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Inventory Model with Demand as a Polynomial Function of Time and Time Dependent Deterioration

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Abstract: ***In the present paper, an inventory model is generated for deteriorating items with shortages which are fully reserved. Demand rate is assumed as polynomial function of time and deterioration rate is dependent of time.***

Keywords: Inventory Model, Demand, Deterioration, Cost

1. Introduction

Inventory management has become the most concerning thing in working of any organization. Deterioration of items has affected inventory management. Deterioration takes place for many reasons like environment, weather, time, etc., for example, some items are only useable in a certain season, some items deteriorate with time, like food products. So there is a need for Inventory management keeping in mind the effect of deterioration. In history, many researchers have worked in this direction and created certain models. Some of them are listed here based on Demand and Deterioration.

Datta & Pal (1988) [3], Lee & Wu (2002) [7], Sharma, Sharma & Ramani (2012) [16] and Sharma & Preeti (2013) [15] considered Power demand pattern for items that deteriorates with time, using varying deterioration in their respective models. Wu (1999) [20], Wu (2002) [19], Lee & Wu (2002) [7], Skouri et. Al. (2009)[18],Sharma et. Al. (2012)[16] considered Weibull distributed deterioration in their respective models.

Sharma et. Al. (2012) [16], Karmakar et.al. (2014) [6], Ibe et. Al. (2016) [5], Shah (2018) [14] considered time varying holding cost in their respective models. Lee (2004) [8] created model with exponential distributed deterioration and Wu (2002) [19] & Ghosh (2004) [4] created model with time varying quadratic demand. Wu (1999) [20] and Skouri (2009)[18]developed models with ramp type demand rate.

Ouyang (2005) [12], Shah (2010) [13] and Aliyu (2020) [1] developed models with exponentially declining demand. Mukherjee (2010) [11] developed a model in which the time of duration of shortages varies directly with deterioration. Bhowmick (2011) [2] et. Al., developed a model with continuous production model for deteriorating items with shortages.

Maragatham (2017) [10] et. Al., presented Model for Items in a single warehouse and assumed constant lead time. Sharma (2018) [17] developed a model for items that deteriorates with time, such as fruits, vegetables, and foodstuffs by considering demand as time-dependent. Long (2019) [9] demonstrated that structural deterioration affects the value of damage detection information. In the present paper, working is done based on the above papers by considering demand as a polynomial function of time and time-dependent deterioration.

2. Assumptions and Notations

2.1 Notations

The following are the notations used here:-

1) C_1 = Inventory Holding Cost per unit per unit time.
2) C_2 = Shortage cost per unit per unit time.
3) C_3 = Deterioration cost per unit per unit time.
4) T = Length of each cycle.
5) I(t) = Inventory at any time t.
6) C(t) = Average total cost.
7) D(t) = Demand Rate
8) I(t) = Deterioration Rate Function
9) S = Initial Inventory

2.2 Assumptions

The following are the assumptions used here:-

1) Demand Rate D(t) is assumed as polynomial function of time, given by $D(t) = t + 2t^2 + 3t^3 + ::: + nt^n$.
2) The deterioration rate function, $\theta_0(t)$ is assumed in the form $\theta(t) = \theta_0 t$; $0 < \theta_0 << 1$; $t > 0$.
3) Replenishment size is constant and the replenishment rate is infinite.
4) The Lead time is zero.
5) Shortages are considered and are totally reserved.
6) During the period T, neither is replacement nor repair of deteriorated units.

3. Analysis of Model

Let Inventory level at any time t be I(t). Inventory level slowly decreases during time interval $(0, t_1)$, $t_1 < T$ and becomes exactly zero at $t = t_1$. Shortages takes place in the interval $(0,t_1)$, which are totally reserved. Differential equations which governs this inventory system during the interval $0 \le t \le T$ using demand and deterioration rate are

$$\frac{dI(t)}{dt} + \theta_0 t\, I(t) = -(t + 2t^2 + 3t^3 + \cdots + nt^n) \qquad (1)$$

and

$$\frac{dI(t)}{dt} = -(t + 2t^2 + 3t^3 + \cdots + nt^n) \qquad (2)$$

Solution of differential equation (1) is

$$I(t)e^{\frac{\theta_0}{2}t^2} = -\int (t + 2t^2 + 3t^3 + \cdots + nt^n)e^{\frac{\theta_0}{2}t^2}dt + C$$

