact this isotropization of the Bianchi metrics is due to the implicit assumption that the DE is isotropic in nature. If the implicit assumption that the pressure of the DE is direction independent is relaxed, the isotropization of the Bianchi metrics can be fine tuned to generate arbitrary ellipsoidality (eccentricity). Therefore, the CMB anisotropy can also be fine tuned, since the Bianchi universe anisotropies determine the CMB anisotropies. The price of this property of DE is a violation of the null energy condition (NEC) since the DE crosses the Phantom Divide Line (PDL), in particular depending on the direction [13].

The simplest candidate for dark energy is the energy density of the quantum vacuum (or cosmological constant) for which $p = -\rho$. However, the inability of particle theorists to complete the energy of the quantum vacuum - contributions from well understood physics amount to 1055 times critical density - casts a dark shadow on the cosmological constant. In addition to this, a number of viable models for DE have been fabricated. These scenarios include, quintessence [14, 15], modified gravity [16]−[22], tachyon [23], cosmological nuclear energy [24], equation of state (EoS) parameter [25]−[35], braneworld [36, 37] and interacting dark energy models [38]−[44]. Therefore some form of dark energy whose fractional energy density is about $\Omega(de) = 0.70$ must exist in the Universe to drive this acceleration. This fact can be put in agreement with the theory, if one assumes that the Universe is basically filled with so-called dark energy. Evolution of the equation of state (EoS) of dark energy $\omega\,(de) = p\,(de)\,\rho(de)$ transfers from $\omega\,(de) > -1$ in the near past (quintessence region) to $\omega\,(de) < -1$ at recent stage (phantom region) [45]−[47]. So another cosmological coincidence problem may be proposed: why $\omega\,(de) = 1$ crossing is occurred at the present time [48]. When SNe results are combined with five-year WMAP, it is found that $-1.38 < \omega(de) < 0.86$ [49]−[51]. For recent review, the readers are advised to see the references of Padmanabhan [52], Copeland et al. [53], Perivolaropoulos [54], Jassal et al. [55] and Miao et al. [56].

In earlier, Xin [57] studied an interacting two-fluid scenario for quintom dark energy. Xin-He et al. [58] considered Friedmann cosmology with a generalized EoS and bulk viscosity to explain DE dominated universe. Recently, several authors [59]−[65] have examined and discussed the DE models in different context of use. Letelier [66] has examined some two-fluid cosmological models, which have similar symmetries to those Bianchi type-III models, where the distinct four-velocity vectors of the two non-interacting perfect fluids generate an axially symmetric anisotropic pressure.

In mixed fluid environment such as dark fluid matter along with usual ordinary matter (Baryonic matter), a number of literature has motivated the researchers to investigate different models in the back drop of General Relativity with different Bianchi forms.

In the present paper we have used anisotropic Bianchi type III space time. Many researchers are using Bianchi type III space time because this space time model allows not only expansion but also rotation and shear. Amirhashchi and Pradhan [67] have investigated Dark Energy model in general theory of relativity. DE models with constant deceleration parameter have been constructed and investigated by Akarsu and Kilinc [68, 69] for Bianchi type I and III space time. With a variable Equation of State (EoS) parameter, Yadav et al. [70] constructed BV DE cosmological models where the deceleration parameter was assumed to be constant. Several theoretical two fluids DE models either interacting or non-interacting have been discussed widely in the literature [71–75]. Mishra et al. [76, 77] have constructed DE cosmological models with two non interacting fluid situations such as DE fluid with cosmic string and nambu string. In both the models, they have shown that the models are mostly dominated by Phantom behaviour. In a similar approach of two fluid, DE cosmological models were constructed in different general scale factors [78]. Pawar and Dagwal [79] investigated Bianchi type IX two fluids cosmological models in general relativity. Most of the models have been studied by considering perfect fluid but it is not sufficient to describe the dynamics of an accelerating universe. This motivates the researchers to consider the models of the universe filled with Bulk Viscous and DE. In the present paper we have investigated Bianchi type III anisotropic space timecoupled with two noninteracting fluids, namely, bulk viscous and dark energy fluid. With this motivation, here we have considered the BIII space time as

$$ds^2 = -dt^2 + A^2dx^2 + e^{-2\alpha x}B^2dy^2 + C^2dz^2, \quad (1)$$

here A, B, C are cosmic scale factors are the functions of cosmic time t.

