Optimal Flight Control System Design Using LQR: A Study on Load Minimum and Drift Minimum Analysis

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Abstract: ***This paper outlines the core principles underlying flight control, including both analysis and design aspects. The Linear Quadratic Regulator (LQR) control methodology is employed to attain performance, stability, and robustness. It delves into essential concepts such as load minimization and drift minimization. The control system's design is subsequently simulated and subjected to analysis using Simulink.***

Keywords: optimal, LQR, flight, control, autopilot

1. Introduction

This paper provides an overview of an initial design for a flight control system for the Ames I crew launch vehicle [1]. It employs LQR control to attain the desired performance and addresses the 'load minimum' and 'drift minimum' concepts. Section 2 presents the mathematical model of the launch vehicle and offers a system overview. Section 3 details the control methodology employed, focusing on parameter selection to balance performance and control effort. This section gives the choice of Q and R matrices for the 'Load minimum' and 'Drift minimum' control design. The entire system and control design are simulated in Simulink in Section 4. Results are presented in Section 5 for both cases, and Section 6 concludes the paper.

2. Launch Vehicle Model

Consider a simplified dynamic model of a launch vehicle [2] as illustrated in Fig. 1, as follows:

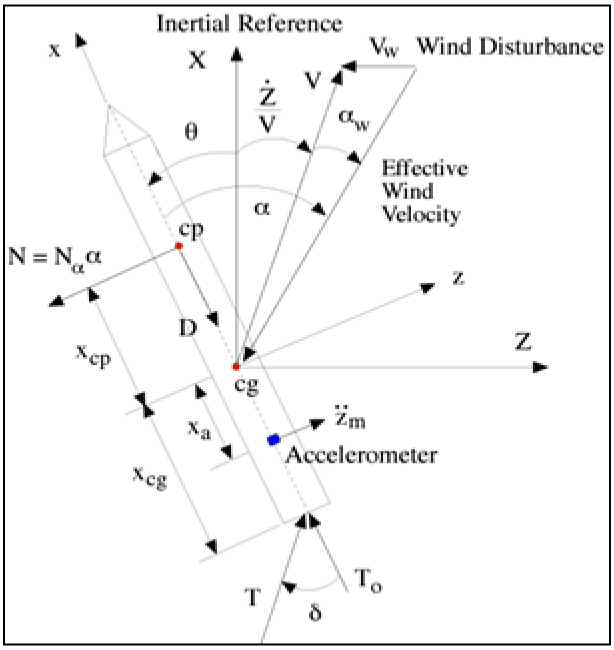

Figure 1: Dynamic model in 2D plane

Where θ is the pitch attitude, α is the angle of attack, Z is the inertial Z-axis drift position of the center-of-mass, Z^. is the inertial drift velocity, m is the vehicle mass, T_0 is the ungimbled sustainer thrust, T is the gimbled thrust, $N = N\alpha\alpha$ the aerodynamic normal (lift) force acting on the centre of pressure, D is the aerodynamic axial force (Drag), F is the total x-axis force, δ the gimbal deflection angle, V is the vehicle velocity, $\alpha_w = V_w/V$ the wind induced angle of attack, V_w is the disturbance velocity and the corresponding 3rd order dynamical model is given by:

$$\begin{bmatrix}\dot{\theta}\\ \ddot{\theta}\\ \ddot{Z}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ M_\alpha & 0 & M_\alpha/V\\ -(F-D+N_\alpha)/m & 0 & -N_\alpha/(mV)\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ \dot{Z}\end{bmatrix}$$

$$+\begin{bmatrix}0\\ M_\delta\\ T_c/m\end{bmatrix}\delta + \begin{bmatrix}0\\ M_\alpha\\ -N_\alpha/m\end{bmatrix}\alpha_w$$

$$C = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \quad D = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

Consider the parameters given by Greensite in [3] as

I_y = 2.43E6 slug-ft^2, m = 5830 slug, T=341,000 lb
F=375,000 lb, x_{cp}=38 ft, x_{cg}=32.3 ft
V=1320 ft/sec, V_w=132 ft/sec, α_w=5.73 deg
N_α=240,000 lb/rad, M_α=3.75 s^{-2}, M_δ=4.54 s^{-2}

Above parameters can be substituted in the above-mentioned model and this system can be further used in the analysis of control system design in Matlab.