$$= -\int (t + 2t^2 + 3t^3 + \cdots + nt^n)\left(1 + \frac{\theta_0}{2}t^2\right)dt + C$$

$$= -\int \left[(t + 2t^2 + 3t^3 + \cdots + nt^n) + \frac{\theta_0}{2}(t + 2t^2 + 3t^3 + \cdots + nt^n)\right]dt + C$$

$$= -\left[\left(\frac{1}{2}t^2 + \frac{2}{3}t^3 + \cdots + \frac{n}{n+1}t^{n+1}\right) + \frac{\theta_0}{2}\left(\frac{1}{4}t^4 + \frac{2}{5}t^5 + \cdots + \frac{n}{n+3}t^{n+3}\right)\right] + C$$

Putting $t = 0$, $I(0) = C$. But $I(0)=S$. Therefore $C = S$. Thus

$$I(t)e^{\frac{\theta_0}{2}t^2} = S - \left[\left(\frac{1}{2}t^2 + \frac{2}{3}t^3 + \cdots + \frac{n}{n+1}t^{n+1}\right) + \right.$$
θ0214t4+25t5+… +nn+3tn+3; 0≤t≤T (3)

Again from (3), $I(t_1)=0$. So

$$0 = S - [\left(\frac{1}{2}t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}t_1^{n+1}\right) + \frac{\theta_0}{2}\left(\frac{1}{4}t_1^4 + \frac{2}{5}t_1^5 + \cdots + \frac{n}{n+3}t_1^{n+3}\right)]$$

Thus

$$S = \left[\left(\frac{1}{2}t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}t_1^{n+1}\right) + \frac{\theta_0}{2}\left(\frac{1}{4}t_1^4 + \frac{2}{5}t_1^5 + \cdots + \frac{n}{n+3}t_1^{n+3}\right)\right] \qquad (4)$$

Putting the value of S in (3), we get

$$I(t)e^{\frac{\theta_0}{2}t^2} = \frac{1}{2}(t_1^2 - t^2) + \frac{2}{3}(t_1^3 - t^3) + \cdots + \frac{n}{n+1}(t_1^{n+1} - t^{n+1}) + \frac{\theta_0}{2}\left(\frac{1}{4}(t_1^4 - t^4) + \frac{2}{5}(t_1^5 - t^5) + \cdots + \frac{n}{n+3}(t_1^{n+3} - t^{n+3})\right)$$

Hence

$$I(t) = \frac{1}{2}(t_1^2 - t^2) + \frac{2}{3}(t_1^3 - t^3) + \cdots + \frac{n}{n+1}(t_1^{n+1} - t^{n+1}) + \frac{\theta_0}{2}\left[\frac{1}{4}(t^2 - t_1^2)^2 + \cdots + \frac{n}{(n+1)(n+3)}[(n+1)t_1^{n+3} - \right.$$
n+3t2t1n+1+2tn+3
(5)

$$I(t) = \sum_1^n\left[\frac{m}{m+1}(t_1^{m+1} - t^{m+1}) + \frac{\theta_0}{2}\frac{m}{(m+1)(m+3)}[(m + \right.$$
1t1m+3−m+3t2 t1m+1 +2tm+3 (6)

Solution of differential equation (2) is

$$I(t) = -\left(\frac{1}{2}t^2 + \frac{2}{3}t^3 + \cdots + \frac{n}{n+1}t^{n+1}\right) + A \qquad (7)$$

Since $I(t_1) = 0$, we have

$$0 = -\left(\frac{1}{2}t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}t_1^{n+1}\right) + A$$

This implies

$$A = \frac{1}{2}t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}t_1^{n+1}$$

Hence

$$I(t) = \frac{1}{2}(t_1^2 - t^2) + \frac{2}{3}(t_1^3 - t^3) + \cdots + \frac{n}{n+1}(t_1^{n+1} -$$
tn+1;t1≤t≤T (8)

Thus the entire amount of deteriorated units = I(0) – stock loss due to demand

$$= S - \int_0^{t_1}(t + 2t^2 + \cdots. + nt^n)dt$$

$$= S - \left(\frac{1}{2}t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}t_1^{n+1}\right)$$

$$= \left(\frac{1}{2}t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}t_1^{n+1}\right) + \frac{\theta_0}{2}\left(\frac{1}{4}t_1^4 + \frac{2}{5}t_1^5 + \cdots + \frac{n}{n+3}t_1^{n+3}\right) - \left(\frac{1}{2}t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}t_1^{n+1}\right)$$

$$= \frac{\theta_0}{2}\left(\frac{1}{4}t_1^4 + \frac{2}{5}t_1^5 + \cdots + \frac{n}{n+3}t_1^{n+3}\right)$$
(9)

