The Oscillations for Delay Impulsive Fractional Partial Differential Equations (IFPDEs)

Suzan. A. A. Abu-harbid[1], Kholoud. W. Saleem[2], Salima Kh. Ahmed[3]

[1]Mathematics Department, Science Faculty, Derna University, Al-guba-Libya

[2]Mathematics Department, Science Faculty, Derna University, Al-guba-Libya

[3]Mathematics Department, Education Faculty, Tobruk University, Tobruk-Libya

Email Address: *sanasuzansana[at]gmail.com, Kholoudwaleed83[at]gmail.com, kasalema980[at]gmail.com*

Abstract: ***This paper investigates the oscillatory properties of impulsive fractional partial differential equation with several delays and establishes sufficient conditions for oscillation of the fractional partial differential equation. Moreover, we study In this paper, based on the modified Riemann-Liouville derivative, we are concerned with The oscillations for impulsive delay fractional partial differential equations. The main results are illustrated by examples.***

Keywords: Oscillation, Impulsive, Fractional partial differential equation, Riemann-Liouville Derivative, Several Delays.

1. Introduction

Fractional differential equations are generalizations of the classical differential equations of integer order, important tool in modeling real life phenomena. It is well-known that fractional differential equations are a more general form of the integer order differential equations, Such processes often occur in the theory of theoretical physics, chemistry, optimal control, medicine, biotechnologies, population dynamics, etc. In the past few years, the theory of impulsive partial differential equations has been investigated extensively. The oscillations for impulsive delay partial differential equations were studied. For example, see [1-3]. Moreover, the research on the oscillatory behavior of solutions of fractional differential equation has been a hot topic and some results have been established. For example, see [4-8]. However, to the best of the author's knowledge, very little is known regarding the oscillatory behavior of fractional partial differential equations which involve the Riemann-Liouville fractional partial derivative up to now [9-11]. The oscillatory behavior of ordinary differential equations, partial differential equations and impulsive partial differential equations have been investigated by many papers in the past, see [12-16].

In recent years, the oscillatory behavior of various classes of fractional ordinary differential equations and fractional partial differential equations have been investigated by many authors, see [17-21]. In [22-27] some authors have studied the oscillatory behavior of solutions of fractional partial differential equations and impulsive fractional partial differential equations.

This paper is organized as follows: In section 2, we introduce problem formulation, related definitions, and lemmas. In Section 3, we discuss the oscillation problem of (2.1) subject to boundary conditions (2.2). We present two examples to illustrate our results in section 4.

2. Formulation of Problems and Preliminaries

Our aim in this paper is to study the oscillation properties of the solutions to a class of (IFPDEs) with several delays of the form

$$\begin{cases} D_{+,t}^{1+\gamma}u(x,t)+a(t)D_{+}^{\gamma}u(x,t)=b(t)g(u(x,t))\Delta u(x,t)- \\ \sum_{k=1}^{m}h_k(x,t)q_k(u(x,t-\delta_k))-v(x,t), t\neq t_i,(x,t)\in R_+\times\Omega\equiv G, \\ D_{+,t}^{\gamma}u(x,t_i^+)-D_{+,t}^{\gamma}u(x,t_i^-)=\sigma(x,t_i)D_{+,t}^{\gamma}u(x,t_i), t=t_i, i\in I_\infty, k\in I_m. \end{cases} \quad \textbf{(2.1)}$$

With the boundary condition

$$\frac{\partial u(x,t)}{\partial N}=\Phi(x,t,u(x,t)),(x,t)\in R_+\times\partial\Omega, t\neq t_i. \quad \textbf{(2.2)}$$

Where $\gamma\in(0,1)$ is a constant, $D_{+,t}^{\gamma}$ is the Riemann-Liouville fractional derivative of order γ of $u(x,t)$ with respect to t, Ω is a boundary domain in R^n with a smooth boundary $\partial\Omega$ and $\overline{\Omega}=\Omega\cup\partial\Omega$, $I_\infty=\{1,2,...\}, I_m=\{1,2,...,m\}$, $R_+=[0,+\infty)$, Δ is the Laplacian in R^n, and N is the unit exterior normal vector to $\partial\Omega$, $a(t),b(t)\in PC[R_+,R_+]$, PC denote the class of functions which are piecewise continuous in t with discontinuities of first kind only at $t = t_i$, $\delta_k > 0$. The solution $u(x,t)$ of the problem (2.1),(2.2) and $D^{\gamma}_{+,t}u(x,t)$ are piecewise continuous with discontinuities first kind only at $t = t_i$, and left continuous at $t = t_i, i \in I_\infty$.