3. Linear Quadratic Regulator

Linear Quadratic Regulator is a control technique used to design optimal control systems for linear time-invariant dynamic systems. It aims to find a state feedback control law that minimizes a quadratic function, typically defined as the sum of two terms:

- The quadratic deviation of the state variables from their desired values (Q matrix).
- The quadratic effort applied in the control input (R matrix).

The objective is to find a control law that minimizes this cost function while ensuring system stability and desired performance.

A set of full state feedback control gains, (k_1,k_2,k_3), can be found by using LQR control method [3-6] as follows:

$$\min_{\delta} \int_0^{\infty} (x^T Q x + \delta^2) dt$$

and
where x= [ ]^T and K= [k_1, k_2, k_3]

The choice of Q and R matrices is crucial in LQR control design. These matrices influence the tradeoff between system performance and control effort.

- The Q matrix is diagonal and defines the importance of each state variable. Larger values in Q indicate that minimizing the deviation of that state variable is more critical for the control objective.
- The R matrix is also diagonal and reflects the control effort. Larger values in R represent a preference for smoother control inputs.
- The values in Q and R are typically chosen based on the designer's knowledge of the system, performance requirements, and engineering intuition. Trial and error or optimization techniques can be used to tune these matrices.
- Higher values in Q lead to a stronger emphasis on tracking the desired state trajectory, resulting in better state tracking performance but potentially requiring more control effort.
- Higher values in R lead to smoother control inputs, reducing control effort but possibly sacrificing tracking performance.

Careful tuning of Q and R is essential to find a balance that meets the control system's performance and effort requirements.

A: Load Minimum

In the context of flight control systems, refers to a design or control strategy aimed at minimizing the loads or forces experienced by the aircraft's structure, especially during maneuvers or turbulent conditions. Load minimum [7-10] control techniques involve adjusting control surfaces and the control law to reduce abrupt changes in aircraft attitude and forces. This can be important for passenger comfort and aircraft structural integrity. In this case the goal is to minimize the angle of attack which is the combination the angle of rotation of the rocket and the drift velocity with respect to the main velocity of the rocket:

$$\alpha = Zdot / V + \alpha_w$$

The choice of the Q matrix for minimizing the angle of attack is

$$Q = \begin{bmatrix} 1 & 0 & 1/V \\ 0 & 0 & 0 \\ 1/V & 0 & 1/V^2 \end{bmatrix}$$

B: Drift Minimum

This refers to the design or control approach that minimizes the lateral or sideways movement (drift) [11-14] of the aircraft during various flight conditions. Lateral drift can occur during turns, crosswind landings, or other flight maneuvers. Minimizing drift is essential for maintaining proper flight path control. The choice of matrix Q for this type of control is

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Choosing a higher value of R penalizes the amount of work that TVC has to do. For this paper the R = 0.1

4. Simulation

The dynamic system of the launch vehicle given in section 2 is modeled in Simulink as shown in Fig.2. LQR control gains are calculated in Matlab by using the lqr command. The resulting values of [k_1, k_2, k_3] are [3.81,3.56,0.001].

