A Deterministic Inventory Model with Demand as a Biquadratic Polynomial Function of Time with Static Rates of Deterioration

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Abstract: In this paper, a Deterministic Model is evolving for the items decline by Demand as well as by Deterioration, where we take Demand as a Biquadratic Polynomial function of Time with the static rate of Deterioration. Here, Shortage is allowing and fully backlogging. An Organization can use this model where demand increases with time biquadratically with a static rate of Deterioration.

Keywords: EOQ Model, Demand, Deterioration, shortage

1. Introduction


2. Assumptions and Notations

Notations
- $\theta(t)$ Inventory Carrying Charge per object per unit time.
- $C_2$ Cost due to deficiency of one object per unit time
- $C_3$ Cost of one Deficient Unit.
- $\beta$ Length of every Production cycle.
- $C(t)$ Average entire cost
- $S$ Inventory at $t = 0$, where $t$ is used for time.
- $I(t)$ Demand rate at any time $t$.
- $D(t)$ Demand rate.
- $\theta(t)$ Deterioration rate function.

Assumptions
1. The Demand rate $D(t)$ is considered as $D(t) = \alpha + \beta t + \gamma t^2 + \delta t^3 + \varepsilon t^4$, where $\alpha, \beta, \gamma, \delta, \varepsilon$ are $> 0$.
2. Deterioration rate function, $\theta(t) = \theta_0$, where $0 < \theta_0 << 1$
3. Lead time has been taken as 0.
4. Shortages are allowed and completely resolved.
5. Refill magnitude is static and refill rate is unbounded.
6. During the time period $T$, there is neither replacement nor repair of deteriorated units.

3. Analysis of Model

Let the no of objects in stock at any time $t$ be $I(t)$. In time period $0 < t < t_1$, $I(t)$ lessens gradually due to requirement and decaying of items and falls to zero at $t = t_1$. In the time period $(t_1, T)$, deficiency of items occurs which are wholly backlogged, where $t_1 < T$. The equations of this process are given by:

\[
\frac{dI(t)}{dt} + \theta(t) I(t) = -\{D(t)\} \quad 0 \leq t \leq t_1
\]

\[
\frac{dI(t)}{dt} = -\{D(t)\} \quad t_1 \leq t \leq T
\]
Put \( \theta = \theta_0 \) and \( D(t) = \alpha + \beta t + \gamma t^2 + \delta t^3 + \varepsilon t^4 \) in (1) and (2), we get
\[
\frac{dI(t)}{dt} + \theta_0 I(t) = \{ \alpha + \beta t + \gamma t^2 + \delta t^3 + \varepsilon t^4 \} \tag{3}
\]
\[
\frac{dI(t)}{dt} = -\{ \alpha + \beta t + \gamma t^2 + \delta t^3 + \varepsilon t^4 \} \tag{4}
\]
Solution of (3) is
\[
I(t) = e^{\theta_0 t} \cdot \{ \alpha + \beta t + \gamma t^2 + \delta t^3 + \varepsilon t^4 \} \tag{5}
\]
\[
I(0) = \{ \alpha + \beta t + \gamma t^2 + \delta t^3 + \varepsilon t^4 \} \tag{6}
\]
\[
I(t) = (\alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\varepsilon t^5}{5}) + \theta_0 \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\varepsilon t^5}{5} \tag{7}
\]
\[
S = \{ \alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\varepsilon t^5}{5} \} \tag{8}
\]
\[
I(t) = (1 - \theta_0) \{ \{ \alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\varepsilon t^5}{5} \} + \theta_0 \{ \alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\varepsilon t^5}{5} \} \} \tag{9}
\]
\[
I(t) = \{ \alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\varepsilon t^5}{5} \} \tag{10}
\]
\[
I(t) = \theta_0 - (\alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\varepsilon t^5}{5}) \tag{11}
\]
\[
I(t) = \theta_0 - (\alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\varepsilon t^5}{5}) \tag{12}
\]
Therefore amount of Deteriorated Items = \( I(0) \) - Stock loss due to Demand
\[
= S - \int_{\theta_0}^{I(t)} (\alpha + \beta t + \gamma t^2 + \delta t^3 + \varepsilon t^4) dt \tag{13}
\]
Using (7)
\[
= \theta_0 \{ \alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\varepsilon t^5}{5} \} \tag{14}
\]
Total amount of Inventory held during \([0, t] \]
\[
I_1 = \int_{0}^{t} I(t) dt = \int_{0}^{t} \{ (\alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\varepsilon t^5}{5}) \} dt \tag{15}
\]
\[
\int_{0}^{t} \{ (\alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\varepsilon t^5}{5}) \} dt + \theta_0 \int_{0}^{t} \{ (\alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\varepsilon t^5}{5}) \} dt - \theta_0 \int_{0}^{t} \{ (\alpha t + \frac{\beta t^2}{2} + \frac{\gamma t^3}{3} + \frac{\delta t^4}{4} + \frac{\varepsilon t^5}{5}) \} dt \tag{16}
\]
\[
= I(t^2 - t^2) - I(t^2 - t^2) + \frac{\beta t^2}{2} - \frac{\gamma t^3}{3} - \frac{\delta t^4}{4} - \frac{\varepsilon t^5}{5} \tag{17}
\]
\[
= I(t^2 - t^2) - I(t^2 - t^2) + \frac{\beta t^2}{2} - \frac{\gamma t^3}{3} - \frac{\delta t^4}{4} - \frac{\varepsilon t^5}{5} \tag{18}
\]
\[
= I(t^2 - t^2) - I(t^2 - t^2) + \frac{\beta t^2}{2} - \frac{\gamma t^3}{3} - \frac{\delta t^4}{4} - \frac{\varepsilon t^5}{5} \tag{19}
\]
For least average cost put \( \frac{dC(t_1)}{dt_1} = 0 \)

