Unsteady MHD Free Convection Flow, Internal Heat Generation, and Bouncy Force for Boundary Layer over a Vertical Plate

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Abstract: ***The present paper's purposes are to study the effects of the magnetic field, internal heat generation, and bouncy force for the boundary layer over a vertical plate. The considered fluid is viscous and electrically conducting as well as incompressible on unsteady free convection heat transfer flow. The non-dimensional partial differential equations of continuity, momentum along energy are considered with appropriate transformations. For numerical solution, an explicit finite difference technique is functional to solve a set of nonlinear dimensionless partial differential equations. The non-similar equations are depending on Magnetic parameter, Grashof number, Prandtl number, Eckert number, and internal heat generation. Dimensionless velocity and temperature profile are also explored due to the effects of the inflowing parameters in the concerned problem. The analysis of the attained results is presented graphically and the flow field is significantly prejudiced by incoming parameters. The stability and convergence analysis are also scrutinized.to complete the formulation of the model.***

Keywords: Free convection, heat transfer flow, magnetic field, heat generation, bouncy force, explicit finite difference method.

1. Introduction

Unsteady MHD Free Convection boundary layer flow with an electrically conducting and viscous incompressible fluid has asignificant effect over vertical plates with internal heat generation/absorption because of their relevance to a wide variety of technical applications, mostly in the manufacture of fibers in glass, combustion modeling, aeronautical plasma flows, meteorology, and polymers industries. In the presence of radiation, the heat transfer analysis of boundary layer flow is also important in astrophysical flows, electrical power generation, fire engineering, solar power technology space vehicle re-entry, and other industrial areas. Convection heat transfer analysis is very significant in involving and controlling high temperatures such as gas turbines, nuclear plants, and thermal energy storage. It is created great interest from both theoretical and practical points of view because of its enormous applications in several engineering and geophysical fields.

Molla et al. [1] studied the natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation or absorption. Magnetohydrodynamic convective flow and heat transfer of a micropolar fluid past a continuously moving vertical porous flat plate in the presence of heat generation or absorption is analyzed numerically by Rahman and Sattar [2]. The steady combined free forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion is numerically analyzed by Alam et al. [3]. Ali, et al. [4] considered numerical analysis of heat and mass transfer along a stretching wedge surface. Unsteady MHD free convective heat transfer flow along with a vertical porous flat plate with internal heat generation is considered by Alam et al. [5]. Singhaand Deka [6] revealed the unsteady natural convection of an electrically conducting fluid between two heated parallel plates in the presence of a uniform magnetic field. The unsteady magnetohydrodynamic free convective flow and heat transfer along with a vertical porous plate with variable suction and internal heat generation is discussed by Sharma and Singh [7]. The free convective flow of heat generating/absorbing fluid between vertical porous plates with periodic heat input is studied by Jha and Ajibade [8]. The effect of heat transfer on peristaltic motion of Oldroyd fluid in the presence of inclined magnetic field is numerically scrutinized by Khan et al. [9]. Sheikholeslami and Bandpy [10] explored free convection of ferrofluid in a cavity heated from below in the presence of an external magnetic field. Rao et al. [11] investigated unsteady MHD Free Convective Heat and Mass Transfer Flow Past a Semi-Infinite Vertical Permeable Moving Plate with Heat Absorption. Vajravelu and Hadjinicolaou [12] studied heat transfer characteristics in the laminar boundary layer of a viscous fluid over a linearity stretching, continuous surface with variable wall temperature on effects of frictional heating and internal heat generation. Hossain et al. [13] investigated the heat transfer response of MHD free convective flow along a vertical plate to surface temperature oscillation. Kim [14] founded an unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Chandrakala et al. [15] studied the radiation effects on the flow of a semi-infinite vertical plate with uniform heat flux in the presence of the transversal magnetic field. Tarammim et al. [16]] analyzed two Dimensional Unsteady MHD Free Convection Flow over a Vertical Plate. Chandrakala et al. [17] studied the thermal radiation effects of an MHD flow past an infinite vertical oscillating plate in the presence of the transverse magnetic field. Senapati et al. [18] analyzed the radiation effects of MHD free convective heat transfer past an oscillating hot vertical surface in the porous medium. Ahmad et al. [19] considered MHD flow and heat transfer through a porous medium over a stretching/shrinking surface with suction. The Effect of Radiation on two dimensional unsteady MHD Free Convection Flow over a Vertical Plate is investigated by Ullah et al. [20]. Jahir and Tarammim [21] analyzed the unsteady MHD free convection, internal heat generation, and bouncy force for boundary layer flow over a vertical plate in the presence of radiation.

Hence the principal objective of this present work is to study the above revealed internal heat generation effects on MHD free convection boundary layer flow over a vertical plate. So, it is necessary to investigate, in detail, the dispersals of velocity and temperature for the flow along with a vertical plate across the boundary layer in the presence of a magnetic field. There is another investigation for the buoyancy effects on the thermal boundary layer over a vertical plate with a convective surface boundary condition.