The exponent $\alpha \neq 0$ in (1) is an arbitrary constant. The total energy momentum tensor (EMT) in presence of both the viscous and DE fluids can be expressed as,

$$T_{ij} = T_{ij}^{vis} + T_{ij}^{de} \quad (2)$$

where, EMT of barotropic bulk viscous fluid is

$$T_{ij}^{vis} = (\rho + \bar{p})u_i u_j - \bar{p}g_{ij} \quad (3)$$

and EMT of DE fluid is

$$T_{ij}^{de} = diag[-\rho_{DE}, p_{DE(x)}, p_{DE(y)}, \ p_{DE(z)}]$$
$$T_{ij}^{de} = diag[-1, \omega_{DE(x)}, \omega_{DE(y)}, \ \omega_{DE(z)}]\rho_{DE}$$
$$T_{ij}^{de} = diag[-1, (\omega_{DE} + \delta), (\omega_{DE} + \gamma), (\omega_{DE} + \eta)]\rho_{DE} \quad (4)$$

Here, u_i is the four velocity vector of the fluid in a comoving coordinate system. ω_{DE} and ρ_{DE} are respectively the EoS parameter of the DE fluid and DE density parameter. The skewness parameters δ on x-axis, γ from y-axis and η on z-axis are deviations from the EoS parameter ω_{DE} on these three directions. With these considerations on the parameters, in the subsequent section, we have developed the mathematical formalism of the problem.

In Sect. 2, the basic equations for BIII space time in presence of viscous fluid and DE fluid are formulated along with the physical and kinematic parameters. In Sect. 3, the scale factor known as hybrid is used to obtain a viable solution. The physical importance of the scale factor also discussed. Moreover, the characteristics of deceleration parameter is presented w.r.t. the hybrid scale factor. The functional form of the skewness parameter and EoS parameter are expressed. At the end, Conclusion is presented in Sect. 4.

2. Basic Formalism of the Model

In the two fluid description of the total energy momentum tensor (EMTs) as discussed in the previous section, Einstein's field equations of General Relativity, for the space–time (1) can be calculated as

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = -p + 3\varsigma H - (\omega_{DE} + \eta)\rho_{DE} \quad (5)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -p + 3\varsigma H - (\omega_{DE} + \delta)\rho_{DE} \quad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -p + 3\varsigma H - (\omega_{DE} + \gamma)\rho_{DE} \quad (7)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = \rho + \rho_{DE} \quad (8)$$

$$\alpha\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0 \quad (9)$$

where an over dot represents the derivatives of corresponding field variable with respect to t. It can be noted that the product of the field variables $A, B\ and\ C$ gives the volume scale factor from where the average scale factor can be deduced as $R = V^{\frac{1}{3}}$. If $H_1, H_2\ and\ H_3$respectively denotes the Hubble parameter in the direction of $x, y\ and\ z$ respectively, then the mean Hubble parameter

$$H = \sum H_i = \frac{\dot{R}}{R}, where\ i = 1, 2, 3. \quad (10)$$

The proper pressure p, in case of barotropic cosmic fluid, is given as, $p = \xi\rho, (0 \leq \xi \leq 1)$. Moreover, the bulk viscosity related to energy density with the help of Hubble's parameter as $3\zeta H = \varepsilon_0\rho$. So, the effective pressure which is the mixture of proper pressure and barotropic bulk viscous pressure can be written as,

$$\bar{p} = p - 3\zeta H = (\xi - \varepsilon_0)\rho = \varepsilon\rho, \quad (11)$$

