Geometric Sequences and Series in Balinese Rebab and Its Potential Applications in Mathematics Learning

I Made Adi Wira Nata Putra[1], I Wayan Puja Astawa[2], I Gusti Putu Suharta[3]

[1, 2, 3]Department of Mathematics Education, Universitas Pendidikan Ganesha, Indonesia

Abstract: ***This study aims to determine the pattern of geometric sequences on the frequency of the chromatic tone in Balinese Rebab, to determine the application of geometric sequences in determining the location of the chromatic tones on Balinese Rebab musical instruments and to know the integration of the application of problems related to Balinese Rebab in learning mathematics. This research is a literature study research. The problems raised in this research will be discussed and studied theoretically. This study obtained the following results. The chromatic scale frequency series forms a geometric sequence with the formula*** $U_i = a \times 2^{\frac{1}{12}(i-1)}$***; the long strings of the Balinese Rebab meet the rules of geometric sequences*** $U_i = l_1 \times \left(\frac{1}{2}\right)^{\frac{1}{12}(i-1)}$***; The application of Balinese Rebab problems in learning mathematics can be carried out by focusing on problem solving, namely finding geometric sequence patterns in Balinese Rebab.***

Keywords: Balinese Rebab, Mathematics Learning

1. Introduction

Rebab is a traditional musical instrument that uses strings as a sound source and is played by swiping (Bandem, 2013). Balinese Rebab has its own uniqueness compared to Rebab from other regions. The uniqueness of the Balinese Rebab that distinguishes it from Rebab from the other regions is the appearance and decoration. Balinese rebab decorations are usually made of cowhide or metal (usually gold, silver, or copper) which are carved in such a way with Balinese carving motifs.

In general, the Balinese rebab has two strings as a sound source and a *babad* (a membrane made from the lining of a cow's stomach) as a resonator (Sukerta, 2011). One of the strings is swiped and then touched on certain parts so that the rebab produces a good tone. How to play the Balinese Rebab is very different from playing a stringed instrument such as a guitar, violin, or something else. On the guitar, there are frets that make it easy for guitar players to determine the location of the tones, but the Balinese Rebab does use fret, so there is no definite determinant to determine the location of the tone on Balinese Rebab. In order for the rebab player to be able to play the rebab well without producing discordant tones, the rebab player or *pangrebab* must have a reference in determining the location of the scales on the rebab. But in reality, in playing the Balinese Rebab, the musician only uses ear sensitivity to determine whether the tone produced by the rebab is appropriate or not.

The difficulty of playing Balinese Rebab can be overcome, if the location of the scales on the Balinese Rebab can be determined with certainty. There are several kinds of scales that are often used in music playing. Some of them are diatonic and pentatonic scales. In traditional musics, there are *pelog* and *selendro* scales that are often used in Balinese Gamelan. However, when it is viewed as a whole, all these scales are part of the chromatic scale. If it is assumed as a set, then the chromatic scale is the universe of the set of tones and the other scales are subsets of the chromatic scale. So it will be very easy to determine the location of the other scales if the location of the chromatic scale is known.

Chromatic scales are scales that have the same interval between tones, namely half a tone or semitone (Hartaya, 2013; Sri Mudjilah, 2010). The existence of the same interval between each two tones on the chromatic scale will cause the chromatic scale to have unique patterns. When we are talking about tones, actually we are talking about frequency. The patterns that emerge from the frequency range of the chromatic scale can be seen by analyzing the frequency of the tones.

To determine the location of the chromatic scale in Balinese Rebab, it is necessary to know how the strings vibrate to produce a certain frequency. The vibration of the strings can be analyzed physically using the theory of standing waves (Chris, 2009). If the patterns of the relationship between the frequency of a tone are obtained, then the patterns on the scale can be used to determine the pattern of string length to produce a certain frequency.

The patterns contained in the strings of tones and string's length will be very interesting if they are discussed further and the results are applied in learning activity. Problems regarding Balinese Rebab will be very interesting if they are raised and applied in learning activity. Raising the issue of musical instruments into learning activity, one of them was done by (Randy, 2013) that is raising the issue of measuring flutes in physics learning. The habit of introducing problems in teaching students and integrating various sciences to solve a problem will familiarize students with creative thinking in solving real problems.

