

Visualization of Spin 1 Divided by N through Spinning Light Clocks

Tao Jia

Email: [tjcoch\[at\]outlook.com](mailto:tjcoch[at]outlook.com)

Abstract: Spinning light clocks are employed to conduct the visualization of spin $1/N$ (where N is a positive integer). An object with spin $1/N$ looks same if it is turned round revolutions of an integer number of N ($N \times 360$ degrees). For a spinning clock, the two mirrors rotate around a common point that is between them. There are two types of spinning light clocks: one is A - type; the other B - type. For an A - type spinning light clock, the distances between the common point (around which the two mirrors rotate) are same, and the distances are different for a B - type spinning light clock. The general method to visualize spin $1/N$ is provided, and specific examples to visualize spin $1/2$ and spin $1/3$ are given.

Keywords: spinning light clock; spin $1/N$; visualization

1. Introduction

Light clock [1 - 2] is a conceptual clock that is usually used to illustrate the time dilation phenomena in uniform moving frames in special relativity. Recent discovery of the ergodicity of the behavior of time is based on the inventions of spinning light clocks [3] and rotational light clocks [4]. The ergodicity means all possibilities including time dilation, time contraction, and time conservation. Time contraction is revealed by spinning light clocks [3], and time conservation is revealed by some rotational light clocks [4], and the time conservation here means that the elapsed time in moving frames is equal to that in stationary frame. For a light clock, its half tick corresponds to the time interval for the light ray to travel from one mirror to the other mirror. If we compare a stationary light clock (shown in fig.1) and a uniform moving light clock (shown in fig.2), we can see that the light ray travels longer in each step (in which the light ray leaves one mirror and reaches the other mirror), so time dilation happens in the uniform moving frame. In fig.1 and fig.2 below, the letter A and B represent the positions of the two mirrors of the light clock.

If we compare a stationary light clock and a spinning light clock [3], we can find that time contraction happens in the spinning frame due to the reason that the light ray travels shorter in each step from one mirror to the other mirror in the spinning light clock.

Quantum mechanics [5 - 6] is another outstanding theory that has been proved to be quite successful in describing the world in atomic and subatomic levels. However, the two giants of relativity and quantum mechanics seem not to be compatible, and scientists are always compelled to find one single mathematical framework to explain all kinds of physical phenomena. The concept of spin [5 - 6] is originated in quantum mechanics, and the visualization of spin $1/2$ seems impossible in our observed world because we never see this; it is hard for us to imagine there is an object that can only retrace itself if it is turned round revolutions of an

integer multiple of two (2×360 degrees).

In this paper, an overview of spinning light clocks will be first given. There are two kinds of spinning light clocks: A - type spinning light clock and B - type spinning light clock, and then the general method on how to visualize spin $1/N$ will be interpreted, and finally specific examples of visualization of spin $1/2$ and spin $1/3$ are given.



Figure 1: Stationary light clock

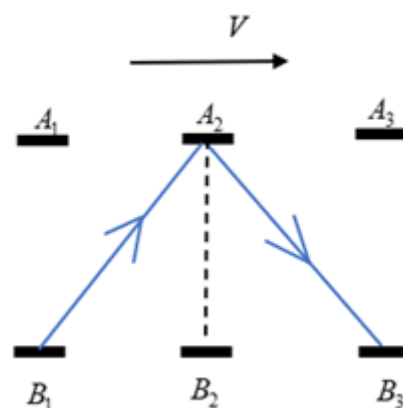


Figure 2: Uniform moving light clock with constant translational velocity

As we know from current college physics books on physics, the relationship between the time interval in uniform moving frame and that in stationary frame is established as the

following:

$$\Delta t_{UMoving} = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}} \Delta t_{Stationary} \quad (1)$$

where $\Delta t_{UMoving}$ and $\Delta t_{Stationary}$ are the time intervals in uniform moving and stationary frames respectively.

2. Spinning Light Clock

2.1 Spinning light clock (A - type)

In fig.3, the two mirrors rotate around a point and the distance between the point and the two mirrors are same. This is called A - type spinning light clock.

