Analysis of a Motion having Opposite Slopes

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Abstract: ***This paper gives an overview of how to analyze a curved path having opposite slopes momentarily and to find a way to know its inherent variable. This is achieved by how we use Weierstrass and Jordan's definition of continuity and making the path function ψ(x) zero. I am considering a particular situation during motion in which the slopes of the curve of motion at two instants become opposite.***

Keywords: Polynomial; Slopes; Path function; Continuity

1. Introduction

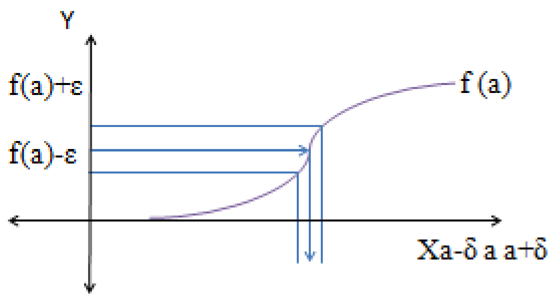
If a portion of a curved motion is defined by a function, let's say a path function, $\psi(x)$ which is defined by any real polynomial, having a particular situation in which their slopes or derivative of the path function becomes opposite to one another than in that portion of the motion for an instant the function certainly becomes zero. To analyze this particular case during a motion between the position $[(x_1, y_1), (x_2, y_2)]$ where the slopes of the path become opposite to the initial slope, let the slope of the curve at $\psi(x_1)$, say m_o and the slope of the curve at $\psi(x_2)$, say m_e such that $(m_o/m_e)<0$ then there is at least one position in between (x_1, y_1) & (x_2, y_2) let it be (x_o, y_o) where path function $\psi(x)$ must be zero momentarily.

2. Calculation

Let $m_o= \Delta Y_o/\Delta X_o$ and $m_e=\Delta Y_e /\Delta X_e$, since we have taken a particular condition when slopes are opposite hence $\Delta Y_o/\Delta X_o = -\eta\Delta Y_e/\Delta X_e$ where η is any proportionality constant such that $\eta \in R$ and $\eta \neq 0$. Let $\Delta Y_o= \psi(x_1)$ and $\Delta Y_e= \psi(x_2)$

Hence, $\psi(x_1) / \Delta X_o = -\eta\psi(x_2)/ \Delta X_e$
$\Rightarrow \Delta X_e\, \psi(x_1) + \eta\, \Delta X_o\psi(x_2) = 0$
$\Rightarrow \psi(x_1) + \eta\, (\Delta X_o/\Delta X_e)\psi(x_2) = 0$
Or, $\psi(x_1) + \Omega\psi(x_2) = 0$
let $\Omega = \eta\, (\Delta X_o/\Delta X_e)$ …………………… [1]

Now let's observe Weierstrass and Jordan definition of continuity for a while,



From above graph a real-valued function f is said to be continuous at an open interval D if for any chosen number $\varepsilon>0$, we can observe a corresponding number $\delta>0$ such that for $x_o-\delta< x < x_o+\delta$ and the value of f(x) satisfies $f(x_o)-\varepsilon < f(x) < f(x_o)+\varepsilon$ or, $|f(x) - f(a)| < \varepsilon$ for all values of x for which $|x - a| < \delta$.

According to the statement $|x - a| <\delta \Rightarrow |f(x) - f(a)| < \varepsilon$ is equivalent to the statement $x\in [a-\delta , a+\delta] \Rightarrow f(x)\in [f(x_o)-\varepsilon , f(x_o)+\varepsilon]$ and this definition can be utilized in our particular situation.

To prove our particular situation of real polynomial, slopes are opposite at the points $[(x_1, y_1), (x_2,y_2)]$, and there is a point 'r' in between these two points 'r'= (x_o ,y_o) such that path function $\psi(x)$ is continuous there.

Now let $\psi(r) \neq 0$ so $|\psi(r)| >0$, here we can take ε as $\{\psi(r)/n\} >0$, here n be any number >1.

If $\psi(r) >0$ then, we can take $\{\psi(r)/n\} > 0$
$\Rightarrow \psi(r)-\psi(r)/n <\psi(x) <\psi(r)+\psi(r)/n$
$\Rightarrow \psi(x)>\psi(r)/n>0$ and $|x - x_o| < \delta$ …………… [2]

If $\psi(r) < 0$ then, we can take $\{-\psi(r)/n\} > 0$
$\Rightarrow \psi(r) + \psi(r)/n <\psi(x) <\psi(r)-\psi(r)/n$
so, $\psi(x) <\psi(r)/n < 0$ and $|x - x_o| < \delta$ …………[3]

From equation [2] and [3] when $\psi(r) \neq 0$ we get $\psi(x) > 0$ and $\psi(r) >0$ for all $x\in\ ]r -\delta , r +\delta[$, let the interval is such that, position [p, q] lies within $[r -\delta , r +\delta]$. But if $\psi(r) \neq 0 \Rightarrow \psi(p) >0 \Rightarrow \exists\ \delta > 0$ such that $\psi(x) > 0$ and $\psi(q) >0 \Rightarrow \exists\ \delta > 0$ such that $\psi(x) > 0$ in the interval $[r -\delta , r +\delta]$, and but for opposite slops situation we have to consider opposite relation $\psi(p) >0$ and $\psi(q) < 0$ or $\psi(p) <0$ and $\psi(q) > 0$. Or, since $|\psi(x)-\psi(r)| <\varepsilon$ for all values in the interval $[r -\delta , r +\delta]$, such that $\psi(p)$ if greater than zero and $\psi(q)$ is less than zero or vice versa for making slopes opposite.

But as we have seen if when $\psi(r) \neq 0$ we get $\psi(p) >0$ and $\psi(x) > 0$. And from equation [1], $\psi(x_1) = -\Omega\psi(x_2)$ and Ω is positive we have seen this above and here we have observed the path, such that $\psi(p) >0$ and $\psi(q) < 0$ or $\psi(p) <0$ and $\psi(q) > 0$, therefore we must have a value of $\psi(r)=0$ for a position 'r' in between points (x_1 ,y_1) & (x_2, y_2).

3. Conclusion

This point 'r' where path function equal to zero, thus can also be regarded as an initial point for further motion. This is the point of half wavelength in simple harmonic motions or a kind of the point of return in SHM. When the path function becomes zero we can solve the polynomial to get values of any variable. And the converse of the above situation is also true if a path function is defined by any real polynomial, making such a curved path where at any instant path function becomes zero then the slopes of the path just before and after coming from that instant will always be opposite to one another.

References

[1] https://en.wikipedia.org/wiki/Continuous_function
[2] https://math.berkeley.edu/~kmill/math1afa2014/poly.pdf