\[
D(t) [ \frac{C_0}{2} t_1^2 + \frac{C_0}{2} + \frac{C_0}{T} t_1 - C_2 ] = 0
\]

\[
[ \frac{C_0}{2} t_1^2 + \frac{C_0}{2} + \frac{C_0}{T} t_1 - C_2 ] = 0
\]

Which is quadratic in \( t_1 \) with last term negative so, it has at least one positive root say \( t_1^* \), and \( \frac{d^2C(t_1^*)}{dt_1^*2} < 0 \). So optimum value of \( t_1 \) is \( t_1^* \). Hence the optimum value of \( S \) is

\[
S^* = (\alpha t_1^* + \beta t_1^{*2} + \gamma t_1^{*3} + \delta t_1^{*4}) + \beta_0 (\frac{\alpha t_1^{*2}}{2} + \beta t_1^{*3})
\]

\[
+ \frac{\gamma t_1^{*4}}{4} + \frac{\delta t_1^{*5}}{5} + \frac{\beta_0 t_1^{*6}}{6}
\]

(20)

Minimum value of \( C(t_1) \) is

\[
C(t_1^*) = \frac{C_0}{2} \left( \frac{\alpha t_1^{*2}}{2} + \beta t_1^{*3} + \frac{\gamma t_1^{*4}}{4} + \frac{\delta t_1^{*5}}{5} + \frac{\beta_0 t_1^{*6}}{6} \right)
\]

\[
+ \frac{C_0}{T} t_1^* - C_2 \left\{ \alpha \left(\frac{T}{2} - t_1^*\right) + \beta \left(\frac{T^2}{3} - t_1^{*2}\right) + \gamma \left(\frac{T^3}{4} - t_1^{*3}\right) + \delta \left(\frac{T^4}{5} - t_1^{*4}\right) + \beta_0 \left(\frac{T^6}{6} - t_1^{*6}\right) \right\}
\]

(21)

Thus (20) gives optimal value of total average cost per unit time.

4. Conclusion

Here, an Inventory model has been created for items depleted due to demand as well as Deterioration by taking demand as a biquadratic polynomial function of time and constant deterioration rate and I have obtained minimum total average cost. This model can be extended further for other values of demand.

References


[16] Srivastava Saurabh and Singh Harendra,“Deterministic Inventory Model for Items with Linear Demand,variable Deterioration and partial backlogging”.Int.J.Inventory Research,Vol.4,No.4, 2017.