The letter $A_i (i = 1, 2, 3 \dots)$ and $B_i (i = 1, 2, 3 \dots)$ represent the positions of the two mirrors in different times. Initially, a light ray starts from point B_1 and then hits the top mirror whose position is at the point A_2 in the next step, and then light ray leaves from point A_2 and reaches point B_3 , and then from B_3 to A_4 , and continue this pattern of leaving from one mirror and reaching the other mirror.

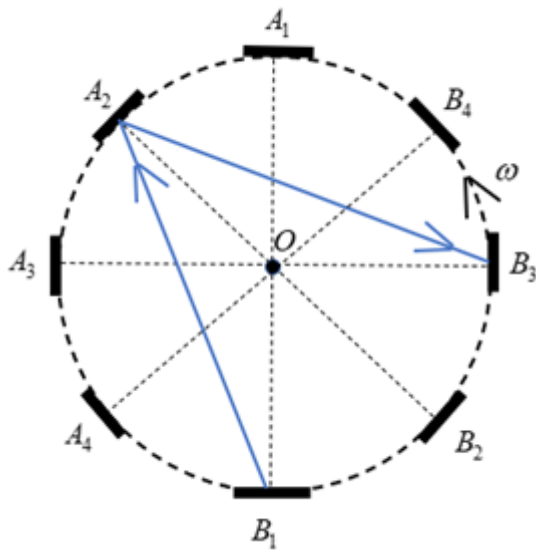


Figure 3: Spinning Light Clock (A - type) with constant angular velocity [6]

The relationship between the time interval in the spinning frame (A - type) is established as the following [3]:

$$\frac{(\Delta t_{Spinning-A})^2}{1 + \cos(\omega \cdot \Delta t_{Spinning-A})} = \frac{1}{2} (\Delta t_{Stationary})^2 \quad (2)$$

where $\Delta t_{Spinning-A}$ and $\Delta t_{Stationary}$ are the time intervals in the spinning (A - type) and stationary frames respectively, and ω is the angular velocity. From eq (2), we know that

time contraction happens in the spinning frame; it is mathematically shown in eq (3):

$$\frac{\Delta t_{Spinning-A}}{\Delta t_{Stationary}} = \sqrt{\frac{1 + \cos(\omega \cdot \Delta t_{Spinning-A})}{2}} \leq 1 \quad (3)$$

2.2 Spinning light clock (B - type)

In fig.4, the two mirrors rotate around a point, and the distance between the point and the two mirrors are different. This is called B - type spinning light clock.

The letter $A_i (i = 1, 2, 3 \dots)$ and $B_i (i = 1, 2, 3 \dots)$ represent the positions of the two mirrors in different times. Initially, a light ray starts from point B_1 and then hits the top mirror whose position is at the point A_2 in the next step, and then light ray leaves from point A_2 and reaches point B_3 , and then from B_3 to A_4 , and continue this pattern of leaving from one mirror and reaching the other mirror.

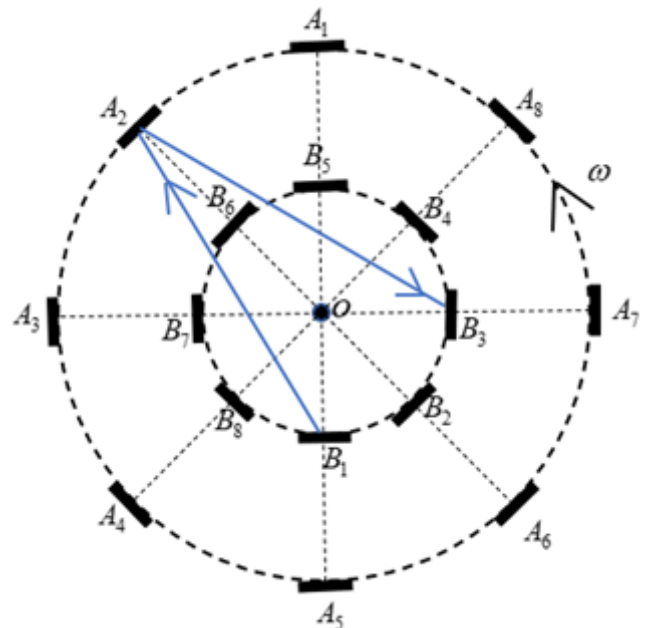


Figure 4: Spinning Light Clock (B - type) with constant angular velocity [6]