Total value of inventory held in $[0,t_1]$ is

$$I_1 = \int_0^{t_1} I(t)dt$$

$$I_1 = \int_0^{t_1}\left[\frac{1}{2}(t_1^2 - t^2) + \frac{2}{3}(t_1^3 - t^3) + \cdots + \frac{n}{n+1}(t_1^{n+1} - t^{n+1})\right]dt + \frac{\theta_0}{2}\int_0^{t_1}[\frac{1}{4}(t^2 - t_1^2)^2 + \cdots + \frac{n}{(n+1)(n+3)}[(n+1)t_1^{n+3} - (n+3)t^2t_1^{n+1} + 2t^{n+3}]]$$

$$I_1 = \left(\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots + \frac{n}{n+2}t_1^{n+2}\right) + \frac{\theta_0}{2}\left(\frac{2}{15}t_1^5 + \cdots + \frac{2n}{3(n+4)}t_1^{n+4}\right)$$

Inventory Holding Cost = C_1 * total inventory

$$= C_1\left[\left(\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots + \frac{n}{n+2}t_1^{n+2}\right) + \frac{\theta_0}{2}\left(\frac{2}{15}t_1^5 + \cdots + \right.\right.$$
2n3n+4t1n+4 (10)

Deterioration Cost = C_3 *the entire amount of deteriorated units

$$= C_3\left[\frac{\theta_0}{2}\left(\frac{1}{4}t_1^4 + \frac{2}{5}t_1^5 + \cdots + \frac{n}{n+3}t_1^{n+3}\right)\right]$$
(11)

Shortage units Quantity $= \int_{t_1}^{T} -I(t)dt$

$$= -\int_{t_1}^{T}\left[\frac{1}{2}(t_1^2-t^2)+\frac{2}{3}(t_1^3-t^3)+\cdots+\frac{n}{n+1}(t_1^{n+1}-t^{n+1})\right]dt$$

$= T\left[\frac{1}{2}\left(\frac{1}{3}T^2-t_1^2\right)+\frac{2}{3}\left(\frac{1}{4}T^3-t_1^3\right)+\cdots+\frac{n}{n+1}\left(\frac{1}{n+2}T^{n+1}-\right.\right.$
t1n+1+13t13+12t14+…+nn+2t1n+2 (12)

Shortage Cost = C_2* shortage units quantity

$$= C_2T\left[\frac{1}{2}\left(\frac{1}{3}T^2-t_1^2\right)+\frac{2}{3}\left(\frac{1}{4}T^3-t_1^3\right)+\cdots+\frac{n}{n+1}\left(\frac{1}{n+2}T^{n+1}-t_1^{n+1}\right)\right]+$$

$$C_2\left[\frac{1}{3}t_1^3+\frac{1}{2}t_1^4+\cdots+\frac{n}{n+2}t_1^{n+2}\right] \quad (13)$$

The Total Cost per unit time
= Inventory Holding Cost + Deterioration Cost + Shortage Cost

$$= C_1\left[\left(\frac{1}{3}t_1^3+\frac{1}{2}t_1^4+\cdots+\frac{n}{n+2}t_1^{n+2}\right)+\frac{\theta_0}{2}\left(\frac{2}{15}t_1^5+\cdots+\frac{2n}{3(n+4)}t_1^{n+4}\right)\right]$$
$$+ C_3\left[\frac{\theta_0}{2}\left(\frac{1}{4}t_1^4+\frac{2}{5}t_1^5+\cdots+\frac{n}{n+3}t_1^{n+3}\right)\right]$$
$$+ C_2T\left[\frac{1}{2}\left(\frac{1}{3}T^2-t_1^2\right)+\frac{2}{3}\left(\frac{1}{4}T^3-t_1^3\right)+\cdots+\frac{n}{n+1}\left(\frac{1}{n+2}T^{n+1}-t_1^{n+1}\right)\right]+$$
$$C_2\left[\frac{1}{3}t_1^3+\frac{1}{2}t_1^4+\cdots+\frac{n}{n+2}t_1^{n+2}\right]$$

