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Mathematical Model on Reliability with Three Units

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Abstract: ***The evolution of the new systems is directly or indirectly connected with betterment in the old systems and hence the efficiency. Thus assessment of reliability of equipment is of great importance in the context of rapidly growing technology and its further development. A large number of studies have been carried out to evaluate the reliability by taking two-unit set-up under different conditions. This paper presents the study of the execution of steam turbine running in electric power-station. Execution of any set-up can be computed with the help of its availability and reliability. In many combined cycle power plants, the turbine generator consists of steam turbines interconnected to each other. Here one steam turbine is considered as one unit. In this paper, a system is considered in which three indistinguishable units (steam turbine) are considered in which two units are functional and third is taken as an understudy. If two units do not perform, the working of third unit is discontinued till the availableness of the other functional unit. Here inspection before failure is conducted to check functional units in a regular and schedule time. Operating units are inspected before failure to analyse whether a functional units required normal maintenance or not, otherwise repair of the failed unit. Here a independent single server system is supposed which execute normal maintenance inspection and reparation. Inspection before failure and maintenance are taken in priority over repair of discontinued unit. Also supposed that unit is failed and under reparation, no pre-failure scheduled inspection is undertaken. In this paper, system will be analyzed to determine various reliability measures by using mathematical tools MTSF/MTBF, Markov chain, Markov process, renewal process etc.***

Keywords: maintenance, availability, busy period, Inspection, steam turbine

1. Introduction

A thermal power station is a power station in which heat energy is converted to electric power. In most of area in the world-wide, the steam turbines are steam driven. A steam turbine is a machine that extricate thermal energy from compressed steam and utilize it to do automated work therefore steam turbines are used to originate a collection of apparatus type pattern of various sizes and speeds such that all production section taken in power generation and gas industries. Although there are considerable dissimilarities in set-up or plan, complexness, steam working order, dimensions of steam turbines and undergo to the identical non-performance or discontinuance technique. To hold reliable turbine performance, there require being an effectual framework, inspecting the functional working order. Here one steam turbine is considered as one unit. In this paper, a system is considered in which three indistinguishable units (steam turbine) are considered in which two units are functional and third is taken as an understudy. If two units do not perform, the working of third unit is discontinued till the availableness of the other functional unit. Here inspection before failure is conducted to check functional units in a regular and schedule time. Operating units are inspected before failure to analyse whether a functional units required normal maintenance or not, otherwise repair of the failed unit. Here a independent single server system is supposed which execute normal maintenance inspection and reparation. Inspection before failure and maintenance are taken in priority over repair of discontinued unit. Also supposed that unit is failed and under reparation, no pre-failure scheduled inspection is undertaken.

1.1 Description of system and Assumption

A working unit analyzed after a bounded or definite time period of its functioning and it is decided whether unit can running further or demand certain maintenance.

- The system having three indistinguishable units - Initially two unit is functional and third unit is kept as an understudy.
- System is supposed to be in Up-state if two units are working and in down state if one or no unit is working.
- Each of the units of the system has two modes-normal operative and failed.
- A normal functional unit is inspected for preventive maintenance before failure.
- The unit which is in understudy cannot fail.
- If an functional unit fails it is reparation by mender and no preventive maintenance inspection is carried out during the repair.
- Maintenance will be preferred over repair.
- Inspection time is too small to go for maintenance of second unit.
- A unit under maintenance would not fail.
- All the random variables are autonomous or freelance.

1.2 Notations

$\mho$: Compose of renewed or regenerative states
$\bar{\mho}$: Compose of non-renewed or non-regenerative states
O: Unit is in working state.
S: Unit is in understudy
O_i : Unit is under inspection
O_I : Continue under inspection from previous state.
m (t): pdf of maintenance time of an unit under inspection
M (t): cdf of maintenance time of an unit under inspection
π: Invariant failure rate of a unit