The relationship between the time interval in the spinning frame (B - type) is established as the following [4]:

$$\frac{(\Delta t_{Spinning-B})^2}{\left(\frac{k^2 + 1 + 2k \cos(\omega \cdot \Delta t_{Spinning-B})}{(k+1)^2} \right)} = (\Delta t_{Stationary})^2 \quad (4)$$

where $\Delta t_{Spinning-B}$ and $\Delta t_{Stationary}$ are the time intervals in the spinning (B - type) and stationary frames respectively,

k is the ratio of the two radiuses, and ω is the angular velocity. Now we explain the meaning of Δt . The distances between the point (around which the two mirrors rotate) and the two mirrors are different, so this results in two different distances (the distances between the point and the two mirrors) that are named two radiuses. From eq (2), we know that time contraction happens in the spinning frame; it is mathematically shown in eq (5):

$$\frac{(\Delta t_{Spinning-B})^2}{(\Delta t_{Stationary})^2} = \frac{k^2 + 1 + 2k \cos(\omega \cdot \Delta t_{Spinning-B})}{(k+1)^2} \leq 1 \tag{5}$$

3. General method to visualize spin 1/N

Spin 1/N means that the object retraces itself after every Nth revolution. A complete revolution corresponds to 360 degrees.

Initially, the photon is at the bottom mirror of the spinning light clock (A-type or B-type), and the bottom mirror is at the lowest point of the circular. The clock is said to have a character of spin 1/N if the photon goes back to the bottom mirror after the clock makes every Nth revolution (N revolutions mean the clock rotates $360 \times N$ degrees).

A concept named half-tick number (denoted as p) is introduced here. Half - tick number is a positive integer, and this number corresponds to the number of time intervals. One time interval is the amount of time corresponding to the half tick of the light clock, and it corresponds to the one - way situation in which the light ray leaves one mirror and reaches the other mirror.

The angle through which the spinning light clock spins in each step (in which the light ray leaves one mirror and reaches the other mirror) is denoted as α , and it is determined as the following eq (6):

$$\alpha = \frac{360 \times N}{2p} \tag{6}$$

The following proves that if and only if p and N are coprime, the spin 1/N can be realized. First, if p and N are coprime, we know that the light clock retraces itself after every Nth revolution. Second, if p and N are not coprime, it

means that $\frac{N}{p}$ can be reduced to $\frac{M}{q}$; the two are equal:

$$\frac{N}{p} = \frac{M}{q} \tag{7}$$

where $M < N$ and $q < p$, M and q are coprime.

So we have:

$$\frac{360 \times N}{2 \times p} = \frac{360 \times M}{2 \times q} \tag{8}$$

From (8) we know that spin 1/M happens to the clock; that means that the clock retraces itself after every Mth revolution, not Nth revolution.

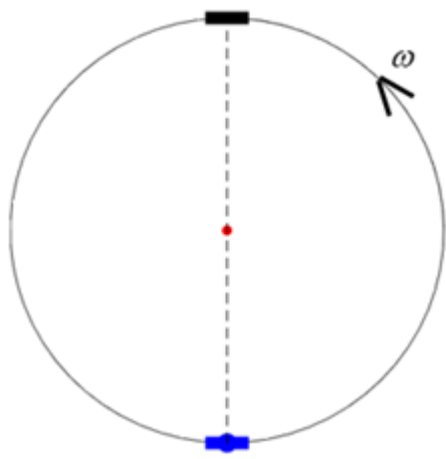
In the next step, two examples of visualization of spin 1/2 and spin 1/3 will be given.

4. Specific examples: Visualization of spin 1/2 and spin 1/3

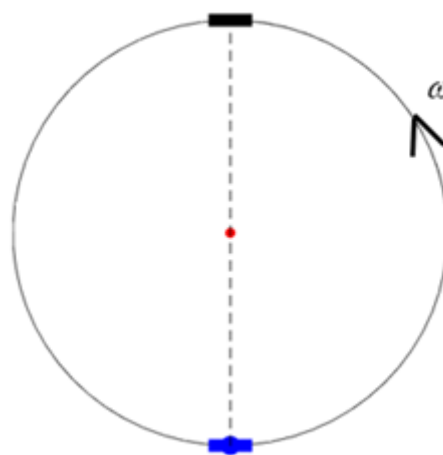
The first example is on spin 1/2. We first let $N = 2$, and then we need a number p that is coprime with 2, and we choose $p = 5$ here. This arrangement means the spinning light clock spins 54 degrees (according to eq. (6)) corresponding to the movement of the photon leaving mirror and reaching the other mirror in each step. In fig.5 and fig.6, the visualizations of spin 1/2 by two types (A-type and B-type) of spinning light clocks are shown respectively.

