# Sidereal Day without Polar Star "A Complete Solution with an Otherwise Explanation" 

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#### Abstract

We are familiar that Earth make rotation about its axis of rotation in 24 hours, but as per microscopic observation Earth completes its rotation cycle just in 23 hrs 56 minutes 04seconds. The periodic time for rotation of earth 24 hours is known as mean solar day but periodic time for rotation cycle of earth 23 hours 56 minutes 04 seconds is known as the sidereal day. However it is a one of the oldest astronomical observation of astronomy, but still till date is not defined. The exact cause behind occurrence of minute difference between mean solar day and sidereal day is still unknown. However some definitions are available in textbook and some online site of astronomy but these definitions are totally based on considering a solar star, which seems very impractical for understand and study this observation for a common student of this subject, and due to this reason it is also called an astronomical puzzle. Because our earth is a integral part of our solar system and occurrence of any kind of discrimination in rotational period of earth should be anyway related with the sun only, not due to existence of any another star in galaxy. Therefore this research paper is written with considering only our Earth and it's any characteristic feature responsible for any discrimination, Sun, and its family. As it is clear from its title, a very peculiar effort has been taken to explain the sidereal day without considering any polar star. As per a very newly introduced assumption regarding orbital period of earth, taken from a published research paper in "International Journal Of Science And Research" and on the basis of this assumption, explanation of sidereal day is done. As per the introduced assumption in that research paper the orbital period of earth is 32.375 years, Earth make an oscillation and an another rotation in 365.2422 days about its axis of rotation which always comply with the shifting tangent of sloping path of revolution. On the basis of this assumption a rotating spherical body on a sloping path of revolution can never completes its rotation cycle having $\mathbf{3 6 0}^{\mathbf{0}}$. For each and every rotation, the angle of rotation will be less than $\mathbf{3 6 0}{ }^{0}$ and angle of rotation cycle will be reduce according to angular displacement of spherical body for one rotation on its sloping path of revolution. This way a very accurate calculation is done to explain the sidereal day, and finally a very simple and applicable definition is introduced in equation form, so that its numerical value could be determine easily by anyone.


## 1. Introduction

As we are familiar that earth make rotation about its axis of rotation in 24 hours covering rotational angle $360^{\circ}$, but on the basis of microscopic observation we find that Earth completes its rotation cycle merely in 23 hours 56 minutes and 04 seconds. The rotation cycle of 24 hours is known as a mean solar day, whereas the rotation cycle of 23 hours 56 minutes and 04 seconds is known as the Sidereal day. It means in Sidereal day Earth does not complete its rotational cycle of $360^{\circ}$. Then a question arise before us, Why does Earth not completes its rotation cycle of $360^{\circ}$ ? Why this rotational cycle completes earlier than 24 Hours? Then the answer of this question may be path of revolution. As per our existing assumption regarding rotation and revolution, earth rotates like a wheel of bicycle about its axis of rotation where axis of rotation always comply with the $23.4{ }^{0}$ tilted normal with the orbit of Earth, and cover a distance about 65 times more than the circumference of Earth on its orbit after each and every rotation, but how and why this large distance dose covered by earth on its orbit after one rotation is still unknown? Till date we only imagine that Earth covers its orbital distance about 25.75 lakh kilo miter per day without any justified and scientific cause, but in this research paper a possible way of rotation and revolution is described, so that basic cause behind occurrence of the Sidereal may be brought in notice. We herewith imagine a geometrical illustration to explain the possible way of rotation and revolution of earth. We will draw a geometrical figure like FIG -1 here is a circle is drown with center $S$, and $A B$ and CD are two tangent of that. let us consider that a spherical body rotate about its axis of rotation in such way that center of axis always overlap the point of contact of tangent, and spherical body rotate in such way that after completion of
rotation cycle it shift from E1 to E2 which is a new point of contact for tangent CD, if the angular displacement from E1 to E2 is $\vartheta$, than the rotation cycle will be just complete when the spherical body completes its $\{360-\vartheta\}$ degree angular rotation, as well as timing of rotation will be also decrease. The rotation of spherical body will only complete $360^{\circ}$, when it rotate only about axis and axis overlap the tangent AB. Probably same happening would also be happen with the case of earth. Earth would be rotate about its axis in such way that after one rotation it would shift from E1 To E2 and because this path is like a slop, therefore we observe that rotation cycle seems complete when it completes just ( 360 $\vartheta)$ degree. However formulation of third law of Kepler is also possible with the help of this assumption and orbital period of earth around the Sun can be determine 32.375 years. Hence on the basis of this assumption and orbital period 32.375 years of Earth, explanation is done for the Sidereal day in this research paper.