where, can be considered as effective viscous coefficient. It can be noted that, $3\zeta H$ is the bulk viscous pressure which is considered in the present work to be a barotropic one. Now, replacing pressure terms $(p - 3\zeta H)$ in field equations as $\bar{p}$ and framing field variables in terms of Hubble parameter, we obtain

$$\frac{6k}{(2k+1)}\dot{H} + \frac{27k^2}{(2k+1)^2}H^2 - \frac{\alpha^2}{A^2} = -\bar{p} - (\omega_{DE} + \eta)\rho_{DE} \quad (12)$$

$$\frac{3(k+1)}{(2k+1)}\dot{H} + \frac{9(k^2+k+1)}{(2k+1)^2}H^2 = -\bar{p} - (\omega_{DE} + \delta)\rho_{DE} \quad (13)$$

$$\frac{3(k+1)}{(2k+1)}\dot{H} + \frac{9(k^2+k+1)}{(2k+1)^2}H^2 = -\bar{p} - (\omega_{DE} + \gamma)\rho_{DE} \quad (14)$$

$$\frac{9k(k+2)}{(2k+1)^2}H^2 - \frac{\alpha^2}{A^2} = \rho + \rho_{DE} \quad (15)$$

The energy conservation equation for viscous fluid, $T^{ij\,(vis)}_{;j} = 0$, and dark energy fluid $T^{ij\,(de)}_{;j} = 0$ can be obtained respectively as

$$\dot{\rho} + 3(\bar{p} + \rho)H = 0 \tag{16}$$

$$\dot{\rho}_{DE} + 3\rho_{DE}(\omega_{DE} + 1)H + \rho_{DE}(\delta H_1 + \gamma H_2 + \eta H_3) = 0 \tag{17}$$

The spatial volume V and the average Hubble's parameter H are defined as

$$V = a^3 = ABC, \tag{18}$$

$$3H = \frac{\dot{V}}{V} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \tag{19}$$

The shear scalar σ and anisotropy parameter Am are defined as follows

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{1}{6}\theta^2 \tag{20}$$

$$Am = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\triangle H_i}{H}\right)^2 \tag{21}$$

where $\triangle H_i = H_i - H, (i = 1, 2, 3)$ and $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters.

From (16), incorporating the relation between Hubble parameter and scale factor, we get the energy density for the matter field as,

$$\rho = \frac{\rho_0}{\left[e^{\int H dt}\right]^{3(\varepsilon+1)}}, \tag{22}$$

where ρ_0 is the integration constant or rest energy density of present time.

From (15) and (22), we can retrieve, the DE density as,

$$\rho_{DE} = \frac{9k(k+2)}{(2k+1)^2}H^2 - \frac{\rho_0}{\left[e^{\int H dt}\right]^{3(\varepsilon+1)}} - \frac{\alpha^2}{A^2} \tag{23}$$

With the help of the second part of conservation equation (17), which corresponds to the deviation of equation of the state parameter, we formalize the EoS parameter of DE as,

$$\omega_{DE} = -\frac{1}{\rho_{DE}}\left[\frac{6k}{(2k+1)}\dot{H} + \frac{27k^2}{(2k+1)^2}H^2 + \frac{\varepsilon\rho_0}{\left[e^{\int H dt}\right]^{3(\varepsilon+1)}} - \frac{\alpha^2}{A^2}\right] \tag{24}$$

With the help of Eqs. (23)–(24), Eqs. (12)–(14) can be formulated as:

$$\delta = \gamma = \frac{1}{(2k-1)\rho_{DE}}\left[\frac{3(k-1)}{(2k+1)}\dot{H} + \frac{9(2k^2-k-1)}{(2k+1)^2}H^2 - \frac{\alpha^2}{A^2}\right] \tag{25}$$

$$\eta = \frac{-2k}{(2k-1)\rho_{DE}}\left[\frac{3(k-1)}{(2k+1)}\dot{H} + \frac{9(2k^2-k-1)}{(2k+1)^2}H^2 - \frac{\alpha^2}{A^2}\right] \tag{26}$$