The following conditions are assumed to hold

C1: $q_k : R \to R$ is a continuous function such that $q_k(u)/u \ge i_k$, for all $u \ne 0$; and i_k is a positive constant, $k \in I_m$

C2: $h_k(x,t) \in PC[R_+ \times \overline{\Omega}, R_+]$,

and $h_k(t) = \min_{1\le k\le m} \min_{x\in\Omega} h_k(x,t)$.

C3: $v(x,t) \in PC[R_+ \times \overline{\Omega}, R_+]$.

C4: $g(u) \in C(R,R); ug'(u) \ge 0; \Phi(x,t,u)$ is a piecewise continuous function, such that

$u\Phi(x,t,u)g(u) \le 0$.

C5: $\sigma : R_+ \times \overline{\Omega} \to R_+$ such that $\sigma(x,t_i) \le \gamma_i$.

C6: The given numbers $0 < t_1 < \cdots < t_i < \cdots$, are such that $\lim_{i\to\infty} t_i = +\infty$.

C7: At the moments of impulsive the following relation is satisfied:

$$D^{\gamma}_{+,t}u(x,t_i^-) = D^{\gamma}_{+,t}(x,t_i)$$

For the sake of convenience, in this paper, we denote:

$$U(t) = \int_\Omega u(x,t)dx \;;\; V(t) = \int_\Omega v(x,t)dx \quad \textbf{(2.3)}$$

Definition 2.1 A nonzero solution $u(x,t)$ of the problem (2.1),(2.2) is said to be nonoscillatory in the domain G if there exists a number $t_0 \ge 0$ such that $u(x,t)$ has a constant sign for $(x,t) \in \Omega \times [t_0, +\infty)$. Otherwise, it is said to be oscillatory.

Definition 2.2 The Riemann-Liouville fractional derivative of order $\alpha > 0$ of a function $y : R_+ \to R$ on the half-axis R_+ is given by

$$D^{\alpha}_{+,t} = \frac{1}{\Gamma(\lceil\alpha\rceil - \alpha)} \frac{d^{\lceil\alpha\rceil}}{dt^{\lceil\alpha\rceil}} \int_0^t (t-\xi)^{\lceil\alpha\rceil-\alpha-1} y(\xi)d\xi \quad \text{for } t > 0 \quad \textbf{(2.4)}$$

Provided the right hand side is pointwise defined on R_+, Γ is gamma function, $\lceil\alpha\rceil$ is the ceiling function of α.

Definition 2.3 The Riemann-Liouville fractional partial derivative of order $0 < \alpha < 1$ with respect to t of a function $u(x,t)$ is given by

$$D^{\alpha}_{+,t}u(x,t) = \frac{1}{\Gamma(1-\alpha)} \frac{\partial}{\partial t} \int_0^t (t-\xi)^{-\alpha} u(x,\xi)d\xi \quad \textbf{(2.5)}$$

Provided the right hand side is pointwise defined on R_+.

Lemma 2.1 Let $E(t) = \int_0^t (t-\xi)^{-\alpha} y(\xi)d\xi$ for $\alpha \in (0,1)$ and $t > 0$. Then

$$E'(t) = \Gamma(1-\alpha)D^{\alpha}_{+}y(\xi). \quad \textbf{(2.6)}$$

Lemma 2.2 Let $0 < \alpha < 1, m \in N$ and $D = d/dx$. If the fractional derivatives $(D^{\alpha}_{+}y)(x)$ and $(D^{m+\alpha}_{+}y)(x)$ exist, then

$$(D^m D^{\alpha}_{+}y)(x) = (D^{m+\alpha}_{+}y)(x) \quad \textbf{(2.7)}$$

3. The Oscillation of the Problem (2.1)-(2.2)

In this section, we establish sufficient conditions for the oscillation of all solutions of the boundary problem (2.1)-(2.2).