Description: In May 2017 a research paper published in an online science journal called International Journal of Science and Research (ijsr.net) article no ART 20173437 with heading "Venus transition, a complete solution with an otherwise explanation ". In which a very unique astronomical observation explained. We hereunder use the conclusion of that research paper in this research paper. The conclusions of that research paper are as follows.

1) The orbital period of Earth is 32.375 years.
2) Earth makes another rotation called Sidereal rotation on its axis of rotation in 365.2422 days and as well as also oscillate about it's axis in 365.2422 days.
3) Earth rotate on its axis in such way that axis of rotation always coincide with the tangent of revolution path in

24 hours and along with every rotation of earth it also cover a path of revolution 1.826 arc minute / day.
4) The orbital period of Venus is calculated 6.41 years.
5) On the basis of above assumption a complete solution of Transit of Venus is possible.
6) As per description of this paper elongation and overlap are quite different in term of astronomy.
7) The possible cause behind SCHROTE'S EFFECT is possible to identify.

Among these 7 conclusions of that research paper we require only three conclusions to explain the Sidereal day in this research paper. No -1, No- 2, and No -3. For clear understanding of this explanation a simple figure for orbit of Earth having 32.375 year showing oscillatory \& Sidereal rotation path of 365.2422 day ( $\mathrm{Fig}-2$ ) As per conclusion No -1, if Earth makes revolution around the Sun in 32.375 year than angular orbital path covered by Earth in one year will be equal to $360^{0} * 60 / 32.375=667.181467176$ Arc minute. $/ 360^{\circ}=1.85328185326$ arc minute. Therefore in one degree of sidereal rotation, Earth will be cover 1.8532818526 arc minute on its path of revolution. It means value of angular displacement of Earth on the path of revolution for one degree Sidereal rotation of Earth will be 1.85328185326 arc minute, and because ratio of rotational periodic time and circumference of Earth is $86400 / 40000=$ 2.16 , hence if 1.8532818526 will be divided by 2.16 , then we find the value 0.85800085768 arc minute, which will be reduced after one degree Sidereal rotation of Earth. If this value will be multiplied by 60 second then this value will be 51.4800514608 arc seconds. Because orbit of Earth is slop therefore this value will be reduced from 1 degree of Sidereal rotation. Now because 1 degree of Sidereal rotation $=60 * 60=3600$ arc second, hence $3600-51.4800514608$ $=3548.51994854$ arc second divided by $60=59.1419991423$ arc minute. Now because for 1 degree Sidereal rotation Earth makes a day rotation of 360 degree, therefore 59.1419991423 arc minute will be reduced from the 360 degree rotation of Earth in 24 hours. Now $369^{0} * 60=21600$ arc minute -59.1419991423 arc minute $=21549.8580009$ arc minute. Now if 21600 arc minute $=24$ hours. Therefore 21540.8580009 arc minute $=24 * 21540.8580009 / 21600=$ 23.9342866676 hours. In which 23 is Hour \& 0.9342866676 is decimal value of hour, hence if it multiplied by $60=$ 56.057200056 minute, in which 56 is minute and 0.057200056 is in decimal form of minute, hence if it multiplied by 60 then we find 3.43200336 seconds. Means 23 hour 56 minute 3.43200336 seconds. Which is just equal to observed value of Sidereal day. NOTE: - Because the value of sidereal is derived on the basis of 32.375 year orbital of Earth, hence some evidences for the same is necessary to describe herewith as a ready reference, however these descriptions are already published in ART 20173437 of ijsr.net.

## Mathematical evidence for 32.4 year orbital period (1):

Let us consider that a spherical body of mass ' $m$ ' very small in compare to central body making revolution around central body with linear velocity ' $v$ ', which is situated at distance' R'. Hence acceleration adopted by this body will be as follow, (see fig ---- 3), $A_{s}=v^{2} / \mathrm{R}$--------------------(-1), (As -acceleration due to Sun). If this spherical body is spinning on its axis also in such way that its direction of revolution
and direction of revolution become perpendicular to each other than the expression for linear velocity in derivative form will be