Scalar Expansion θ and Shear Scalar σ can be defined respectively as

$$\theta = 3H \tag{27}$$

$$\sigma^2 = \frac{1}{2}\left[\frac{(2k^2+1)}{(2k+1)^2} - \frac{1}{6}\right]9H^2 \tag{28}$$

3. Hybrid Scale Factor and Anisotropy Effect

The variation of Hubble law may not be consistent with observations. However, it has the advantage of providing simple functional form of the time evolution of the scale factor. Also, in order to explain the late time cosmic acceleration a simple parametrization of Hubble parameter is needed. Hence, in order to construct the cosmological models of the universe, in this section, we have considered the scale factor which is a combination of two factors: one is the generalized form of exponential function and the second one is hyperbolic function. The scale factor can be defined as

$$a = e^{mt}\,\mathrm{sech}(nt) \tag{29}$$

So, the Hubble parameter becomes

$$H = m - n\tanh(nt) \tag{30}$$

The directional Hubble parameters take the form

$$H_1 = H_2 = \frac{3k}{(2k+1)}[m - n\tanh(nt)], \quad H_3 = \frac{3}{(2k+1)}[m - n\tanh(nt)] \tag{31}$$

Using (29) and (30) Bianchi type III cosmological model in (1) takes the form

$$ds^2 = -dt^2 + \beta^{2k}e^{\frac{6mkt}{(2k+1)}}[\mathrm{sech}(nt)]^{\frac{6k}{(2k+1)}}dx^2 + e^{-2\alpha x}\beta^{2k}e^{\frac{6mkt}{(2k+1)}}[\mathrm{sech}(nt)]^{\frac{6k}{(2k+1)}}dy^2 + \beta^2 e^{\frac{6mt}{2k+1}}[\mathrm{sech}(nt)]^{\frac{6}{(2k+1)}}dz^2 \tag{32}$$

With the hybrid scale factor the energy density of the matter from (22) would be

$$\rho = \frac{\rho_0[\cosh(nt)]^{3(\varepsilon+1)}}{e^{3(\varepsilon+1)mt}} \tag{33}$$

Subsequently, DE density and the effective EoS parameter can be written as,

$$\rho_{DE} = \frac{9k(k+2)}{(2k+1)^2}[m - n\tanh(nt)]^2 - \frac{\rho_0[\cosh(nt)]^{3(\varepsilon+1)}}{e^{3(\varepsilon+1)mt}} - \frac{\alpha^2}{\beta^{2k}e^{\frac{6mkt}{(2k+1)}}[\mathrm{sech}\,(nt)]^{\frac{6k}{(2k+1)}}} \tag{34}$$

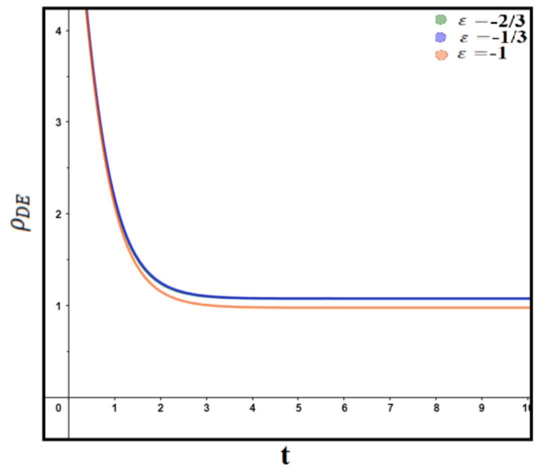

Figure 1: DE density parameter vs. Time

In fig (1), We have observed, respectively, that the dark energy density ρ_{DE} remain positive till the phase of cosmic evolution for the representative value of the constants $(m = 1.5, n = 0.9, k = 1.1, \ (\varepsilon = -1, -\frac{1}{3}, -2/3), \rho_0 = 0.1, \alpha = 0.1, \beta = 0.5)$. Hence, it indicates that both weak energy condition (WEC) and null energy condition (NEC) are satisfied in the derived model. Further, ρ_{DE} decrease with increase in time and slowly reach small positive values in the present epoch, which indicates that the considered two fluids affect the dark energy density. It is worth noting here that, irrespective of the value of the viscous coefficient ε, the behaviour of ρ_{DE} remains alike. So, in fig (1), we have chosen the value of the viscous coefficient to be $-1. -1/$