Theorem 3.1 If impulsive fractional differential inequality

$$D^{1+\gamma}_{+,t}U(t) + a(t)D^{\gamma}_{+,t}U(t) \le -V(t), \quad (3.1)$$

$$D^{\gamma}_{+,t}U(t^+) \le (1+\alpha_i)D^{\gamma}_{+,t}U(t), i = 1,2,..., \quad (3.2)$$

Has no eventually positive solutions and impulsive fractional differential inequality

$$D^{1+\gamma}_{+,t}U(t) + a(t)D^{\gamma}_{+,t}U(t) \ge -V(t), \quad \textbf{(3.3)}$$

$$D^{\gamma}_{+,t}U(t^+) \ge (1+\alpha_i)D^{\gamma}_{+,t}U(t), i = 1,2,..., \quad \textbf{(3.4)}$$

Has no eventually negative solutions, then every nonzero solution $u(x,t)$ of the problem (2.1) and (2.2) is oscillatory in the domain G.

Proof Suppose to contrary that $u(x,t)$ be a nonzero solution of the problem (2.1) and (2.2) which is nonoscillatory in the domain G. Without loss of generality, we assume that $u(x,t)$ is an eventually positive solution of problem (2.1) and (2.2) in the domain G. Then there exists a $t_0 \ge 0$ such that $u(x,t) > 0, u(x,t-\delta_i) > 0$, for $(x,t) \in \Omega \times [t_0, +\infty)$.

Case I: $t \ne t_i$. Integrating the first equation of problem (2,1) with respect to x over the domain Ω, we have

$$\int_\Omega D_{+,t}^{1+\gamma}u(x,t)dx + a(t)\int_\Omega D_{+,t}^{\gamma}u(x,t)dx = b(t)\int_\Omega g(u(x,t))\Delta u(x,t)dx$$
$$-\int_\Omega \sum_{k=1}^{s} h_k(x,t)q_k(u(x,t-\delta_k))dx - \int_\Omega v(x,t)dx \quad (3.5)$$

By using Green's formula, combing boundary condition (2.2) and assumption (C4), we obtain

$$\int_\Omega g(u)\Delta u(x,t)dx = \int_\Omega g(u)\frac{\partial u(x,t)}{\partial N}dx - \int_\Omega g'(u)|gradu|^2 dx$$
$$= \int_\Omega g(u)\Phi(x,t,u)dx - \int_\Omega g'(u)|gradu|^2 dx \le 0 \quad (3.6)$$

According to assumption (C1) and (C2), we have

$$\int_\Omega \sum_{k=1}^{m} h_k(x,t)q_k(u(x,t-\delta_k))dx \ge \sum_{k=1}^{m} h_k(t)\int_\Omega u(x,t-\delta_k)dx \ge 0, t \ge t_0 \quad (3.7)$$

And according to Lemma 2.2, combing (3.5) and (3.7), we can easily obtain

$$D_{+,t}^{1+\gamma}U(t) + a(t)D_{+,t}^{\gamma}U(t) \le -\sum_{k=1}^{m} i_k h_k(t)\int_\Omega u(x,t-\delta_k)dx$$
$$\le -\sum_{k=1}^{m} i_k h_k(t)\int_\Omega u(x,t-\delta_k) - G(t) \le -G(t), t \ge t_0. \quad (3.8)$$

Case II: $t = t_i$. Integrating the second equation of problem (2.1) with respect to x over the domain Ω, and according to assumption (C5), we have

$$D_{+,t}^{\gamma}U(t_i^+) = D_{+,t}^{\gamma}\int_\Omega u(x,t_i^+)dx \le (1+\gamma_i)D_{+,t}^{\gamma}\int_\Omega u(x,t_i)dx = (1+\gamma_i)D_{+,t}^{\gamma}U(t) \quad (3.9)$$

Thus impulsive fractional differential inequality (3.8) and (3.9) imply that the function $t \ne t_i$ is an eventually positive solution of fractional impulsive differential inequality (3.1) and (3.2) which contradicts the conditions of the theorem.

On the other hand, if $u(x,t)$ is an eventually negative solution of the problem (2.1) and (2.2) in the domain G, then using the similar method, we can easily obtain that $t \ne t_i$ is an eventually negative solution of the fractional impulsive differential inequality (3.3) and (3.4) which contradicts the conditions of the theorem. The proof is completed.