2. Mathematical Model and Governing Equations

Let us consider unsteady MHD free convection, internal heat generation, and bouncy force for boundary layer flow with electrically conducting incompressible viscous fluid along with a vertical plate which is illustrated in **Figure 1**. All the fluid properties except that the influence of the density variations in the buoyancy force term is constant. We introduce in the Cartesian coordinate system, the X-axis is taken along the plate in the vertically upward direction and the Y-axis is normal to the plate directed into the fluid. T_w is the temperature of the surface plate and T_∞ is the outside of the surface plate separately. A uniform magnetic field $\mathbf{B} = (0, \mathrm{B}_0, 0)$ is enacted normally to the plate, and the magnetic field is anticipated to be negligible while B_0 is constant. It is also implicit that the free stream velocity U_∞ parallel to the vertical plate which is constant. Also, a heat source is retained within the flow to support potential heat generation or absorption effects.

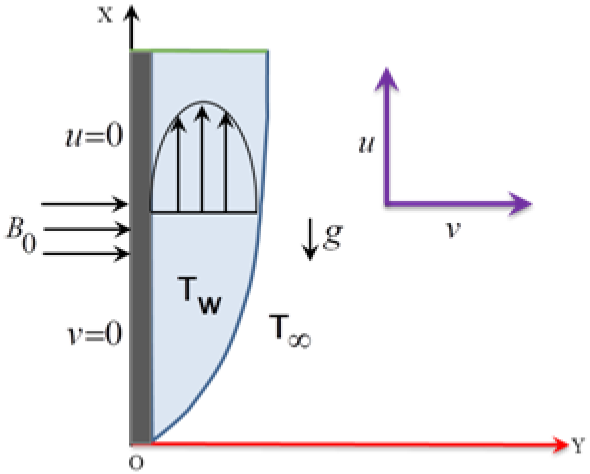

Figure: Physical configuration and coordinates system

The equations for unsteady MHD heat transfer flow, internal heat generation over a vertical plate with boundary conditions are given below:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(\mathrm{T} - \mathrm{T}_\infty) - \frac{\sigma \mathrm{B}_0^2}{\rho}u \quad (2)$$

Energy equation

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p}.\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q_0}{\rho C_P}(T - T_\infty) \quad (3)$$

The appropriate boundary condition for velocity and temperature is assumed by

$$\begin{aligned} &u = U_0, v = 0,\ T = T_w,\ \ at\ \ y = 0 \\ &u \to 0,\ T \to T_\infty,\ \ as\ \ y \to \infty \end{aligned} \quad (4)$$

where β is the co-efficient of volumetric expansion, ν is the kinematic viscosity, g is the acceleration due to gravity, T is the temperature of the fluid inside the thermal boundary layer, T_w is the temperature of the plate, T_∞ is the temperature in the free stream, σ is the electric conductivity, B_0 is a constant magnetic field, ρ is the fluid density, κ is the kinematic viscosity, C_p is the specific heat with constant pressure, Q_0 is the heat release per unit mass, U_0 is a constant measure of the uniform velocity of the fluid.

3. Mathematical Formulation

Applying the ensuing usual transformations, the system of partial differential equations with boundary conditions transformed into a non-dimensional equation.

$$u = U_0 U, v = VU_0, Y = \frac{yU_0}{\nu}, X = \frac{xU_0}{\nu}, \eta = \frac{tU_0^2}{\nu},$$

$$T = T_\infty + (T_w - T_\infty)\overline{T}.$$

Applying the above transformation in eq. (1), (2), (3), and with corresponding boundary conditions (4), after simplification we obtain, the following non-linear differential equations in terms of dimensionless variables such as:

Continuity equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (5)$$

Momentum equation

$$\frac{\partial U}{\partial \eta} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + Gr\overline{T} - MU \quad (6)$$

Energy equation

$$\frac{\partial \overline{T}}{\partial \eta} + U\frac{\partial \overline{T}}{\partial X} + V\frac{\partial \overline{T}}{\partial Y} = \frac{1}{Pr}\frac{\partial^2 \overline{T}}{\partial Y^2} + Ec\left(\frac{\partial U}{\partial Y}\right)^2 + H\overline{T} \quad (7)$$

with boundary conditions

$$U = 1,\ V = 0,\ \overline{T} = 1,\ at\ Y = 0$$
$$U = 0,\ \overline{T} = 0,\ as\ Y \to \infty \quad (8)$$

where,

Magnetic parameter, $M = \frac{\sigma \nu B_0^2}{\rho U_0^2}$

Grashof number, $Gr = \frac{\nu g\beta(T_w - T_\infty)}{U_0^3}$

Prandtl number, $Pr = \frac{\nu}{\alpha}$

Eckert number, $Ec = \frac{U_0^2}{C_p(T_w - T_\infty)}$

Heat Generation parameter, $H = \frac{Q_0 \nu}{\rho C_p U_0^2}$

4. Numerical Solution

A set of nonlinear partial differential dimensionless governing equations is to solved numerically with related boundary conditions along with an explicit finite difference method which is tentatively stable. The region of the flow is divided into a grid or mesh of lines parallel to X - and Y - axes, where X - axis indicates the plate in the upward direction and Y - axis is normal to the plate.we measured the height of plate X_{max} (=100), i.e., X varies from 0 to 100 and supposed Y_{max} (=25) as taken to $Y \to \infty$, it means that Y varies from 0 to 25.