$3, -2/3$. However, a small effect of viscous fluid in dark energy density cannot be ruled out.

$$\omega_{DE} = -\frac{1}{\rho_{DE}}\Bigg[-\frac{6kn^2}{(2k+1)}[\text{sech}(nt)]^2 + \frac{27k^2}{(2k+1)^2}[m - n\tanh(nt)]2+\varepsilon\rho0[\cosh(nt)]3(\varepsilon+1)e3(\varepsilon+1)mt-\alpha2\beta 2ke6mkt(2k+1)\text{sech}\, nt6k(2k+1) \quad (35)$$

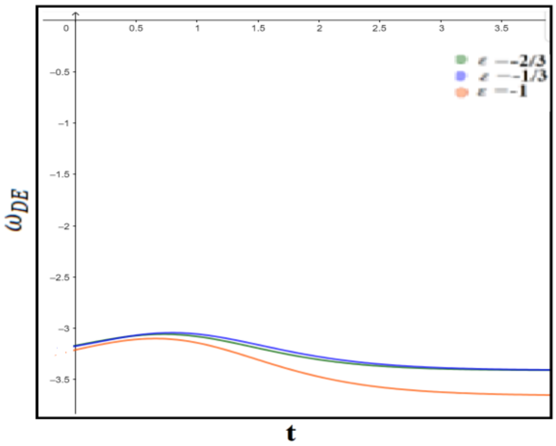

Figure 2: DE EoS parameter vs. time for different viscous coefficient

Fig.(2) represents variation of EoS parameter (ω_{DE}) w.r.t appropriate choices of bulk viscous coefficients $\varepsilon = -1, -\frac{1}{3}, -2/3$. The behaviour of EoS parameters are directly proportional to the increasing values of viscous coefficient. At early phase of cosmic evolution, the EoS parameters gathered some amount of energy; however, at late phase, they behave differently. The reason why the dynamics of EoS are greatly affected at early phases is that the bulk viscous fluid has a substantial contribution to the density parameter at that corresponding phase. But, at late phase, the dark energy dominates in spite of the presence of bulk viscous fluid. Hence, cosmic bulk viscous fluid has a very little impact on the dynamics of EoS parameter. The EoS parameter starts in the negative region and stays in the acceptable range [80]. Hence, it can be infer that the matter evolves in the quintessence region at early time and remains in the same domain till late phase of cosmic time and approaching towards phantom barrier. The parameter evolves dynamically with the expansion of the universe. This is basically governed by the rest energy density that appears in the denominator of eqn. (35).

Similarly, the skewness parameters can be expressed as,

$$\delta = \gamma = \frac{1}{(2k-1)\rho_{DE}}\Bigg[\frac{-3(k-1)n^2}{(2k+1)}[\text{sech}(nt)]^2 + \frac{9(2k^2-k-1)}{(2k+1)^2}[m - n\tanh(nt)]2-\alpha2\beta2ke6mkt(2k+1)\text{sech}\, nt6k(2k+1) \quad (36)$$

$$\eta = \frac{-2k}{(2k-1)\rho_{DE}}\Bigg[\frac{-3(k-1)n^2}{(2k+1)}[\text{sech}(nt)]^2 + \frac{9(2k^2-k-1)}{(2k+1)^2}[m - n\tanh(nt)]2-\alpha2\beta2ke6mkt(2k+1)\text{sech}\, nt6k(2k+1) \quad (37)$$