The Average Total Cost per unit time,

$$C(t_1) = \frac{1}{T}\,[\,Total\ Cost\ per\ unit\ time\,]$$

$$C(t_1) = \frac{C_1}{T}\left[\left(\frac{1}{3}t_1^3+\frac{1}{2}t_1^4+\cdots+\frac{n}{n+2}t_1^{n+2}\right)+\frac{\theta_0}{2}\left(\frac{2}{15}t_1^5+\cdots+\frac{2n}{3(n+4)}t_1^{n+4}\right)\right]$$
$$+\frac{C_3}{T}\left[\frac{\theta_0}{2}\left(\frac{1}{4}t_1^4+\frac{2}{5}t_1^5+\cdots+\frac{n}{n+3}t_1^{n+3}\right)\right]$$
$$+ C_2\left[\frac{1}{2}\left(\frac{1}{3}T^2-t_1^2\right)+\frac{2}{3}\left(\frac{1}{4}T^3-t_1^3\right)+\cdots+\frac{n}{n+1}\left(\frac{1}{n+2}T^{n+1}-t_1^{n+1}\right)\right]$$
$$+\frac{C_2}{T}\left[\frac{1}{3}t_1^3+\frac{1}{2}t_1^4+\cdots.+\frac{n}{n+2}t_1^{n+2}\right]$$

For minimum average total cost , the necessary and sufficient conditions are $\frac{dC(t_1)}{dt_1}=0$ *and* $\frac{d^2C(t_1)}{dt_1^2}>0$.

Now $\frac{dC(t_1)}{dt_1}=0$ *gives*

$$(t_1+2t_1^2+3t_1^3+\cdots+nt_1^n)\left[\frac{C_1\theta_0}{3T}t_1^3+\frac{C_3\theta_0}{2T}t_1^2+\frac{(C_1+C_2)}{T}t_1-C_2\right]=0$$

Which further implies

$$\left[\frac{C_1\theta_0}{3T}t_1^3+\frac{C_3\theta_0}{2T}t_1^2+\frac{(C_1+C_2)}{T}t_1-C_2\right]=0 \quad (14)$$

Since (14) is a cubic equation in t_1 having last term negative, thus it has at least one positive root. Also $\frac{d^2C(t_1)}{dt_1^2}>0$. Let t_1^* be the positive root of (14). So optimum value of t_1 is t_1^*. Substituting it in (4), the optimized value of S is

$S^* = \left[\left(\frac{1}{2}t_1^{*2}+\frac{2}{3}t_1^{*3}+\cdots+\frac{n}{n+1}t_1^{*(n+1)}\right)+\frac{\theta_0}{2}\left(\frac{1}{4}t_1^{*4}+\frac{2}{5}t_1^{*5}+\right.\right.$
…+nn+3t1(n+3)* (15)

Minimum value of $C(t_1)$ is

$$C(t_1^*) = \frac{C_1}{T}\left[\left(\frac{1}{3}t_1^{*3}+\frac{1}{2}t_1^{*4}+\cdots+\frac{n}{n+2}t_1^{*(n+2)}\right)+\frac{\theta_0}{2}\left(\frac{2}{15}t_1^{*5}+\cdots+\frac{2n}{3(n+4)}t_1^{*(n+4)}\right)\right]$$
$$+\frac{C_3}{T}\left[\frac{\theta_0}{2}\left(\frac{1}{4}t_1^{*4}+\frac{2}{5}t_1^{*5}+\cdots+\frac{n}{n+3}t_1^{*(n+3)}\right)\right]$$
$$+ C_2\left[\frac{1}{2}\left(\frac{1}{3}T^2-t_1^{*2}\right)+\frac{2}{3}\left(\frac{1}{4}T^3-t_1^{*3}\right)+\cdots+\frac{n}{n+1}\left(\frac{1}{n+2}T^{n+1}-t_1^{*(n+1)}\right)\right]$$
$$+\frac{C_2}{T}\left[\frac{1}{3}t_1^{*3}+\frac{1}{2}t_1^{*4}+\cdots.+\frac{n}{n+2}t_1^{*(n+2)}\right] \quad (16)$$

Thus equation (16) gives optimal value of total average cost per unit time. These equations can be further solved for different values of variables used here, using software's like Matlab and Mathematica.

4. Conclusion

In this paper, an inventory model is generated for items that deteriorate with time by considering demand as polynomial function of time with time dependent deterioration.

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