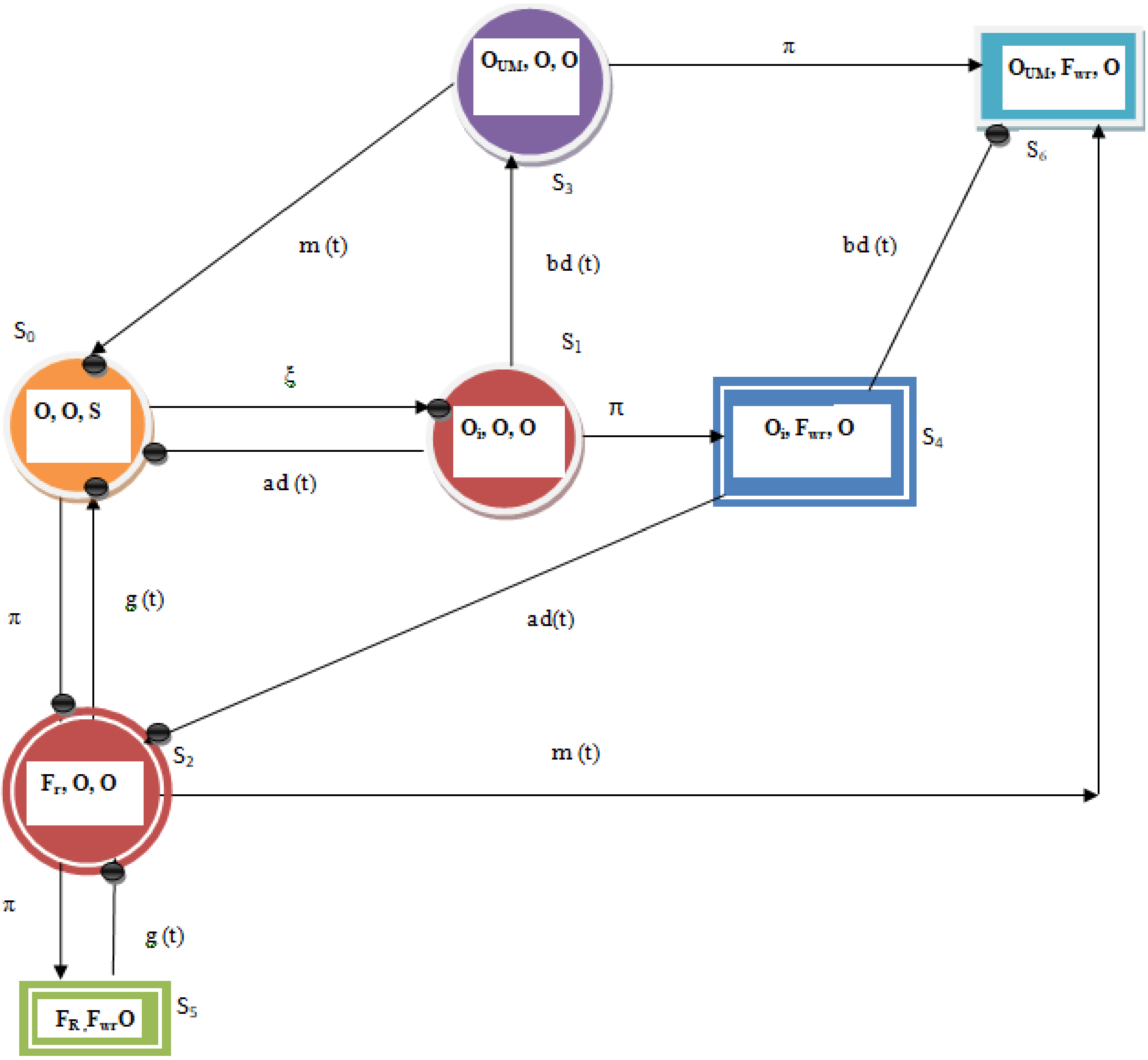
a: Chance of unit(steam turbine) required no maintenance
b: Chance of unit(steam turbine) required maintenance
d (t): pdf of observed period of a unit
D (t): cdf of observed period of a unit
α: Rate of inspection before failure
g (t), G (t): pdf and cdf of repair time period of a unsuccessful unit
O_{um}: Inspected unit under maintenance
O_{UM}: Maintenance of inspected unit continuous from previous state
F_r: unsuccessful unit under repair
F_R: Repair of unsuccessful unit continuous from previous state
F_{wr}, F_{WR}: A unsuccessful unit waiting for repair
©: Symbol for Convolution
Δ Symbol for Laplace convolution
ʘ Symbol for Laplace Stieltjes Convolution
Down-state