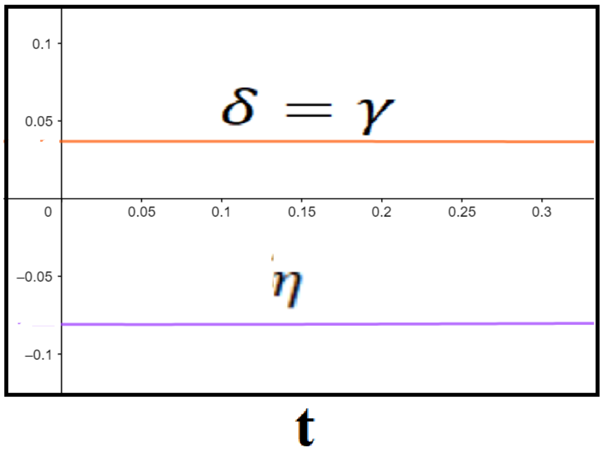

Figure 3: DE skewness parameters vs. time

The plot for the skewness parameters and cosmic time has been represented in fig (3). The corresponding skewness parameters are shown for bulk viscous coefficient $\varepsilon = -2/3$.It is also very much notable that the skewness parameters γ and η are found to be like mirror image of each other. It is certain that, the behaviour of η is just opposite to that of γ. γ is positive whereas η is negative all through the cosmic evolution. This may be due to the fact that the anisotropic parameter $k = 1.1$, which will describe the small anisotropic nature of the universe. The constants (model parameters) are adjusted in such a way that the present numerical values of the cosmological parameters lie in the neighbourhood of predicted values by the observational data.δ, γ 's are less affected by the presence of cosmic fluid compared to η's . The DE pressures along directions x and z are mostly affected. The reason behind the sensitivity may be due to the consideration of assuming mean Hubble parameter the same as directional Hubble parameter along x, y axis. Due to presence of bulk viscous fluid, the anisotropy in DE pressure continues along with the cosmic expansion and decreases slowly at the later period as shown in fig (3).

The scalar expansion and shear scalar are respectively given by,

$$\theta = 3[m - n\tanh(nt)] \quad (38)$$

The shear scalar σ and anisotropy parameter Am can be expressed as

$$\sigma^2 = \frac{9(8k^2-4k+5)}{2(2k+1)^2}[m - n\tanh(nt)]^2 \quad (39)$$

$$Am = 6\left[\frac{(k-1)}{(2k+1)}\right]^2 \quad (40)$$

The deceleration parameter $q = -\left[\frac{\dot{H}+H^2}{H^2}\right]$describes the cosmic dynamics of universe. Positive value of it indicates decelerating universe where as negative values confirms the accelerated expansion of the universe. In view of the observations of high red shift supernova, Type Ia supernova observations combined with BAO and CMB, models transiting from early decelerating universe to late time accelerating universe gained much importance in recent times. The deceleration parameter for the hybrid scale factor, turns to be

$$q = -\left[\frac{(m^2-n^2)+2n\tanh(nt)(n\tanh(nt)-m)}{[m-n\tanh(nt)]^2}\right] \quad (41)$$

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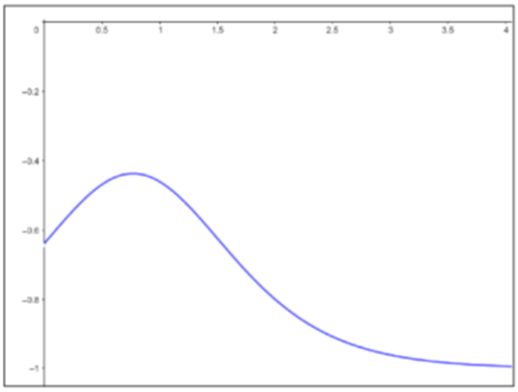