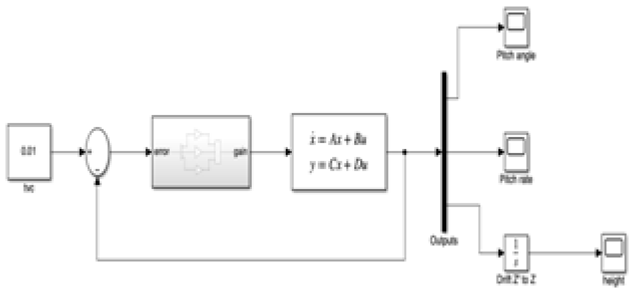

Figure 2: Simulink implementation

5. Results

Outputs are shown for both load minimum and drift minimum cases. The values of Q and R used in the simulation are given in sections 3.1 and 3.2. The plots for pitch angle, pitch rate, and drift are shown below. X-axis is time in all plots

Load Minimum

For this scenario, the attitude of the aircraft of penalized, and the choice of Q matrix is given in Section 3.1. As observed pitch angle converges as seen in Fig. 3, and the pitch rate remains small giving a stable load to the aircraft shown in Fig. 4. However, Fig. 5, represents the large drift of the aircraft.

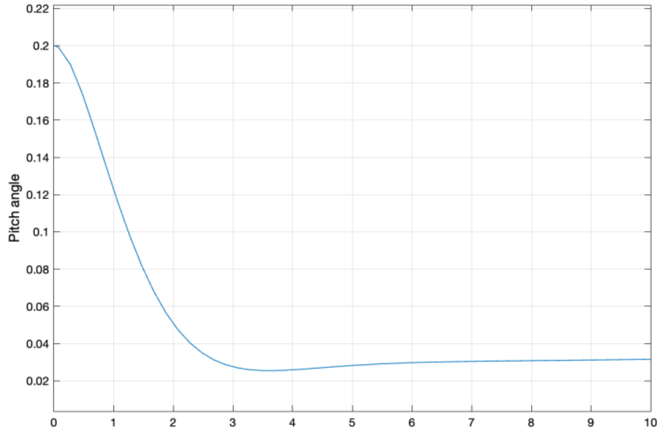

Figure 3: Pitch Angle

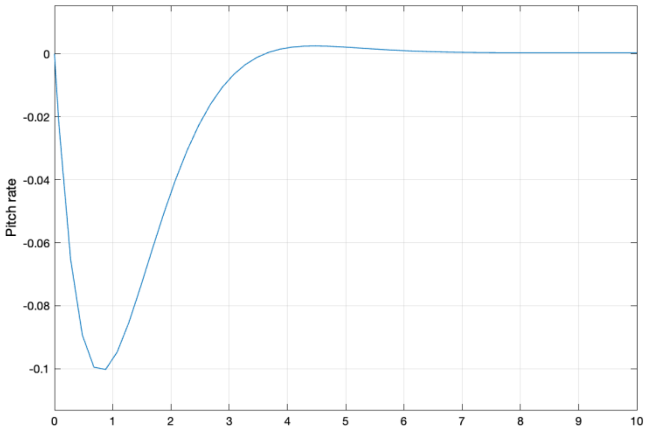

Figure 4: Pitch Rate

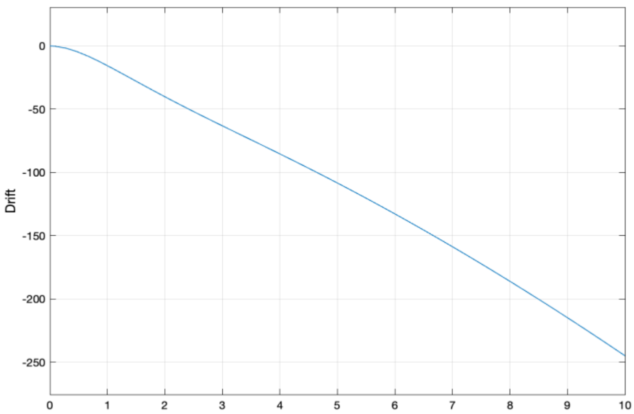

Figure 5: Drift

Drift Minimum

In this scenario, the drift is penalized in the Q matrix which was discussed in section 3.2, and as a result aircraft drift is lower as seen in Fig. 6 than the previous case. However, the pitch angle overshoots represented in Fig. 7, and the pitch rate observed in Fig. 8, are more compared to the load minimum case.