The second example is on spin 1/3. We first let $N = 3$, and then we need a number p that is coprime with 3, and we choose $p = 10$ here. This arrangement means the spinning light clock spins 54 degrees (according to eq (6)) corresponding to the movement of the photon leaving mirror and reaching the other mirror in each step. In fig.5 and fig.6, the visualizations of spin 1/3 by two types (A-type and B-type) of spinning light clocks are shown respectively.

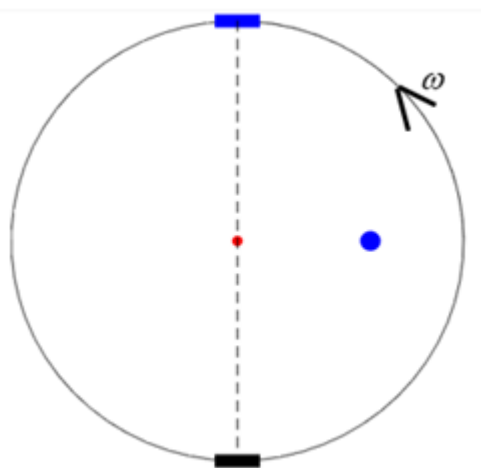
In fig.5- 8, the two mirrors are marked by two different colors of blue and black, the blue disk represents the photon, and the red spot represents the point around which the two mirrors rotate. Initially, the photon and one of the two mirrors (marked by blue color) are at the same position, and after the clock rotates two revolutions (as shown in fig.5-6) or three revolutions (as shown in fig.7-8), the photon and the blue mirror go together at the bottom position; this means the clock retraces its original state after two revolutions (in fig. 5 - 6) or three revolutions (in fig.7- 8).



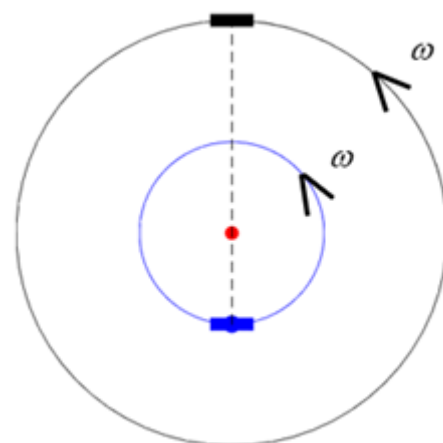
(a) Initial State



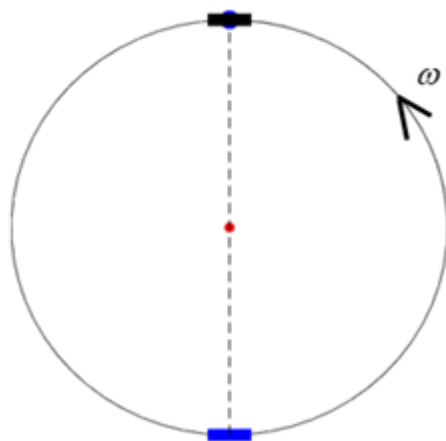
(d) After two revolutions (going back to initial state)