Learning mathematics that focuses on problems can have a positive impact on students. This is in accordance with research conducted by (Sumartini, 2016) which states that problem-based learning is able to improve students' problem abilities. Learning with an emphasis on problems solving will certainly open students' insight into the theories that have been learned. Solving everyday problems is not enough just to use one mathematical equation. Daily problems that tend to be complex will require studies from various disciplines to solve them. Students' ability in connecting the dot is needed to integrate all the knowledge they have learned to solve a problem. This is where the importance of habituation of problem recognition and problem solving in the classroom so that students are accustomed to dealing with real problems.

Using Balinese Rebab as a problem in learning mathematics will also makes students aware with their own culture. It will introduce them that they have a unique culture. Also they can learn that their knowledge in science and mathematics can be used to solve a real problem such as the problem that is available in their culture.

2. Literature Survey

1) Research conducted by Randy Worland (2013), that is Flute Measurements in a Physics of Music Lab
2) Research conducted by Purwoko (2010), that is “Barisan Geometri pada Tangga Nada Diatonis*”*
3) Tina Sri Sumantri (2016) “Peningkatan Kemampuan Pemecahan Masalah Matematis Siswa melalui Pembelajaran Berbasis Masalah”
4) Ika Mustika Sari (2016) Sundanese Flute : from Art and Physics Perspective.

3. Methods / Approach

This research uses literature study method. The problems raised in this research will be discussed and studied theoretically. Various relevant theories will be collected and then used to solve the problems raised in this research. First, researchers will study theories related to standing waves, geometric sequences, music theory and Balinese Rebab theory which will then be mixed to find patterns found in Balinese Rebab. Furthermore, these theories will be the basis for this research which will then be used to build concepts to understand the existing problems (Wiratna, 2014).

Then to find the tone patterns and the location of the tones on the Balinese Rebab strings, the writer will first conduct a literature study, both literature on Balinese Rebab, mathematics, physics and music theory. For literature on Balinese Rebab the author uses the book "Learn Balinese Rebab 2" by Mr. Pande Made Sukarta, and the book "Gambelan Bali on the Stage of History" by Prof. I Made Bandem. Then literature on mathematics will be used books related to number theory, geometric sequences and others. For the study of standing wave theory, Chris Vuille (2009)'s physics book entitled "College Physics Eighth Edition" will be used as well as several other books that explain theories on string waves, one-dimensional waves and standing waves. For the chromatic tone theory, books by Hanna Sri Mudjilah, Agus Krisno, Kari Hartaya and several other books on music theory will be used. However, in the course of the research, it is possible that the author will explore information from various other books related to Balinese Rebab, both from the artistic and mathematical aspects.

After the various literatures have been obtained and studied, the writer then mixes and formulates and uses these sources to solve problems related to Balinese Rebab. The author then tries to find mathematical patterns found in Balinese Rebab, both in terms of tone and in terms of the length of the strings. Furthermore, the author examines aspects that can be raised and then applied to everyday mathematics education. The author will look for the right integration pattern between the Balinese Rebab problems with relevant mathematical concepts. Then based on the data obtained, conceptually formulated about ways to integrate problems in Balinese Rebab into mathematics learning.

In its application to learning, the author will formulate the integration of the Balinese Rebab problem in theoretical learning. The author will examine various sources of literature related to learning theory which can be adapted and applied to integrate the Balinese Rebab problem in learning mathematics. Of course, in the learning that will take place, students will be faced with problems that are not enough to only use mathematical tools to solve these problems, but also students need understanding from other branches of science besides mathematics so that the Balinese Rebab problem can be solved.

This type of research is a literature study research, where the researcher collects data in the form of theories, writings and studies that are relevant to the problems that will be discussed in this study. This research is descriptive which focuses on explaining the problems encountered systematically (Sanusi, 2016). The data collection method used in this research is documentation and recording of the sources used as a reference in this study. The data sources used are literature books on Balinese Rebab theory, Balinese Gambelan theory, music theory, standing wave theory, string theory and geometric sequences. The data analysis technique used in this research is content analysis technique. Namely, the author analyzes the contents of a reading source and literature to then draw a conclusion to solve the problem to be studied.

4. Results / Discussion

4.1 Determining Tone Frequency Using Geometric Sequences

Physically the frequency of the tone in the upper octave is twice the frequency of the tone in the previous octave. This implies that while the C1 tone in the first octave has a frequency of a Hz, then the C2 tone in the second octave has a frequency of 2a Hz. Based on this, it can be seen that the intervals of tones that are one octave apart form a geometric sequence with a ratio of 2.