Regenerative Point

The system can be in any of the following states with respect of the above symbols:

RS_0 = (O,O,S)
RS_1 = (O_i,O,O)
RS_2 = (F_r, O,O)
RS_3 = (O_{um},O,O)
RS_4 = (O_I, F_{Wr},O)
RS_5 = (F_R, F_{wr},O)
RS_6 = ($F_{wr,}$ O,O_{UM})

State Transition Diagram



2. Transition Probabilities

The era of entering into states {RS_0 , RS_1, RS_2 , RS_3} are regenerative states. The transition probabilities from the states RS_i to RS_j are given by Q_{ij} and in the steady states Tp_{ij} denotes the transition probability from states RS_i to RS_j are given under

$Tp_{01} = \xi/(\xi+\pi)$
$Tp_{02} = \pi/(\xi+\pi)$
$Tp_{10} = a\, d^*(\pi)$
$Tp_{13} = b\, d^*(\pi)$
$Tp_{14} = \{1- d^*(\pi)\}$
$Tp_{20} = g^*(\pi)$
$Tp_{30} = m^*(\pi)$

$Tp_{25} = \{1- g^{*}(\pi)\}$
$Tp_{36} = \{1- m^{*}(\pi)\}$
$Tp_{42} = a$
$Tp_{46} = b$
$Tp_{52} = 1$
$Tp_{62} = 1$
$Tp_1{}^{(4)}{}_2 = a\,\{1- d^{*}(\pi)\}$
$Tp_1{}^{(4)}{}_6 = b\,\{1- d^{*}(\pi)\}$
$Tp_2{}^{(5)}{}_2 = \{1- g^{*}(\pi)\}$

It can be easily verified that
$Tp_{01} + Tp_{02} = 1$
$Tp_{10} + Tp_{13} + Tp_{14} = 1$
$Tp_{20} + Tp_{25} = 1$
$Tp_{30} + Tp_{36} = 1$
$Tp_{42} + Tp_{46} = 1$
$Tp_1{}^{(4)}{}_2 + Tp_1{}^{(4)}{}_6 = Tp_{14}$
$Tp_2{}^{(5)}{}_2 = Tp_{25}$

Mean Sojourn Times

To compute the mean sojourn time $\mu_i(t)$ for state RS_i, let T_i be sojourn time for state RS_i. Then

$$\mu_i(t) = \lim_{t\to\infty} \int_0^t P[t: 0 < t < T]\,dt$$

So that in steady state we have following relations
$\mu_0(t) = 1/(\xi+\pi)$
$\mu_1(t) = \{1- d^{*}(\pi)\}/\pi$
$\mu_2(t) = [1- g^{*}(\pi)]/\pi$
$\mu_3(t) = [1- m^{*}(\pi)]/\pi$

The unconditional mean time taken by the system to transit from any states RS_i to RS_j is mathematically given by

$$m_{ij} = \int_0^\infty t\,dQ_{ij}(t) = -q_{ij}^{*}(s)'/_{\text{at } s=0}$$

So that
$m_{01} = \xi / (\xi + \pi)^2$

$m_{02} = \pi / (\xi + \pi)^2$
$m_{10} = a\,d^{**'}(\pi)$
$m_{13} = b\,d^{**'}(\pi)$
$m_{14} = [\{1 - d^{*}(\pi)\} / \pi] + d^{*'}(\lambda)$
$m_{20} = - g^{*'}(\pi)$

