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Markov Model for Distinguishing COVID-19 Positive and Negative Persons

Dr. Rajalekshmi .V .G

Associate Professor, Department of Mathematics, S.D College, Alapuzha, Kerala, India

rajisreeramam[at]gmail.com

Abstract: ***Corona virus, named as COVID-19 outbreak that began in china causes mild to severe respiratory illness leading to death of the affected patients. The best prevention method is to identify the people suffering from this virus and isolate them so as to avoid further spreading. Here a three stage compartmental model is developed by applying stochastic method to find the probability of the number of persons in each compartment.***

Keywords: Continuous time Markov Chain, Corona Virus, Compartmental Model, Stochastic Model

1. Introduction

The COVID-19 pandemic is an ongoing global pandemic of corona virus disease 2019, causes severe respiratory syndrome. Lab test enable a person to see whether he is affected by the virus. People who have symptoms of the disease are advised COVID test by the health department for effective control of disease and prevention of further spreading. All symptomatic people need not be COVID patients. Some People with symptoms of COVID-19 ,may have test result negative. This pandemic may affect people at any time. Here a compartmental mathematical model is created in the context of result obtained from the COVID test done in a symptomatic group. Persons with test result negative may not think they are safe from COVID-19 in future. This virus will affect any one at any time.

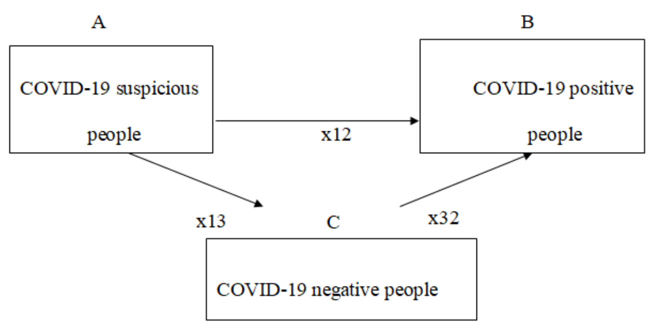
This model describes the movement of people in a region in two compartments according to the effect of COVID-19. Let there is a compartment where people who are suspicious to be affected by corona virus, arriving for COVID-19 test. From this compartment there is a movement to compartment B and C. Compartment C represent people who are free from the corona virus. There may be chance that the people in compartment C may be affected by corona virus after some time (Here time may be days or month) then there occur a movement of people from C to B.

Compartment models have been in vogue for a number of years to represent real world applications as a number of real hypothetical compartments. Here a three stage compartment is used for describing transition of COVID negative to positive. This mathematical model is a three stage compartmental stochastic model applying in a Marko chain with continuous time Poisson arrivals. It can be used to estimate the behavior of COVID spreading in the society. Consider the suspicious people arriving for COVID test as a Poisson process. There are many Mathematical models about the spread of COVID in the society. Here we are actually mining the entire person from a suspicious group who are COVID positive.

2. Related Works

Bagozzi, 1983[1] Engel, Black well and Kollat, 1978[2] describes various theories of purchase intention behavior that helps to predict subsequent purchase. Daniel and Gates, 1991[8] observed that information about purchase intention is typically drawn from a purchase intent scale. Juster, 1966, [6] introduce an 11 – point purchase probability scale which is designed to elicit a response to the question whether an item will be purchased within a specific time period. Ferber and Priskie 1965, [4], Kalwani and silk, 1982, [7] Mullet and Karson, 1985, [9] describe the relationship between purchse intention and purchase behavior. Ercan Tirtiroglu and Matt Elbeck 2008, [3] describe purchase intention using queuing theory. Jones, 1995, [5] gives a stochastic model for the high demand CCRS's insurance which also help in this paper. Taylor, Clean and Millard, 1998, [10] explain the movement of patients in hospitals by compartmental models which is very helpful in this paper.

Model Description



Here compartment A represent collections of people who are suspicious of COVID. From A those people confirmed as COVID positive move to compartment B and those in A are COVID negative move to compartment 'C'. If the people in compartment 'C' affected by COVID, then they may be moving to compartment B. Let x_{ij} is the transition rate of people moving from compartment i to j. Let 'λ' be the arrival rate of people in compartment 'A'.

Let $f_{pqr}^{(t)}$ be the probability that there are 'p' person in compartment 'A' 'q' in compartment 'B' and 'r' in compartment 'C' at time t.

f_{pqr} (t+Δt)= f_{pqr} (t) $(1-(px_{11}+qx_{22}+rx_{33}+\lambda)\Delta t)$
$+(p+1)x_{12 f_{p+1,q-1,r}}$ (t) $\Delta t+(p+1)x_{13} f_{p+1,q,r-1}$(t) $\Delta t+$ $(r+1)x_{32}$ $f_{p,q-1,r+1}$ (t) $\Delta t+$ $\lambda f_{p-1,q,r}$(t) Δt
$\frac{d}{dt}$(f_{pqr} (t)) $=$ $-(px_{11}+qx_{22}+rx_{33}+\lambda)$ f_{pqr} (t) + $(p+1)x_{12 f_{p+1,q-1,,r}}$ (t) + $(p+1)x_{13} f_{p+1,q,r-1}$(t) + $(r+$
$1x32fp,q-1,r+1$ $t+$ $\lambda fp-1,q,r$(t)

We shall now use the p.g.f in order to calculate differential equations for the moments about the origin.