$$
d s / d t
$$

As per differential calculus any derivative represents a tangent form of curve. Hence its normal form is also possible as $-1 / d s / d t$, if $d s / d t$ represents a tangent form of velocity, whose direction of movement is perpendicular to it means rotation. Hence $-1 / d s / d t=-d t / d s$ represents a normal form of velocity whose direction of movement is parallel to it mean revolution, If this spherical body is our Earth. Hence the linear velocity of Earth for perpendicular direction $d s / d t=2 \pi r / t$---------------(2), where ' $r$ ' $=$ radius of Earth $\&, \mathrm{t}^{\prime}=$ periodic time of revolution. $d s / d t=$ $2 * 3.14 * 6370 / 86400=0.46 \mathrm{~km} / \mathrm{sec}$. The normal form of velocity may be represents as $-1 / d s / d t=-d t / d s=t / 2 \pi \mathrm{r}=$ $86400 / 2 * 3.14 * 6370=2.16 \mathrm{sec} / \mathrm{km}-------(3)$. Because this is an expression of velocity of revolution for Earth, hence its dimension should also be $\mathrm{km} / \mathrm{sec}$, But the equation (3) represents sec/km (just reverse to the dimension of velocity) , hence Earth does not adopt this magnitude of velocity unless its dimension become dimension of velocity like $\mathrm{km} / \mathrm{sec}$, which possible only when magnitude become unit, like $86400 / 86400$ or $40000 / 40000$. Now because the time quantity is independent variable and displacement is dependent variable of velocity derivative. Hence we replace the 40000 by 86400 , so that the magnitude become unit. And in this case meaning of expression --(3) will not change if $864000 / 86400 \mathrm{~km} / \mathrm{sec}$ or $86400 / 86400 \mathrm{sec} / \mathrm{km}$. The velocity $86400 / 86400 \mathrm{~km} / \mathrm{sec}$ represents the velocity with respect to the Sun. Hence Earth adopts $1 \mathrm{~km} / \mathrm{sec}$ as the velocity of derivative expression ---(3) $d t / d s$ for revolution around the Sun. If T is periodic time of revolution for Earth and R is the distance between Earth and the Sun hence $d t / d s=1 \mathrm{sec} / \mathrm{km}$ $=\mathrm{T} / 2 \pi \mathrm{R}=1 \mathrm{~km} / \mathrm{sec}=d T / d S$ linear velocity of revolution by Earth. If Earth adopts velocity $1 \mathrm{~km} / \mathrm{sec}$ of expression---(3) (Note - Because minor and major axis are just near to equal), then the acceleration due to Sun for revolution of Earth will be $A_{s}=v^{2} / \mathrm{R}=(1)^{2} / \mathrm{R}=\left(\frac{T}{2 \pi R}\right)^{2} / \mathrm{R}=T^{2} / 4 \pi 2 R^{3}$ or $T^{2}=4 A_{S} \pi 2 R^{3}$
$T^{2}$ is directly proportion to $R^{3}$ $\qquad$ which is Kepler's third Law.

Here T is the periodic time for revolution of Earth around the Sun, however the expression of equation -(4) is very similar to the Kepler's third law of planetary motion, hence the orbital period of the Earth can be derive, if we have a correct value for the distance of the Sun from Earth. As we are familiar with the existing knowledge of distance from the Earth to the Sum 149.6 million K.M, but the same is not utilized for derive orbital period of Earth in this research paper, A separate method is introduced here to determine the distance of the Sun as follow. Direct observation method to determine the distance of the Sun from Earth.: If we observe carefully the rising interval of the Sun daily with following way then we find that rising period of the Sun is 182 to 183 seconds or between them on horizon at the time of rising,
which may be verified easily daily- wise by direct observation method as follow. See figure ---(4). If we imagine a linear plane in east horizon at the time of rising Sun in morning in such way that just when the Sun seems to be rise, the upper edge of the Sun touches the imagined linear plane and switch on a stopwatch and observe carefully the rising event of the Sun till bottom edge of the Sun coincide with the imagined linear plane like given in fig (4), and note down the periodic interval of this event by switch off the stopwatch. If we repeat same observation many times then we will find that the periodic interval 182 sec or 183 sec or any other between them. As we are familiar that the sun is fixed on a particular place or having negligible displacement in space and rising event takes place due to spine of Earth about its axis, therefore in which time period Sum seems to cross the linear plane, Earth spines some degree about its axis, and value of this angle in 182.5 sec will be 0.7604166 degree. Again draw a figure like (fig ---5 ) as straight line AC and CD are drown touching the upper and bottom edge of the Sun respectively then we find a triangle ACD. Now if a straight line CB drown on AD in such way that it bisect the $\angle A C D$ and meet at $B$ on $A D$ making right angle, newly formed triangle ABC will be a right angle triangle. In which $\mathrm{AB}=690200 \mathrm{~K} . \mathrm{M}$ (radius of the Sun), $\angle \mathrm{ACB}=0.382083^{\circ}$ and $\angle \mathrm{CAB}=89.619792^{\circ}$. If these values are placed in equation for law of triangle, than distance of the Sun from the observer location on Earth can be easily determine. $\angle \mathrm{A} / \mathrm{a}=<\mathrm{B} / \mathrm{b}=<\mathrm{C} / \mathrm{c}$ or $89.619792^{\circ} / a=$ $90^{0} / b=0.3802083^{0} / 690200$ or $\mathrm{a}=162688660$ K.M $\& \mathrm{~b}=$ 163378850 K.M. The minimum distance of the Sun is 162688660 K.M. Now according equation (4) $T^{2}=4 \pi 2$ $A_{s} R^{3}=4 \pi 2 \frac{1}{R} R^{3}=4 \pi 2 R^{2}$ or $\mathrm{T}=2 \pi \mathrm{R}=2 * 3.14 * 162688660$ $=1021684784.8 / 60 * 60 * 24 * 365.2422=32.3759285742$ year.