$m_{25} = [\{1- g^{*}(\pi)\} / \pi] + g^{*'}(\pi)$

$m_{30} = - m^{*'}(\pi)$
$m_{36} = [\{1- m^{*}(\pi)\} / \pi] + m^{*'}(\pi)$

It can be easily verified that
$m_{01} + m_{02} = \mu_0(t)$
$m_{10} + m_{13} + m_{14} = \mu_1(t)$
$m_{20} + m_{25} = \mu_2(t)$
$m_{30} + m_{36} = \mu_3(t)$

Mean Time to System Failure

Let $\Omega_i(t)$ be the cdf of the first and foremost transition time from regenerative state i to a failed state, respecting the failed state as absorbing state. So the recursive relations for the Mean Time to System Failure (MTSF) are given by the following equations

$\Omega_0(t) = Q_{01}(t) \odot \Omega_1(t) + Q_{02}(t) \odot \Omega_2(t)$
$\Omega_1(t) = Q_{10}(t) \odot \Omega_0(t) + Q_{13}(t) \odot \Omega_3(t) + Q_{14}$
$\Omega_2(t) = Q_{20}(t) \odot \Omega_0(t) + Q_{25}(t)$
$\Omega_3(t) = Q_{30}(t) \odot \Omega_0(t) + Q_{36}(t)$

Above these equation can be Solving by taking Laplace Stieltjes transformations and solving for $\Omega_0^{**}(s)$,we get

$$\Omega_0^{**}(s) = U(s) / V(s)$$

Where
$U(s) = q_{14}\,q_{01} + q_{02}q_{25} + q_{01}q_{36}\,q_{13}$
$V(s) = - q_{01}q_{13}q_{30} + 1 - q_{01}q_{10} - q_{02}q_{20}$

$$MTSF = \Omega_0 = \lim_{s\to 0}[\{1-\Omega_0^{**}(s)\}/s] = \{V'(0)-U'(0)\}/V(0) = \frac{U}{V}$$

Where
$U = m_{01} + m_{02} + (m_{13} + m_{14})\,p_{01} + m_{10}p_{13}p_{30} + p_{02}\,(m_{20} + m_{25}) + p_{01}p_{13}\,(m_{30} + m_{36})$
$V = p_{01}p_{13}p_{36} + p_{02}p_{25} + p_{01}p_{14}$

Availability of the system - (Av)-

Let $Av_i(t)$ be the chance or probability that the system which is in upstate at instant of time't' given that the system entering in the regenerative state i at $t=0$. Then the recursive relations for the point wise availability $Av_i(t)$ of the system is given by
$Av_0(t) = M_0(t) + q_{01}(t)\,\Delta\,Av_1(t) + q_{02}(t)\,\Delta\,Av_2(t)$
$Av_1(t) = M_1(t) + q_{10}(t)\,\Delta\,Av_0(t) + q_1{}^{(4)}{}_2(t)\,\Delta\,Av_2(t) + q_{13}(t)\,\Delta\,Av_3(t) + q_1{}^{(4)}{}_6(t)\,\Delta\,Av_6(t)$
$Av_2(t) = M_2(t) + q_{20}(t)\,\Delta\,Av_0(t) + q_2{}^{(5)}{}_2\,\Delta\,Av_2(t)$
$Av_3(t) = M_3(t) + q_{30}(t)\,\Delta\,Av_0(t) + q_{36}(t)\,\Delta\,Av_6(t)$
$Av_6(t) = q_{62}(t)\,\Delta\,Av_2(t)$

Where
$M_0(t) = \int_0^t e^{-(\xi+\pi)t}$
$M_1(t) = \int_0^t e^{-\pi t}\,\overline{D(t)}$
$M_2(t) = \int_0^t e^{-\pi t}\,\overline{G(t)}$
$M_3(t) = \int_0^t e^{-\pi t}\,\overline{M(t)}$

Now solving these equations by taking Laplace transform and solving for $Av_0^{*}(s)$,we get

$$Av_0^{*}(t) = U_1(s)/V_1(s)$$

The steady states availability is given by
$Av_0^{**} = (\lim_{s\to 0} Av_0^{*}(s)) = U_1(0)/V_1(0)$
here
$U_1(0) = - \mu_2\,[p_{01}p_{13}p_{36} + p_{01}p_{14} + p_{02}] + \mu_1 p_{01}p_{20} - \mu_3 p_{01}p_{13}p_{20} - p_{20}\mu_0$