Because in one octave tone interval there are 12 tones that have the same interval between adjacent tones, then mathematically the meaning is the same as inserting 11 other terms in a geometric sequence that has a ratio of 2. So it is obtained as follows.

$$C, C\#, D, D\#, E, F, F\#, G, G\#, A, A\#, B, c$$

$$a, ar, ar^2, ar^3, ar^4, ar^5, ar^6, ar^7, ar^8, ar^9, ar^{10}, ar^{11}, 2a$$

Based on this, there is a similarity, namely:

$$\frac{ar}{a} = \frac{2a}{ar^{11}}$$

$$r = \frac{2}{r^{11}}$$

$$r^{12} = 2$$

$$r = \sqrt[12]{2} = 2^{\frac{1}{12}}$$

So that the following table is obtained.

Table 1: Tone Frequency Pattern

n^{th} Term	Tone	Frequency
1	C	A
2	C#	$2^{1/12}a$
3	D	$2^{2/12}a$
4	D#	$2^{3/12}a$
5	E	$2^{4/12}a$
6	F	$2^{5/12}a$
7	F#	$2^{6/12}a$
8	G	$2^{7/12}a$
9	G#	$2^{8/12}a$
10	A	$2^{9/12}a$
11	A#	$2^{10/12}a$
12	B	$2^{11/12}a$
13	C	$2^{12/12}a$

From the table a formula can be made for the i^{th} term of the frequency sequence of the tones in that one octave.

$$U_i = a \times 2^{\frac{1}{12}(i-1)}$$

By setting the frequency of tone A at 440 Hz, we can determine the frequency of the other tones. This is shown in the following table.

Table 2: Tone Frequency Sequence Count Using Geometric Sequences

n^{th} Term	Tone	Frequency
1	C	261.6256
2	C#	277.1826
3	D	293.6648
4	D#	311.127
5	E	329.6276
6	F	349.2282
7	F#	369.9944
8	G	391.9954
9	G#	415.3047
10	A	440
11	A#	466.1638
12	B	493.8833
13	C	523.2511

4.2 Determining the Location of the Tone on the Balinese Rebab

The Balinese Rebab is played by swiping one of the strings, and in order to produce a harmonious tone, the fingers of the *pangrebab* must be placed on the strings at a certain distance. Traditionally, the exact distance between tones on the rebab strings is not determined. However, this can be overcome by calculating using geometric sequences.

Physically the relationship between string length, string tension, and the resulting frequency is shown by the following formula.

$$f = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

Or it can be written as

$$f = \frac{1}{2L}\sqrt{\frac{F}{\rho \times A}}$$

Dengan:

f : the frequency of the resulting tone,
L : the length of the string,
F : the tention of the string,
μ : mass of string per unit length,
ρ : density of the string,
A : cross-sectional area of the string.

Because only one string on Balinese Rebab that is played, the tension and mass per unit length are the same, so they can be ignored. This implies that in the Balinese Rebab game the most influential in determining the location of the tone is the length of the strings. So when viewed from the above equation, it is found that the resulting frequency is inversely proportional to the length of the string.

In accordance with the theory of physics, namely the frequency of the octave of a tone equal to twice the frequency of the tone, it can be written as follows. Suppose the frequency of a tone is then the frequency of the top octave tone (say) will be equal to. Suppose the length of the strings needed to produce a tone with a frequency is then the length of the string needed to produce a tone with a frequency is as follows.

$$f_1 = \frac{1}{2l_1}\sqrt{\frac{F}{\mu}} \text{...pers(1)}$$

$$f_2 = \frac{1}{2l_2}\sqrt{\frac{F}{\mu}}$$

$$2f_1 = \frac{1}{2l_2}\sqrt{\frac{F}{\mu}} \text{...pers(2)}$$

Then, substitute eq.1 and eq.2

$$2\frac{1}{2l_1}\sqrt{\frac{F}{\mu}} = \frac{1}{2l_2}\sqrt{\frac{F}{\mu}}$$

$$2\frac{1}{2l_1} = \frac{1}{2l_2}$$

$$2l_2 = l_1$$

$$l_2 = \frac{1}{2}l_1$$

From this equation we get that the length of the string to produce the top octave of the basic tone is half the length of

the string that produces the basic tone. If a tone in the first octave has a string length l_1, the second octave tone has a string length $\frac{1}{2}l_1$ in the third octave has a string length $\frac{1}{4}l_1$.
Thus, the length of the string producing tones that are one octave apart can be determined by a geometric sequence with the ratio between adjacent sequences being.