Lemma 3.1 Assume that

$$\omega'(t) \le g_1(t)\omega(t) + g_2(t), t \ne t_k, t \ge t_0,$$
$$\omega(t_k^+) \le (1+a_k)\omega(t_k), k = 1,2,\cdots, \quad (3.10)$$

Where $0 < t_1 < \cdots < t_k < \cdots$ and $\lim_{k\to\infty} t_k = +\infty$;

$\omega \in PC'[R_+, R]$; $g_1, g_2 \in [R_+, R]$ and a_k are constants. Then

$$\omega(t) \le \omega(t_0)\prod_{t_0<t_k<t}(1+a_k)\exp\left(\int_{t_0}^{t} g_1(s)ds\right)$$
$$+\int_{t_0}^{t}\prod_{s<t_k<t}(1+a_k)\exp\left(\int_s^t g_1(\sigma)d\sigma\right)g_2(s)ds, t \ge t_0 \quad (3.11)$$

Lemma 3.2 If $U(t)$ is a solution of the impulsive fractional differential inequality (3.1) and (3.2) (or (3.3) and (3.4)), and

$$E(t) = \int_0^t (t-\xi)^{-\gamma}U(\xi)d\xi, \text{ for } \gamma \in (0,1), \text{ and } t > 0,$$

Then

$$E'(t) = \Gamma(1-\gamma)D_{+,t}^{\gamma}U(\xi). \quad (3.12)$$

Theorem 3.1 If we assume, for some $\xi_2 > 0$,

$$\int_{\xi_2}^{\infty}\exp\left(-\int_{t_0}^{t} a(\sigma)d\sigma\right)ds = \infty, \quad (3.13)$$

Further we assume, for some $\xi_1 > 0$,

$$\limsup_{t\to\infty}\frac{\int_{\xi_1}^{t}\prod_{s<t_k<t}(1+\gamma_i)\exp\left(-\int_s^t a(\sigma)d\sigma\right)V(s)ds}{\prod_{\xi_1<t_k<t}(1+\gamma_i)\exp\left(-\int_{\xi_1}^{t} a(s)ds\right)} = \infty, \quad (3.14)$$

And

$$\liminf_{t\to\infty}\frac{\int_{\xi_1}^{t}\prod_{s<t_k<t}(1+\gamma_i)\exp\left(-\int_s^t a(\sigma)d\sigma\right)V(s)ds}{\prod_{\xi_1<t_k<t}(1+\gamma_i)\exp\left(-\int_{\xi_1}^{t} a(s)ds\right)} = -\infty, \quad (3.15)$$

Then each nonzero solution of problem (2.1) and (2.2) is oscillatory in the domain G.

Proof To prove the theorem, it is sufficient to prove that the impulsive fractional differential inequality (3.1) and (3.2) has no eventually positive solutions, and the impulsive fractional differential inequality (3.3) and (3.4) has no eventually negative solutions. Suppose on contrary, the impulsive fractional differential inequality (3.1) and (3.2) has an eventually positive solution $U(t)$, then there exists $\xi_1 > 0$ such that $U(t) > 0$, $U(t-\delta_k) > 0, V(t) > 0, t \ge \xi_1$.

Let

$$w(x,t) = \exp\left(\int_{t_0}^{t} a(s)ds\right).$$

By using lemma 2.2 and the impulsive fractional differential inequality (3.1), we obtain

$$[(D^{\gamma}_{+,t}U)w(t)]' = (D^{\gamma}_{+,t}U)'(t)w(t) + a(t)(D^{\gamma}_{+,t}U)(t)w(t) \le -V(t)w(t) < 0$$

Thus $(D^{\gamma}_{+,t}U)(t)w(t)$ is strictly decreasing for $t \ge \xi_1$ and is eventually of constant sign. Since $w(t) > 0$, for $t \ge \xi_1$, we see that $(D^{\gamma}_{+,t}U)(t)$ is eventually of constant sign. Then, $(D^{\gamma}_{+,t}U)(t) > 0$ for $t \ge \xi_1$. Otherwise, $(D^{\gamma}_{+,t}U)(t) < 0$ for $t \ge \xi_1$, there exists $\xi_2 \in [\xi_1, \infty)$, such that $(D^{\gamma}_{+,t}U)(\xi_2)w(\xi_2) < 0$.

Since $(D^{\gamma}_{+,t}U)(t)w(t)$ is strictly decreasing for $t \ge \xi_1$, so $(D^{\gamma}_{+,t}U)(t)w(t) < (D^{\gamma}_{+,t}U)(\xi_2)w(\xi_2) = C < 0$ for $t \ge \xi_2$. From Lemma 3.2, we have

$$\frac{E'(t)}{\Gamma(1-\gamma)} = (D^{\gamma}_{+,t}U)(t) < C\exp(-\int_{t_0}^{t} a(s)ds), t \ge \xi_2$$
.