And
$D_1(0) = 0$
$D_1'(0) = - m_{20}\,(p_{01}p_{14} + p_{01}p_{13}p_{36} - p_{02}) - [p_{10}p_{25} + p_{20}\,(1 - p_{10})]$
$m_{01} - m_{02}p_{20} - (m_{10} + m_{13} +$
$m_1{}^{(4)}{}_2 + m_1{}^{(4)}{}_6)\,p_{01}p_{20}$
$+ m_2{}^{(5)}{}_2(p_{01}p_{10} + p_{01}p_{13}p_{30} - 1)$
$- m_{62}\,(p_{01}p_{20}\,p_1{}^{(4)}{}_6 + p_{01}p_{20}p_{13}p_{36})$
$- p_{01}p_{13}p_{20}\,(m_{30} + m_{36})$

Busy Period Analysis

The recursive relations for the busy period $BP_i(t)$ of the system is given by

$BP_0(t) = q_{01}(t) \Delta\ BP_1(t) + q_{02}(t) \Delta\ BP_2(t)$
$BP_1(t) = q_{10}(t) \Delta\ BP_0(t) + q_1{}^{(4)}{}_2\ (t) \Delta\ BP_2(t) + q_{13}(t) \Delta\ BP_3(t) + q_1{}^{(4)}{}_6\ (t) \Delta\ BP_6(t)$
$BP_2(t) = S_2(t) + q_{20}(t) \Delta\ BP_0(t) + q_2{}^{(5)}{}_2(t) \Delta\ BP_2(t)$
$BP_3(t) = q_{30}(t) \Delta\ BP_0\ (t) + q_{36}(t) \Delta\ BP_6(t)$
$BP_6(t) = q_{62}(t) \Delta\ BP_2\ (t)$

Where
$U_2(t) = \int_0^t e^{-\pi t}\, G(t)\, dt + \int_0^t (\pi e^{-\pi t} © 1) G(t)\, dt$

Now solving these equations by taking Laplace transform and find $BP_0{}^*(s)$, we get
$BP_0{}^*(s) = U_2\ (s)/V_1(s)$

Then for steady states
$BP_0{}^{**} = \lim_{s \to 0}\ (s\, BP_0{}^*(s)) = U_2(0) / V_1`(0)$
Where $U_2(0) = -(p_{01}p_{14} + p_{02} + p_{01}p_{13}p_{36})\ \mu_2$
$D_1{}'(0)$ is already defined

Maintenance Time

Let K_i is the Maintenance time starting from a regenerative states S_i at t=0 is given by
$K_0(t) = q_{01}(t) \Delta\ K_1(t) + q_{02}(t) \Delta\ K_2(t)$
$K_1(t) = q_{10}(t) \Delta\ K_0(t) + q_1{}^{(4)}{}_2(t) \Delta\ K_2(t) + q_{13}(t) \Delta\ K_3(t) + q_1{}^{(4)}{}_6(t) \Delta\ K_6(t)$
$K_2(t) = q_2{}^{(5)}{}_2(t) \Delta\ K_2(t) + q_{20}(t) \Delta\ K_0(t)$
$K_3(t) = X_3(t) + q_{30}(t) \Delta\ K_0(t) + q_{36}(t) \Delta\ K_6(t)$
$K_6(t) = X_6 + q_{62}(t) \Delta\ K_2(t)$
Where
$X_3\ (t) = \mu_3$
$X_6\ (t) = \mu_6$

Now solving these equations by taking Laplace transform and solving for $K_0{}^*(s)$, we get
$K_0{}^*(t) = U_3(s) / V_3(s)$
$K_0{}^{**} = \lim_{s \to 0}\ (s\, K_0{}^*(s)) = U_3(0) / V_1`(0)$
Where
$U_3(0) = (p_{13}p_{36}\ \mu_6 + p_{13}\mu_3 + p\ _1{}^{(4)}{}_6\ \mu_6)\ (p_{01}p_{25} - p_{01})$
$V_1{}'(0)$ is already defined