## $2^{\text {nd }}$ evidence of 32.40 year ' $s$ orbital period of Earth with the help of analyzing the Venus transition

Table 1: Chart of previous occurrence of Venus transition since 1631)
A1 A2 B1 B2 CI C2 D1 D2A1---8 YEAR---A2 ---121.5YEAR--- B1--8YEAR-- B2---105.5YEAR----C1----8YEAR-----C2 ---121.5 YEAR----D1----8YEAR-----D2

| Location On Line | Date of transition |
| :---: | :---: |
| A1 | $7^{\text {th }}$ Dec 1631 |
| A2 | $4^{\text {th }}$ Dec 1639 |
| B1 | $6^{\text {th }}$ June 1761 |
| B2 | $3^{\text {rd }}$ and $4^{\text {th }}$ June 1769 |
| C1 | $9^{\text {th }}$ Dec 1864 |
| C2 | $6^{\text {th }}$ Dec 1882 |
| D1 | $5^{\text {th }}$ June 2004 |
| D2 | $5^{\text {th }}$ June 2012 |

Difference between two Consecutive Years

| A1 to B1 | 129.5 years |
| :---: | :---: |
| A2 to B2 | 129.5 years |
| B1 to C1 | 113.5 years |
| B2 to C2 | 113.5 years |
| C1 to D1 | 129.5 years |
| C2 to D2 | 129.5 years |