Integrating from ξ_2 to t, we obtain

$$E(t) < E(\xi_2) + \Gamma(1-\gamma)C\int_{\xi_2}^{t} \exp(-\int_{t_0}^{t} a(\sigma)d\sigma)ds.$$

Letting $t \to \infty$, and using condition (3.13), we get $\lim_{t\to\infty} E(t) = -\infty$, which is contradiction to the fact that $E(t) > 0$. Hence $(D^{\gamma}_{+,t}U)(t) > 0$ for $t \ge \xi_1$. Let $\omega(t) = (D^{\gamma}_{+,t}U)(t)$. From Lemma 2.2 and impulsive fractional differential inequality (3.1) and (3.2), we have

$$\omega'(t) \le -a(t)\omega(t) - V(t) \ge t_0, t \ne t_t,$$

$$\omega(t^+) \le (1+\gamma_i)\omega(t_i).i = 1,2,\cdots.$$

By using Lemma 3.1, we have

$$\omega(t) \le \omega(\xi_1)\prod_{\xi_1<t_k<t}(1+\gamma_i)\exp\left(\int_{\xi_1}^{t} a(s)ds\right)$$

$$-\int_{\xi_1}^{t}\prod_{s<t_k<t}(1+\gamma_i)\exp\left(\int_{s}^{t} a(\sigma)d\sigma\right)V(s)ds$$

By using condition (3.14) and taking $t \to \infty$, it follows from above equation

$$\liminf_{t\to\infty}\frac{\int_{\xi_1}^{t}\prod_{s<t_k<t}(1+\gamma_i)\exp\left(-\int_{s}^{t} a(\sigma)d\sigma\right)V(s)ds}{\prod_{\xi_1<t_k<t}(1+\gamma_i)\exp\left(-\int_{\xi_1}^{t} a(s)ds\right)} = -\infty,$$

Which contradicts $\omega(t) > 0$.

Secondly, suppose on contrary that the impulsive fractional differential inequality (3.3) and (3.4) has an eventually positive solution $U(t)$, then there exists $\xi_1 > 0$ such that $U(t) < 0$, $U(t-\delta_k) < 0, V(t) < 0, t \ge \xi_1$. Then using similar methods, according the condition (3.15) and taking $t \to \infty$, we can obtain

$$\limsup_{t\to\infty}\frac{\int_{\xi_1}^{t}\prod_{s<t_k<t}(1+\gamma_i)\exp\left(-\int_{s}^{t} a(\sigma)d\sigma\right)V(s)ds}{\prod_{\xi_1<t_k<t}(1+\gamma_i)\exp\left(-\int_{\xi_1}^{t} a(s)ds\right)} = \infty,$$

Which contradicts $\omega(t) < 0$. This completed the proof.

4. Examples

In this section, we provide two examples to illustrate results.

Example 4.1 Consider the following problem

$$\begin{cases} D^{\frac{5}{3}}_{+,t}u(x,t) + \dfrac{1}{t}D^{\frac{2}{3}}_{+,t}u(x,t) = e^{-t}u^2(x,t)\Delta u(x,t) - \\ (x^2+t^2+1)u(x,t-\dfrac{2\pi}{3})e^{\left[u(x,t-\frac{2\pi}{3})\right]^2} - t\cos x, t \ne t_i, \\ D^{\frac{2}{3}}_{+,t}u(x,t_i^+) - D^{\frac{2}{3}}_{+,t}u(x,t_i^-) = t_i^{-3}\cos x D^{\frac{2}{3}}_{+,t}u(x,t_i), t = t_i. \end{cases}$$

(4.1)

With the boundary condition

$$\frac{\partial u(x,t)}{\partial N}=\Phi(x,t,u(x,t))=-u^3(x,t),(x,t)\in R_+\times\partial\Omega,t\neq t_i. \quad \textbf{(4.2)}$$

Here

$$\gamma=\frac{2}{3},\Omega=(0,\frac{\pi}{2}),m=1,a(t)=\frac{1}{t},b(t)=e^{-t},g(u)=u^2,h_1(x,t)=x^2+t^2+1,$$

$$q_1(u)=ue^{u^2},\delta_1=\frac{2\pi}{3},v(x,t)=t\cos x,\sigma(x,t)=t_i^{-3}\cos x,\gamma_i=i_i^{-3},(x,t)\in(0,\frac{\pi}{2})\times R_+.$$