Inspection Time before Failure

Let Γ_i is the inspection time starting from a regenerative states S_i at t=0 is given by
$\Gamma_0(t) = q_{01}(t) \Delta\ \Gamma_1(t) + q_{02}(t) \Delta\ \Gamma_2(t)$
$\Gamma_1(t) = N_1(t) + q_{10}(t) \Delta\ \Gamma_0(t) + q_1{}^{(4)}{}_2(t) \Delta\ \Gamma_2(t) + q_{13}(t) \Delta\ \Gamma_3(t) + q_1{}^{(4)}{}_2(t) \Delta\ \Gamma_2(t)$
$\Gamma_2(t) = q_{20}(t) \Delta\ \Gamma_0(t) + q_2{}^{(5)}{}_2 \Delta\ \Gamma_2(t)$
$\Gamma_3(t) = q_{30}(t) \Delta\ \Gamma_0(t) + q_{36}(t) \Delta\ \Gamma_6(t)$
$\Gamma_6(t) = q_{62}(t) \Delta\ \Gamma_2(t)$
Where $N_1\ (t) = \mu_1$

Now solving these equations by taking Laplace transform and solving for $\Gamma_0{}^*(s)$, we get
$\Gamma_0{}^*(s) = U_4(s) / V_4(s)$

In the steady states
$\Gamma_0{}^{**} = \lim_{s \to 0}\ (s\Gamma_0{}^*(s)) = U_4(0) / D_1`(0)$
Where
$U_4(0) = -\ p_{01}p_{20}\ \mu_1$
$D_1{}'(0)$ is already defined

Particular cases:

If we take repair rate and inspection time as negative binomial distributions as

$g(t) = \gamma e^{-(\gamma)t}$ $i(t) = \delta e^{-(\delta)t}$

Then we get,
$p_{01} = \xi / \xi + \pi$
$p_{02} = \pi / \xi + \pi$
$p_{10} = a\delta / \delta + \pi$
$p_{13} = b\delta / \delta + \lambda$

$p_{14} = \pi / \pi + \delta$
$p_{20} = \sigma / \pi + \sigma$
$p_{25} = \pi / \pi + \sigma$
$p_{30} = m/m+\pi$
$p_{36} = \pi / m+\pi$
$p_{42} = a$
$p_{46} = b$
$p_{62} = 1$
$p_{52} = 1$
$p_1{}^{(4)}{}_2 = a\pi / \delta + \pi$
$p_1{}^{(4)}{}_6 = b\,\pi / \pi + \delta$
$p_2{}^{(5)}{}_2 = \pi / (\pi + \delta)$

Mean Sojourn Time:

$\mu_0 = 1 / \xi + \pi$
$\mu_1 = 1 / \delta + \pi$
$\mu_2 = 1/\pi + \sigma$
$\mu_3 = 1 / m + \pi$
$\mu_4 = 1 / \delta$
$\mu_4 = 1 / \sigma$
$\mu_4 = 1 / m$

Unconditional Mean Time:

$m_{01} = \xi / (\xi + \pi)^2$
$m_{02} = \pi / (\xi + \pi)^2$
$m_{10} = a\delta / (\delta + \pi)^2$
$m_{13} = b\delta / (\delta + \pi)^2$
$m_{14} = \pi / (\delta + \pi)^2$
$m_{20} = \sigma / (\pi + \sigma)^2$
$m_{25} = \pi / (\pi + \sigma)^2$
$m_{30} = m / (\pi + m)^2$
$m_{36} = \pi / (\pi + m)^2$
$m_{42} = a/\delta$
$m_{46} = b/\delta$
$m_{52} = 1/\sigma$
$m_{62} = 1/m$

The expectable outcomes based on above particular cases can be explained with the graphs as following:

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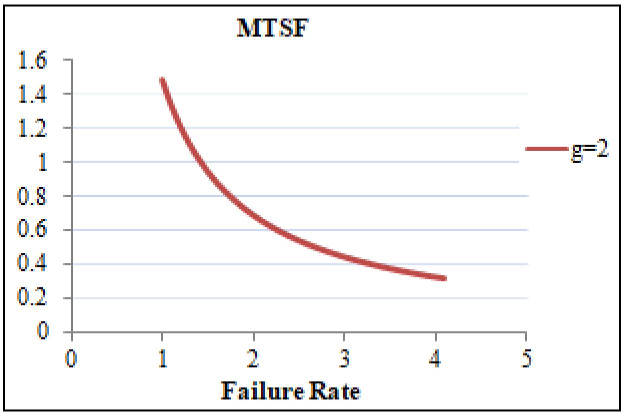

Figure 4.12: MTSF vs Failure rate

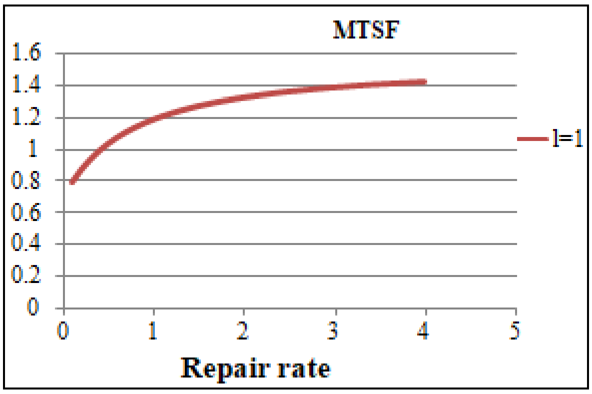

Figure 4.13: MTSF vs Repair rate

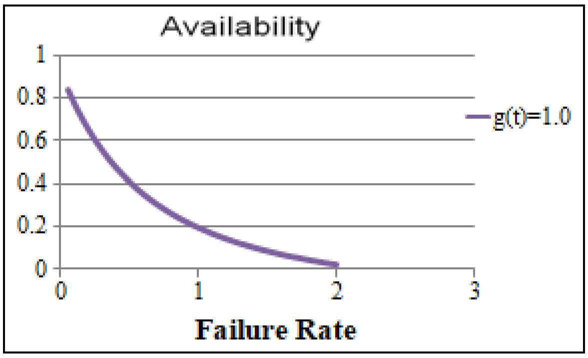

Figure 4.14: Availability vs Failure rate

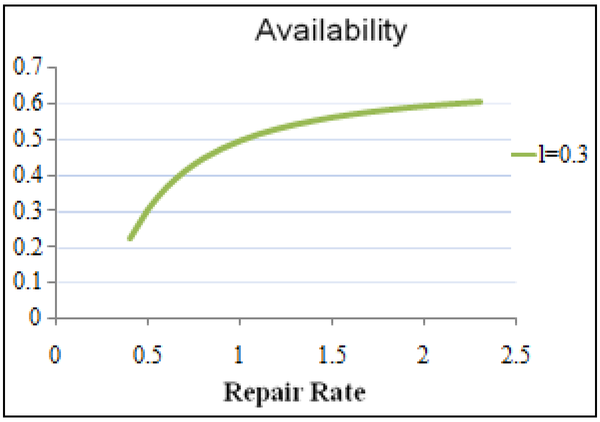

Figure 4.15: Availability vs Repair rate

References

[1] Jui-Hsiang Chian and John Yuan, (2001) , optimal maintenance policy for a deteriorating production system under inspection,,

[2] Tuteja, R.K., and Gulshan, T. "cost – benefit analysis of a two- server-two unit warm standby system with different type of failure", Microelectron. Reliab., 1992, VOL.32,p1353-1359.

[3] Brown, M., Proschan, F. (1983) “Imperfect repair”, Journal of Applied Probability, 20: 851–859.

[4] Jiang, R. and Jardine, A.K.S. (2005) “Two Optimization Models of the Optimum Inspection Problem”,Journal of the Operational Research Society,56: 1176–1183.

[5] H. Wang. A survey of maintenance policies of deteriorating systems. European Journal of Operational Research, 139(3):469–489, 2002