Description : - Watching Venus over several year and noted down the period of maximum elongation NICOLUS COPERNICUS at first become capable to determine the orbital period of Venus as well as overlap period of Earth and Venus by the help of derived formula for overlapping period by ordinary watch describe as follow. If we consider that minute needle and hour needle overlapping at 12.00 Hour then, when will be the next overlap again? The answer of this question may be given by following formula derived as follow. Let the period of revolution for Hour needle is $T_{1}$ means it cover $360^{\circ}$ in $T_{1}$. Let the period of revolution of minute needle is $T_{2}$ means it cover $360^{\circ}$ in $T_{2}$. Therefore in interval of " t • Hour needle will cover angle $360^{\circ} \mathrm{t} / T_{1}$ and Minute needle will cover angle $360^{\circ} \mathrm{t} / T_{2}$ and overlap will be only possible when difference in the cover angle will be $360^{0}$. Means $360^{\circ} \mathrm{t} / T_{1}-360^{\circ} \mathrm{t} / T_{2}=360^{0}$ or $\mathrm{t} / T_{1}-\mathrm{t} /$ $T_{2}=1$ or $\mathrm{t}=T_{1} T_{2} / T_{2}-T_{1}$, ----------------(5) This same formula is utilized for overlap period of Earth and Venus as $\mathrm{t}=T_{E} T_{V} / T_{E}-T_{V}$ (6) $\left(T_{E}=\right.$ orbital period of Earth $\& T_{V}=$ orbital period of Venus). With the help of this formula and its extended shape COPERNICUS could be able to determine the orbital period of Venus $=224.7$ day, and value for overlap Venus and Earth determined as, $\mathrm{t}=584$ days, but in practical and actual direct observation we find that minimum interval for overlap of Venus and Earth is 8 year, then why not we utilize this overlap formula of COPERNICUS to determine the orbital time of Earth considering 8 year overlap period for Earth and Venus ? Because difference between angle of revolution covered by Earth and Venus is always $360^{\circ}$. Then we may imagine the following different condition for every time 8 year overlap. Condition (1) $30^{\circ} \& 390^{\circ}$ angle covered by Earth and Venus respectively. Condition (2) $60^{\circ} . \& 420^{\circ}$ angle covered by Earth and Venus respectively. Condition (3) $90^{\circ} \& 450^{\circ}$ angle covered by Earth \& Venus respectively. Condition (4) $120^{\circ} \& 480^{\circ}$ angle covered by Earth and Venus respectively. Condition (5) $180^{\circ} \& 540^{\circ}$ angle covered by Earth and Venus respectively. Meaning for Condition (1) If Earth covers $30^{\circ}$ angle in 8 year then the orbital period of the Earth will be 96 year, and if Venus covers $390^{\circ}$ of orbit in 8 year then orbital period of Venus will be $96 / 13=$ 7.384 year. Meaning of condition (2) If Earth covers $60^{\circ}$ of its orbit in 8 year then the orbital period of Earth will be 48 year \& if Venus covers $420^{\circ}$ of its orbit in 8 year then the orbital period of the Venus will be $48 / 7=$ 6.859 year. Meaning of condition (3) If Earth covers $90^{\circ}$ of its orbit in 8 year then the orbital period of the Earth will be 32 year \& if Venus covers $450^{\circ}$ of its orbit in 8 year then the orbital period of the Venus will be $32 / 5=$ 6.4 year. Meaning of condition (4) If Earth covers $120^{\circ}$ of its orbit in 8 year then the orbital period of the Earth will be 24 year \& if Venus covers $480^{\circ}$ of its orbit in 8 year then the orbital period of Venus will be $24 / 4=6$ year. Meaning of condition (5) If Earth covers $180^{\circ}$ of its orbit in 8 year then the orbital period of Earth will be 16 year \& if Venus covers $540^{\circ}$ of its orbit in 8 year then the orbital period of Venus will be $16 / 3=5.33$ year. Among these above (5) conditions for overlap of Earth \& Venus, we find that condition (3) is most appropriate
than the others and its results directly related with our determined orbital period of Earth as well as the synodic period 584 day as follow. $584 * 4=2336 / 365.242=$ 6.39575 year $=T_{v}, 584 * 5=2920$ day $/ 365.2422=8$ year $=\mathrm{t}, 584 * 20=11680$ day $/ 365.2422=32$ year . Hence we take in consideration the $3^{\text {rd }}$ condition for analysis the following data of Venus transition. However the orbital period for Earth ( 32 year) and Venus ( 6.4 year) are near by approximate. Hence we will analysis the data of 129.5 year, 113.5 year, \& 121.5 year respectively for overlap period of Venus transitions , then we find the following if we divide these data by 32 year. $1^{\text {st }}-129.5 / 32=4.046875 \cdot 2^{\text {nd }}-113.5 / 32=$ 3.546875. $3^{\text {rd }}-121.5 / 32=3.796875$. Now we find all these out- comes are numerically .046875 more than the round figure $4,3.5$, \& 3.75 respectively. If these outcomes represent the number of revolution of Earth as follow, 129.5/ $T_{E}=4$ Than $T_{E}=32.375$ year, $113.5 / T_{E}=$ 3.5, $\operatorname{ThanT}_{E}=32.428$ Year, $121.5 / T_{E}=3.75 \mathrm{Than} T_{E}=$ 32.40 year. Hence average of above three are equal to 32.40 year. which is very similar to our previous calculation for orbital period of Earth.


Figure 2


Figure 3


Figure 4


Figure 5

## 2. Conclusion

1) As per above description we come on a conclusion that the sidereal day may be define as "The periodic time required for completion of $\left\{360^{\circ}-\left(1-\frac{1}{T * P C}\right)\right\}$ degree rotational cycle on angular circumference of Earth in 24 Hours. "(where T = orbital period of Earth calculated in this research paper \& $\mathrm{PC}=$ projection constant, which has shown 2.16 in earlier part of this research paper). Calculation of sidereal day $=\left\{360-\left(1-\frac{1}{32.375 * 2.16}\right)\right\}$ degree $=\left(359+\frac{1}{69.93}\right)$ degree $=(359+0.0143000143)$ degree $=359.0143000143^{\circ}$. Now 359.0143000143 ${ }^{\circ} *$ $86400 / 360^{\circ}=86163.432003 \mathrm{sec}=23$ Hours 56 Minutes 3.432 Seconds. This is the calculated SIDEREAL DAY. [ NOTE - However this value is. 01199 second less than
the required period of sidereal day, because as completion of 365.2422 days sidereal rotation on orbital path of 32.375 year, the value of sidereal day should be 86163.444 seconds if we want to tally this period with 365.2422 day.
2) Some outcomes of this research paper is also important to explain The Precession of Equinoxes.

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