We can easily see that

$$\int_{\xi_2}^{\infty}\exp(-a(\sigma)d\sigma)ds=\int_{\xi_2}^{\infty}\exp(-\int_{t_0}^{t}\frac{1}{\sigma}d\sigma)ds=\int_{\xi_2}^{\infty}\frac{t_0}{t}dt=\infty,$$

$$\limsup_{t\to\infty}\frac{\int_{\xi_1}^{t}\prod_{s<t_k<t}(1+\gamma_i)\exp\left(-\int_s^t a(\sigma)d\sigma\right)V(s)ds}{\prod_{\xi_1<t_k<t}(1+\gamma_i)\exp\left(-\int_{\xi_1}^{t}a(s)ds\right)}=\infty,$$

And

$$\liminf_{t\to\infty}\frac{\int_{\xi_1}^{t}\prod_{s<t_k<t}(1+\gamma_i)\exp\left(-\int_s^t a(\sigma)d\sigma\right)V(s)ds}{\prod_{\xi_1<t_k<t}(1+\gamma_i)\exp\left(-\int_{\xi_1}^{t}a(s)ds\right)}=-\infty,$$

The conditions of theorem 3.2 are satisfied, thus every solution of the problem (4.1) and (4.2) oscillates.

Example 4.2 Consider the following problem

$$\begin{cases} D_{+,t}^{\frac{8}{5}}u(x,t)+\frac{1}{t+1}D_{+}^{\frac{3}{5}}u(x,t)=e^{-t}u^2(x,t)\Delta u(x,t)- \\ \frac{t^2}{2}u(x,t-\frac{2\pi}{3})e^{\left[u(x,t-\frac{2\pi}{3})\right]^2}-t\cos x,t\neq t_i, \\ D_{+,t}^{\frac{3}{5}}u(x,t_i^+)-D_{+,t}^{\frac{3}{5}}u(x,t_i^-)=t_i^{-3}\cos xD_{+,t}^{\frac{3}{5}}u(x,t_i),t=t_i. \end{cases} \quad \textbf{(4.3)}$$

With the boundary condition

$$\frac{\partial u(x,t)}{\partial N}=\Phi(x,t,u(x,t))=-u^3(x,t),(x,t)\in R_+\times\partial\Omega,t\neq t_i. \quad \textbf{(4.4)}$$

Here

$$\gamma=\frac{3}{5},\Omega=(0,\frac{\pi}{2}),m=1,a(t)=\frac{1}{t+1},b(t)=e^{-t},g(u)=u^2,h_1(x,t)=\frac{t^2}{2},$$

$$q_1(u)=ue^{u^2},\delta_1=\frac{2\pi}{3},v(x,t)=t\cos x,\sigma(x,t)=t_i^{-3}\cos x,\gamma_i=i_i^{-3},(x,t)\in(0,\frac{\pi}{2})\times R_+.$$

We can easily see that

$$\int_{\xi_2}^{\infty}\exp(-a(\sigma)d\sigma)ds=\int_{\xi_2}^{\infty}\exp(-\int_{t_0}^{t}\frac{1}{\sigma+1}d\sigma)ds=\int_{\xi_2}^{\infty}\frac{t_0+1}{t+1}dt=\infty,$$

$$\limsup_{t\to\infty}\frac{\int_{\xi_1}^{t}\prod_{s<t_k<t}(1+\gamma_i)\exp\left(-\int_s^t a(\sigma)d\sigma\right)V(s)ds}{\prod_{\xi_1<t_k<t}(1+\gamma_i)\exp\left(-\int_{\xi_1}^{t}a(s)ds\right)}=\infty,$$

And

$$\liminf_{t\to\infty}\frac{\int_{\xi_1}^{t}\prod_{s<t_k<t}(1+\gamma_i)\exp\left(-\int_s^t a(\sigma)d\sigma\right)V(s)ds}{\prod_{\xi_1<t_k<t}(1+\gamma_i)\exp\left(-\int_{\xi_1}^{t}a(s)ds\right)}=-\infty,$$

The conditions of theorem 3.2 are satisfied, thus every solution of the problem (4.3) and (4.4) oscillates.

5. Conclusion

In this paper, we have identified some new sufficient conditions for all solutions of impulsive fractional partial differential equations to be oscillatory, which has a scope beyond the available results in the existing literature. Also we presented two examples to illustrate results.

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