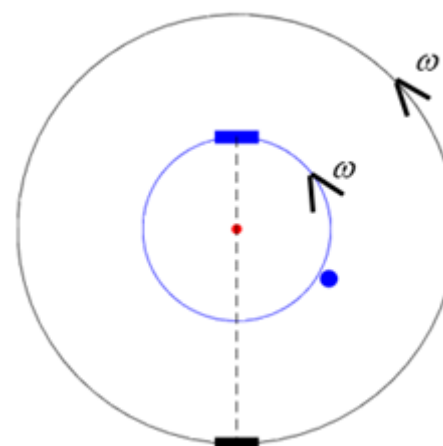
(b) After half revolution



(a) Initial State

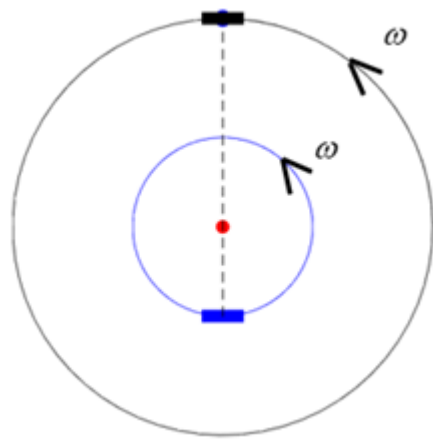


(c) After one revolution

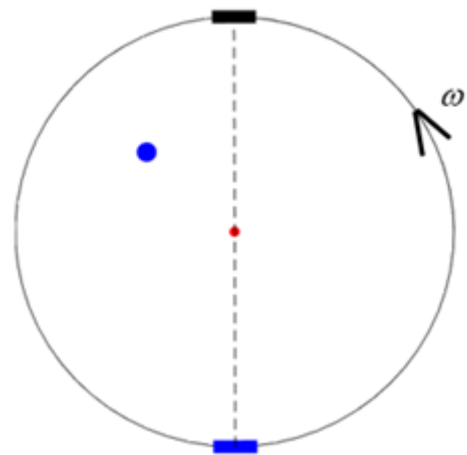


(b) After half revolution

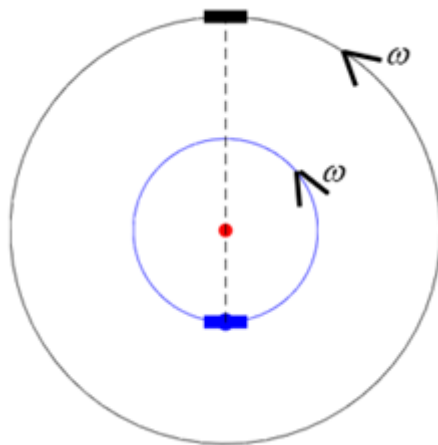
Figure 5: Visualization of spin 1/2 by A-type spinning light clock



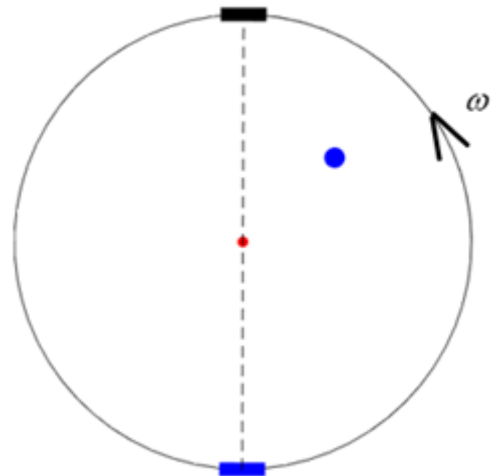
(c) After one revolution



(b) After one revolution

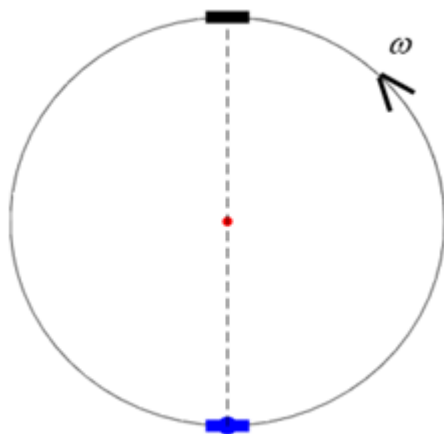


(d) After two revolutions (going back to initial state)