When we insert another 11 tones that have the same interval, i.e. tones in the sequence of tones that are one octave apart, then it is the same as inserting 11 terms in a geometric sequence. And because every adjacent tone has the same interval, the result of its insertion must also be a geometric sequence. If it is patterned it will form a pattern like the following.

$$l_1, rl_1, r^2l_1, r^3l_1, r^4l_1, r^5l_1, r^6l_1, r^7l_1, r^8l_1, r^9l_1, r^{10}l_1, r^{11}l_1, \frac{1}{2}l_1$$

From that sequence, it can be seen that.

$$\frac{\frac{1}{2}l_1}{r^{11}l_1} = \frac{rl_1}{l_1}$$

$$\frac{\frac{1}{2}}{r^{11}} = r$$

$$\frac{1}{2} = r^{12}$$

$$r = \sqrt[12]{\frac{1}{2}} = \left(\frac{1}{2}\right)^{\frac{1}{12}}$$

From this, a table of long strings on the Balinese Rebab can be made. The table is as follow

Table 3: Sequence of The Length of Rebab String Patterns

n^{th} Term	Tone	The Length of The String
1	C	l_1
2	C#	$\left(\frac{1}{2}\right)^{\frac{1}{12}} l_1$
3	D	$\left(\frac{1}{2}\right)^{\frac{2}{12}} l_1$
4	D#	$\left(\frac{1}{2}\right)^{\frac{3}{12}} l_1$
5	E	$\left(\frac{1}{2}\right)^{\frac{4}{12}} l_1$
6	F	$\left(\frac{1}{2}\right)^{\frac{5}{12}} l_1$
7	F#	$\left(\frac{1}{2}\right)^{\frac{6}{12}} l_1$
8	G	$\left(\frac{1}{2}\right)^{\frac{7}{12}} l_1$
9	G#	$\left(\frac{1}{2}\right)^{\frac{8}{12}} l_1$
10	A	$\left(\frac{1}{2}\right)^{\frac{9}{12}} l_1$
11	A#	$\left(\frac{1}{2}\right)^{\frac{10}{12}} l_1$
12	B	$\left(\frac{1}{2}\right)^{\frac{11}{12}} l_1$
13	C	$\frac{1}{2} l_1$

From the table it can be determined a formula for the ith term of the long row of rebab strings in one octave as follows.

$$U_i = l_1 \times \left(\frac{1}{2}\right)^{\frac{1}{12}(i-1)}$$

The length of the Balinese Rebab strings is an average of 42 cm, so the distance for each tone from the bottom end of the string is as follows.

Table 4: The Sequences of the Length of Rebab Strings Calculated Using Geometric Sequences

n^{th} Term	Tone	The Length of The String (cm)
1	C	42
2	C#	39.64272
3	D	37.41775
4	D#	35.31765
5	E	33.33542
6	F	31.46445
7	F#	29.69848
8	G	28.03164
9	G#	26.45834
10	A	24.97335
11	A#	23.5717
12	B	22.24872
13	C	21

Thus it can be formulated that if the length of the rebab string is *l* cm then the *i*th tone is located as far as $l \times \left(\frac{1}{2}\right)^{\frac{1}{12}(i-1)}$ cm from the bottom end of the string. By knowing the length of the strings of the rebab to produce a chromatic scale as in the table above, someone who wants to learn the Balinese Rebab instrument will find it easier to determine the location of the chromatic tones in the Balinese Rebab.

4.3 The Aplication of Balinese Rebab on Mathematics Learning

Mathematics learning that is integrated with Balinese Rebab problems can be applied in 11th grade SMA when studying geometric sequences and series. Students are expected to be able to relate the material for geometric sequences and series that they are studying, with the physics of string waves that they have studied in high school, and music theory that they have learned in junior high school. In addition, students are also expected to be able to use various useful applications to perform related calculations, such as Microsoft Excel for processing data and the tuner application (a measuring instrument for tone frequency) which can be used to measure the tone produced by the rebab.

The essence of the implementation of this learning is that students learn to solve Balinese Rebab problems using various materials they learn, mainly geometric sequences and series. Learning integrated with Balinese Rebab can be started by forming students into several groups. Furthermore, each group is assigned to solve problems regarding the calculation to determine the location of the chromatic scales in Balinese Rebab.

The tools and materials they need to do this learning are not difficult. The teacher does not need to prepare many rebabs, but just one is enough as an example. However, if you can prepare more than one rebab, of course it will be very good. The rebab shown by the teacher is only an example so that students know the parts of the rebab, the source of the sound and how the rebab is able to produce sound. Next, students will perform analytical calculations by modeling the rebab strings as standing waves. Then students also need a ruler or meter as a measuring tool for length, a tuner as a tone meter, a Microsoft Excel application or a calculator as a calculating tool.