Figure 4: Deceleration parameter vs. time

According to recent observational data at present time, the most favourable value for q to be -0.81 ± 0.14. The deceleration parameter in fig (4) lies. Though initially, it rises rapidly but at late times it decreases rapidly. At present time, the deceleration parameter value found to be in $(-0.4 \leq q \leq -1)$, which is in alignment with the observational data.The geometrical behaviour of the DE model can be assessed through the state finder diagnostic pair (r, s). The acceptability of corresponding DE Hybrid (DEH) model can be decided through the (r, s) diagnosis comparing with the standard ΛCDM model. Hence, we have analysed the evolutionary behaviour of both the parameters r and s for the DE universe along with ΛCDM universe. Both parameters evolve continuously with time from big bang time (t ⟶ 0) to large value at late time (t ⟶ ∞). The pair can be obtained as,

$$r = 1 - \frac{3n^2[\text{sech}\,(nt)]^2}{[m - n\tanh\,(nt)]^2} + \frac{2n^3[\text{sech}\,(nt)]^2\tanh\,(nt)}{[m - n\tanh\,(nt)]^3} \qquad (43)$$

$$s = \frac{2}{3}\,\frac{[3n^2[\text{sech}\,(nt)]^2[m - n\tanh\,(nt)]^2 - 2n^3[\text{sech}\,(nt)]^2\tanh\,(nt)]}{[m - n\tanh\,(nt)][2(m^2 - n^2) + 4\tanh\,(nt)(\tanh\,(nt) - mn) + (m - n\tanh\,(nt))^2]} \qquad (44)$$

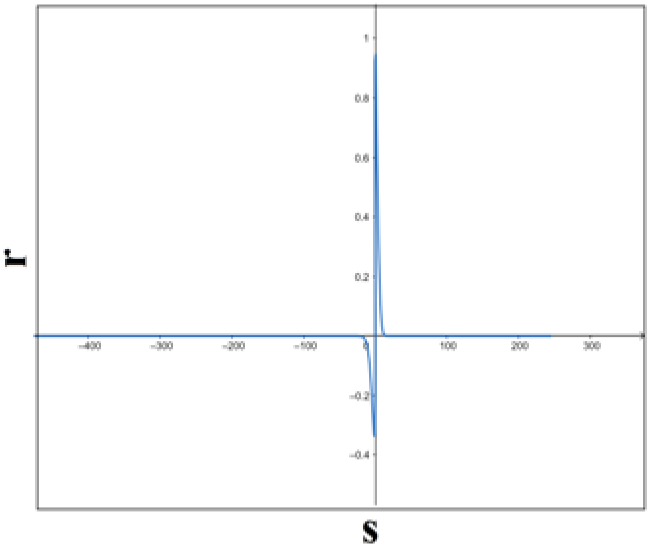

Figure 5: The variation of r vs. s

From Fig. 5, we observe that s is negative when $r \geq 1$. The figure shows that the universe starts from an asymptotic Einstein static era $(r \rightarrow \infty, s \rightarrow -\infty)$ and goes to the ΛCDM model $(r = 1, s = 0)$.

4. Conclusion

The article represent two Fluid cosmological scenario with the use of exponential and hyperbolic scale factor.We have investigated the anisotropic behaviour of the cosmological model constructed in a two fluid situations: the usual bulk viscous fluid and DE fluid. A more systematic mathematical formulation has been developed to express the physical, kinematical parameters as well the metric potentials involved in the study. Along, x, y direction, the anisotropy in DE pressure increases on the pressure anisotropy and along z-direction decreases at late times. Presence of viscous fluid substantially affects the DE density at early phase of cosmic evolution; however at late phase DE density dominates over viscous fluid. Also, our model is scale factor dependent and may change its behaviour in different scale factors; however, the formalism developed here clearly indicates the accelerating behaviour of the expanding universe. Moreover, there is resemblance of data of considered model with standard ΛCDM model; our model found here also aligned with the present day observational outcomes.

The main features of the model are as follows:

- For $k = 1$ the anisotropic parameter Am tends to zero. Hence, the present model is isotropic at $k = 1$.
- The derived DE model represents an acceleration universe (see, fig (4)) which is in good agreement with recent observations [3].
- The dark energy density ρ_{DE} approaches to 1 for sufficiently large time (see, fig (1)) which is reproducible with current observations.
- The Statefinder pair $\{r, s\}$ enable the behaviour of different stages of the evolution of the universe i.e. the universe starts from asymptotic Einstein static era $(r \rightarrow \infty, s \rightarrow -\infty)$ and goes to Λ CDM model $(r = 1, s = 0)$.

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