# Proof and Solution to Fermat’s Last Theorem Using Gap Analysis or Difference Analysis

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**Abstract:** ***The author is not much aware of Gap Analysis or Difference Analysis. This is the first time he will use it. This is to solve Fermat’s theorem.***

**Keywords:** Difference Analysis, Gap Analysis, Fermat’s Conjecture, Fermat’s Last Theorem, Simple Solution

## 1. Introduction

In number theory, Fermat's Last Theorem (sometimes called Fermat's conjecture, especially in older texts) states that no three positive integers *a*, *b*, and *c* satisfy the equation $a^n + b^n = c^n$ for any integer value of *n* greater than 2. The cases $n = 1$ and $n = 2$ have been known since antiquity to have infinitely many solutions. (Anonymous 2021)

Only one relevant proof by Fermat has survived, in which he uses the technique of infinite descent to show that the area of a right triangle with integer sides can never equal the square of an integer. His proof is equivalent to demonstrating that the equation

$$x^4 - y^4 = z^2$$

has no primitive solutions in integers (no pairwise coprime solutions). In turn, this proves Fermat's Last Theorem for the case $n = 4$, since the equation $a^4 + b^4 = c^4$ can be written as $c^4 - b^4 = (a^2)^2$.

(Freeman 2005)

A proof was published by Andrew Wiles 1995.

**Body**

| | | | | | |
|---|---|---|---|---|---|
| $0^3$ = 0 | | | | | |
| $1^3$ = 1 | $11^3$ = 1331 | $21^3$ = 9261 | $31^3$ = 29,791 | $41^3$ = 68,921 | $51^3$ = 132,651 |
| $2^3$ = 8 | $12^3$ = 1728 | $22^3$ = 10,648 | $32^3$ = 32,768 | $42^3$ = 74,088 | $52^3$ = 140,608 |
| $3^3$ = 27 | $13^3$ = 2197 | $23^3$ = 12,167 | $33^3$ = 35,937 | $43^3$ = 79,507 | $53^3$ = 148,877 |
| $4^3$ = 64 | $14^3$ = 2744 | $24^3$ = 13,824 | $34^3$ = 39,304 | $44^3$ = 85,184 | $54^3$ = 157,464 |
| $5^3$ = 125 | $15^3$ = 3375 | $25^3$ = 15,625 | $35^3$ = 42,875 | $45^3$ = 91,125 | $55^3$ = 166,375 |
| $6^3$ = 216 | $16^3$ = 4096 | $26^3$ = 17,576 | $36^3$ = 46,656 | $46^3$ = 97,336 | $56^3$ = 175,616 |
| $7^3$ = 343 | $17^3$ = 4913 | $27^3$ = 19,683 | $37^3$ = 50,653 | $47^3$ = 103,823 | $57^3$ = 185,193 |
| $8^3$ = 512 | $18^3$ = 5832 | $28^3$ = 21,952 | $38^3$ = 54,872 | $48^3$ = 110,592 | $58^3$ = 195,112 |
| $9^3$ = 729 | $19^3$ = 6859 | $29^3$ = 24,389 | $39^3$ = 59,319 | $49^3$ = 117,649 | $59^3$ = 205,379 |
| $10^3$ = 1000 | $20^3$ = 8000 | $30^3$ = 27,000 | $40^3$ = 64,000 | $50^3$ = 125,000 | $60^3$ = 216,000 |

**Figure 1:** Partial Table for Cubes

**Analysis**

**We could look for counterexamples.**

$X^3 + Y^3 \neq Z^3$
Which is also equal to $X^3 \neq Z^3 - Y^3$
Let X be the smallest integer among the three.
Let Y be an integer.
Ley Z be the greatest integer among the three.

Substitute 1 for X, 2 for Y and 3 for Z.
1 ≠ 27 – 8, difference of 19
As we observe in the table as Z increases and Y increases, the gap or difference is more.
Substitute 1 for X, 3 for Y and 4 for Z.
1 ≠64 – 27, difference of 37
So it is not possible for 1 to be X.

Let us try 2
Substitute 2 for X, 3 for Y and 4 for Z.
8≠ 64 – 27, difference of 37
As we observe in the table as Z increases and Y increases, the gap or difference is more.
Substitute 2 for X, 4 for Y and 5 for Z.
8≠125 – 64, difference of 61
So it is not possible for 2 to be X.

Let us try 3
Substitute 3 for X, 4 for Y and 5 for Z.
27≠125 – 64, difference of 61
As we observe in the table as Z increases and Y increases, the gap or difference is more.
Substitute 3 for X, 5 for Y and 6 for Z.
*27* ≠216 – 125, difference of 91
Substitute 3 for X, 1 for Y and 2 for Z.

$27 \neq (1 - 8)$ – interchange becomes $(8 - 1)$, difference of 20
As we observe in the table as Z decreases and Y decreases, the gap or difference is more.
So it is not possible for 3 to be X.

It is not possible for 4, 5, 6, … also to be X.
…
.
.
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The table could be extended to any number so is the analysis.

The analysis for higher superscripts or power is the same (use gap analysis or difference analysis).

Use another table then perform the analysis.
$X^4 \neq Z^4 - Y^4$
$X^5 \neq Z^5 - Y^5$
…

## 2. Conclusion

The Solution is correct. Fermat’s Last Theorem or Conjecture is correct. This is the simplest solution to Fermat’s. A Philippine journal rejected this. I hope this could be accepted internationally.

## References

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