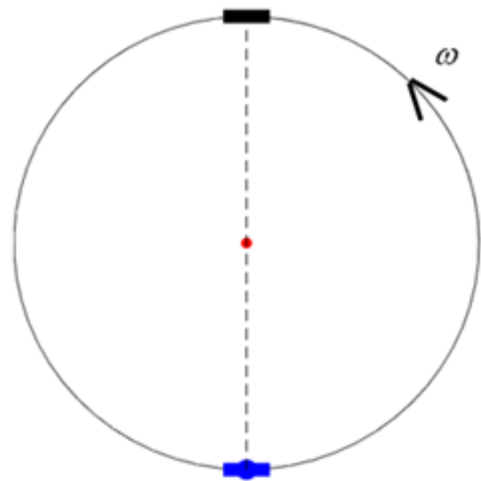


(c) After two revolutions

Figure 6: Visualization of spin 1/2 by B-type spinning light clock

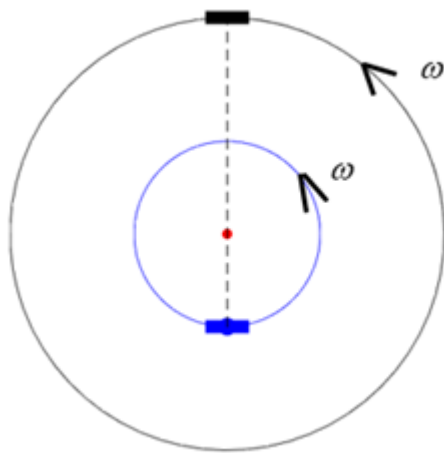


(a) Initial State

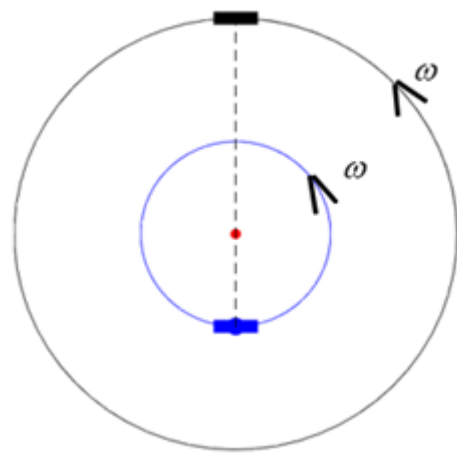


(d) After three revolutions (going back to initial state)

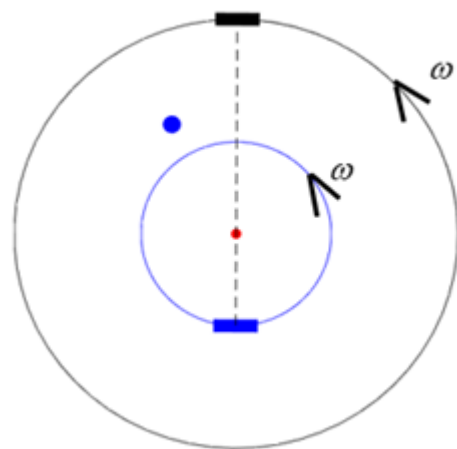
Figure 7: Visualization of spin 1/3 by A-type spinning light clock



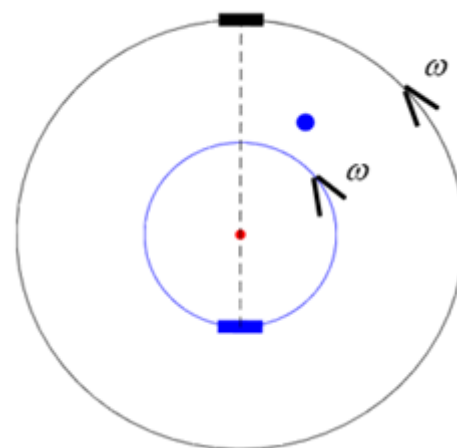
(a) Initial State



(d) After three revolutions (going back to initial state)



(b) After one revolution



(c) After two revolutions

Figure 8: Visualization of spin 1/3 by B-type spinning light clock

5. Conclusion

A general method to visualize spin 1/N is given and specific examples of visualization of spin 1/2 and spin 1/3 are shown. The visualization method is based on spinning light clock in which the two mirrors rotate around a point that is between them.

References

- [1] G. F. R. Ellis, R. M. Williams, Flat and Curved Space Times, Oxford University Press, 2000.
- [2] R. Ferraro, Einstein's Space - Time: An Introduction to Special and General Relativity, Springer Science, 2007.
- [3] T. Jia, Spinning Light Clocks Reveal Time Contraction in Spinning Noninertial Frames, International Journal of Science and Research, 10 (5), 2021.
- [4] T. Jia, Rotational Light Clocks and Visualization of Spins, International Journal of Science and Research, 10 (5), 2021.
- [5] J. Polkinghorne, Quantum theory: A Very Short Introduction, Oxford University Press, 2002.
- [6] S. W. Hawking, A Brief History of Time, Bantam Books, 1996.