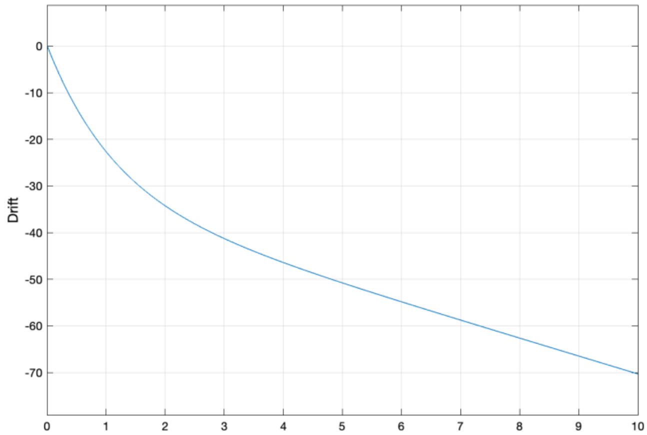

Figure 6: Drift

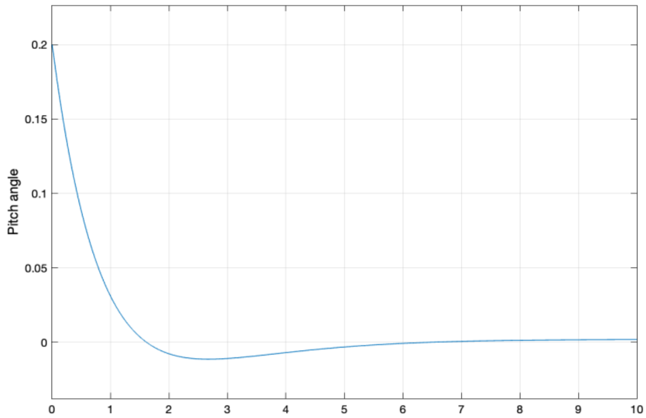

Figure 7: Pitch Angle

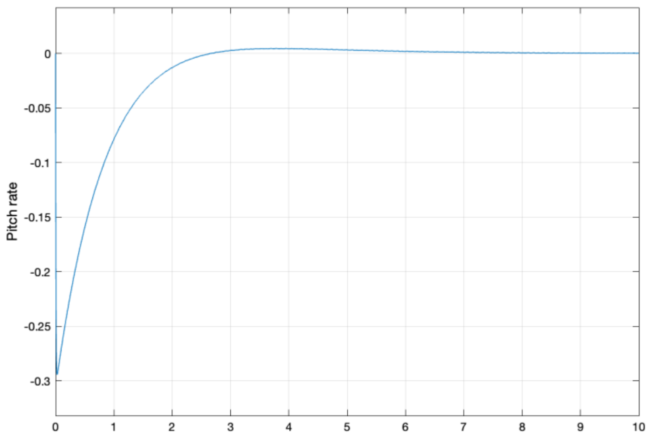

Figure 8: Pitch Rate

6. Conclusion

In summary, this paper has applied Linear Quadratic Regulator (LQR) control to enhance the design of a launch vehicle flight control system. Through extensive analysis, it has successfully achieved the desired performance while simultaneously enhancing system stability and robustness. The study's examination of load minimum and drift minimum scenarios has provided valuable insights, showcasing the efficacy of the LQR approach in reducing excessive loads and minimizing drift during flight maneuvers. These findings represent a significant stride toward improving safety, passenger comfort, and structural integrity in launch vehicle operations. Future research can build upon these insights, focusing on real-world implementations to further advance launch vehicle technology and ensure the secure and efficient transportation of astronauts and payloads to their intended destinations.

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