Let F (z_1, z_2, z_3, t) the joint probability generating function. Then

F(z_1, z_2, z_3, t) = $\sum_{p,q,r=0}^{\infty} f_{pqr}$ (t)) $z_1^p z_2^q z_3^r$

$$\frac{dF}{dt}-(x_{11}z_1+x_{12}z_2+x_{13}z_3)\frac{dF}{dz_1}-x_{22}z_2\frac{dF}{dz_2}-(x_{32}z_2+x_{33}z_3)\frac{dF}{dz_3}=F\lambda(z_1)$$

Solving by Lagrange's method, the auxiliary equations are

$$\frac{dt}{1}=\frac{-dz_1}{x_{11}z_1+x_{12}z_2+x_{13}z_3}=\frac{-dz_2}{x_{22}z_2}=\frac{-dz_3}{x_{32}z_2+x_{33}z_3}=\frac{dF}{F\lambda(z_1)} \text{\_\_\_\_\_(1)}$$

Writing the auxiliary equation in the matrix notation we obtain

$$\frac{\partial z}{\partial t}=RZ$$

Where R = $\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ 0 & x_{22} & 0 \\ 0 & x_{32} & x_{33} \end{bmatrix}$

Which has solution T = $e^{-Rt}z$

Where z = {z_i}

$\frac{dt}{1}=\frac{dF}{Fz\lambda}$ from (1)

i.e. $\frac{1}{F}\frac{dF}{dt}=\sum_{i=1}^{3}\lambda_i z_i=AZ=A_e^{Rt}T$ _______ (2)

Where A = [λ 0 0] be the arrival rate from outside to each compartment, λ being the poisson arrival rate.

$$\frac{1}{F}dF=A_e^{Rt}\ T\ \ dt$$

Integrating

log F =A $\frac{T}{R}e^{Rt}$ + a constant

F = $e^{AzR^{-1}}\phi(e^{-Rt}$) z ....(3)

Where ϕ is an arbitrary function of three variables z_1, z_2, z_3

For calculating ϕ take t = 0 and assume the initial sizes of each compartment are (a_1, a_2, a_3)

F(z_1, z_2, z_3, 0) = ${z_1}^{a_1}\ {z_2}^{a_2}\ {z_3}^{a_3}$

∴(3) be come ${z_1}^{a_1}\ {z_2}^{a_2}\ {z_3}^{a_3}=e^{AR^{-1}}$z ϕ(z)

By the use of the matrix notation then we get

$$\phi(e^{-Rt}z)=\prod_{i=1}^{3}(1+R_i e^{-RT}Z)^{a_1}e^{AR^{-1}}e^{-RT}Z$$

The joint probability generating function for the number of products in each compartment is given by

$$F = \prod_{i=1}^{3} (1 + R_i e^{-RT} Z)^{a_1} e^{AR^{-1}} (1 - e^{-Rt})Z$$

By letting *t* tends to infinity then we get the equation for the steady state.

$$F = e^{AR^{-1}Z}$$

$$\left[\frac{dF}{dZ}\right]_{z=0} = AR^{-1} \dots\dots\dots\dots (4)$$

$$\left[\frac{d^2F}{dZ^2}\right]_{z=0} = [AR^{-1}]^2 \dots\dots\dots\dots (5)$$

(4) an d(5) gives the first and second moments. The mean and variance for the steady state solution may be then expressed by the following formulae

E(F) = Var(F) = AR^{-1}

Where A = [λ 0 0]

$$R^{-1} = \frac{1}{x_{11}x_{22}x_{33}} \begin{bmatrix} x_{22}x_{33} & x_{13}x_{32} - x_{12}x_{33} & -x_{13}x_{22} \\ 0 & x_{11}x_{33} & 0 \\ 0 & -x_{11}x_{32} & x_{11}x_{22} \end{bmatrix}$$

$$\therefore AR^{-1} = \frac{1}{x_{11}\ x_{22}x_{33}} \begin{bmatrix} \lambda x_{22}x_{33} \\ \lambda(x_{13}x_{32} - x_{12}x_{33}) \\ -\lambda x_{13}x_{22} \end{bmatrix}$$

∴Expected number of persons in compartment 1 = variance of number of persons in compartment 1

$= \lambda \frac{x_{22}x_{33}}{x_{11}x_{22}x_{33}} = \frac{\lambda}{x_{11}}$

Expected number of persons in compartment 2 = Variance of number of persons in compartment 2

$= \lambda \frac{x_{13}x_{32} - x_{12}x_{33}}{x_{11}x_{22}x_{33}}$

= Expected number of persons in compartment 3 = Variance of number of persons in compartment = $\lambda \frac{x_{13}x_{22}}{x_{11}x_{22}x_{33}} = \lambda \frac{x_{13}}{x_{11}x_{32}}$

From the above equations we get the mean and variance of peoples who are COVID19 patients and the people who are unaffected by corona virus.

3. Conclusion

Compartmental modeling describes the movement of a substance from one compartment to another. Its origins are based on the metabolism of tracer-labeled compound studies in the 1920s. As we will see, compartmental modeling is a special case of physiological modeling.

Stochastic modeling of epidemics is very important. There are different stochastic models of compartmental system which deals with different subjects. Many researchers have done with application of compartmental model. By using compartmental model we can discuss many health issues.

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