First, students are formed into several groups. Next, the teacher explains how the rebab works when it produces sound. If possible, the teacher can show how to play the rebab. Or maybe, to make it more interesting, students who can play the rebab are welcome to play the rebab in front of the class. From the explanation of the rebab game, students are expected to be able to relate the concepts of standing waves to the strings. By using student worksheet students are provoked to be able to explore and innovate to solve the rebab problem. As an inducement, the student worksheet can explain a little about the chromatic scale, as well as the standing wave. Furthermore, students are expected to be able to relate all of these concepts so that an understanding arises that on the chromatic scale and the location of the tones on the rebab strings will form a geometric sequence.

In line with the explanation in the previous sub-chapter, it is hoped that students will be able to find that, on the rebab there will be a geometric sequence with the nth term of the chromatic scale, that is $U_i = a \times 2^{\frac{1}{12}(i-1)}$ and on the rebab string there is a geometric sequence and series for the location of the chromatic scale, that is $U_i = l \times \left(\frac{1}{2}\right)^{\frac{1}{12}(i-1)}$. Then, when the strings and series on the rebab strings are analyzed further, it will be found a convergent infinite series. This is because the ratio of the sequence is $\left(\frac{1}{2}\right)^{\frac{1}{12}}$ and where $0 < \left(\frac{1}{2}\right)^{\frac{1}{12}(i-1)} < 1$.

5. Conclusion

Based on the discussion above, it can be concluded as follows.

1) The formula for the ith term of the frequency of the tones belonging to the chromatic scale when calculated using a geometric sequence is as follows.

$$U_i = a \times 2^{\frac{1}{12}(i-1)}$$

Where a is the frequency of the fundamental tone

2) The formula for the ith term of the long string of Balinese Rebab strings when calculated using a geometric sequence is as follows.

$$U_i = l_1 \times \left(\frac{1}{2}\right)^{\frac{1}{12}(i-1)}$$

With is the length of the strings of the Balinese Rebab.

3) Balinese Rebab can also be used to support mathematics learning, as a problem to help student understanding mathematics contextually.

6. Future Scope

This integration of learning should be investigated further so that conclusions can be drawn on how effective this learning is if it is applied to students. Experiments need to be done so that the effectiveness of this learning can be known more clearly

References

[1] Afifudin, et.al. 2012. Metodelogi Penelitian Kualitatif. Bandung: Pustaka Setia
[2] Bandem, I Made. 2013. Gambelan Bali di Atas Panggung Sejarah. Denpasar: BP STIKOM BALI
[3] Chris Vuille. 2009. College Physics Eigth Edition.Canada: Nelson Education, Ltd.
[4] Hartaya, Kari. 2013. PIP PIANO 1. Jakarta: Kementerian Pendidikan dan Kebudayaan, Direktorat Pembinaan Sekolah Menengah Kejuruan
[5] Purwoko. 2010. Barisan Geometri dalam Tanga Nada Diatonis. Jurnal Pendidikan Matematika, Volume 1, NO.2, Juli 2007. Palembang: Universitas Sriwijaya.
[6] Randy Worland. 2013. Flute Measurements in a Physics of Music Lab. The Journal of the Acoustical Society of America 134, 4019; https://doi.org/10.1121/1.4830671
[7] Sanusi, Anwar. 2016. Metodologi Penelitian Bisnis. Jakarta: Salemba Empat
[8] Sri Mudjilah, Hanna. 2010. Teori Musik 1. Yogyakarta: Universitas Negeri Yogyakarta
[9] Sukerta, Pande Made. 2011. Belajar Rebab Bali Edisi 2. Surakarta: ISI Press Surakarta.
[10] Sumartini, Tina Sri. 2016. Peningkatan Kemampuan Pemecahan Masalah Matematis Siswa melalui Pembelajaran Berbasis Masalah. Garut: STKIP Garut. https://media.neliti.com/media/publications/226581-peningkatan-kemampuan-pemecahan-masalah-360cbfca.pdf
[11] Wiratna Sujarweni.V. 2014. Metode Penelitian. Yogyakarta: Pustaka Baru Perss

Author Profile

I Made Adi Wira Nata Putra born in Gianyar, September, 10th 1994. Email: adiwiranata104@gmail.com