We have taken $m = 250$ and $n = 250$ grid spacing in the X and Y directions correspondingly and as follows $\Delta x = 0.4(0 \le x \le 100)$ and $\Delta Y = 0.1(0 \le Y \le 25)$ with the minor time step $\Delta\eta = 0.005$. Let U', $\overline{T'}$ indicate the values of U, $\overline{T}$ at the end of a time-step separately.

Applying the explicit finite difference method into the partial equations (5-7) with boundary conditions (8) we get,

$$\frac{U_{i,j} - U_{i,j-1}}{\Delta X} + \frac{V_{i,j} - V_{i,j-1}}{\Delta Y} = 0 \quad (9)$$

$$\frac{U'_{i,j} - U_{i,j}}{\Delta\eta} + U_{i,j}\frac{U_{i,j} - U_{i-1,j}}{\Delta X} + V_{i,j}\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} = \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + Gr\overline{T}_{i,j} - MU_{i,j}$$

$$\Rightarrow U'_{i,j} = U_{i,j} + \Delta\eta\left(-U_{i,j}\frac{U_{i,j} - U_{i-1,j}}{\Delta X} - V_{i,j}\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} + \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + Gr\overline{T}_{i,j} - MU_{i,j}\right) \quad (10)$$

$$\frac{\overline{T}'_{i,j} - \overline{T}_{i,j}}{\Delta\eta} + U_{i,j}\frac{\overline{T}_{i,j} - \overline{T}_{i-1,j}}{\Delta X} + V_{i,j}\frac{\overline{T}_{i,j+1} - \overline{T}_{i,j}}{\Delta Y} = \frac{1}{Pr}\frac{\overline{T}_{i,j+1} - 2\overline{T}_{i,j} + \overline{T}_{i,j-1}}{(\Delta Y)^2} + Ec\left(\frac{U_{i,j+1} - U_{i,j}}{\Delta Y}\right)^2 + H\overline{T}_{ij}$$

$$\Rightarrow \overline{T}'_{i,j} = \overline{T}_{i,j} + \Delta\eta\left(-U_{I,J}\frac{\overline{T}_{i,j} - \overline{T}_{i-1,j}}{\Delta X} - V_{i,j}\frac{\overline{T}_{i,j+1} - \overline{T}_{i,j}}{\Delta Y} + \frac{1}{Pr}\frac{\overline{T}_{i,j+1} - 2\overline{T}_{i,j} + \overline{T}_{i,j-1}}{(\Delta Y)^2} + Ec\left(\frac{U_{i,j+1} - U_{i,j}}{\Delta Y}\right)^2 + H\overline{T}_{ij}\right) \quad (11)$$

The boundary conditions with the finite difference techniques are as follows:

$$U_{i,0}^n = 1,\ V_{i,0}^n = 0,\ \overline{T}_{i,0}^n = 1 \quad (12)$$

$$U_{i,L}^n = 0,\ \overline{T}_{i,L}^n = 0,\ \text{where } L \to \infty.$$

Here, the subscripts i and j indicate the grid points with X and Y coordinates respectively and $\overline{T}$ is the temperature.

5. Convergence and Stability Analysis

Anynumerical computation will remain inadequate without the stability and convergence analysis. It's always supportive of tangible computations. So, it established numerical solutions (finite difference scheme) that are dependable and consistent.

For the constant mesh sizes, the stability criteria of the scheme may be established are as follows.

The general terms of the Fourier expansion for U, θ, φ at time arbitrary say $\eta = 0$ are $e^{i\bar{\alpha}\bar{x}}$ and $e^{i\bar{\beta}\bar{y}}$ apart from a constant, where $i = \sqrt{-1}$.

Then

$$U : \Psi(\eta) e^{i\bar{\alpha}\bar{x}} . e^{i\bar{\beta}\bar{y}}$$
$$\bar{T} : \theta(\eta) e^{i\bar{\alpha}\bar{x}} e^{i\bar{\beta}\bar{y}} \qquad (13)$$

And after the time step, these terms will adapts

$$U : \Psi'(\eta) e^{i\bar{\alpha}\bar{x}} . e^{i\bar{\beta}\bar{y}}$$
$$\bar{T} : \theta'(\eta) e^{i\bar{\alpha}\bar{x}} . e^{i\bar{\beta}\bar{y}} \qquad (14)$$

Putting eq. (13) and eq. (14) into equations (10-11), we get the following equations by simplification.

$$\frac{\Psi' - \Psi}{\Delta\eta} + U \frac{\Psi\left(1 - e^{-i\bar{\alpha}\Delta X}\right)}{\Delta X} + V \frac{\Psi\left(e^{i\bar{\beta}\Delta Y} - 1\right)}{\Delta Y} = \frac{2\Psi(\Delta Y.\cos\beta - 1)}{(\Delta Y)^2} + Gr\,\theta' - M\Psi$$

$$\Rightarrow \Psi' - \Psi + \frac{\Delta\eta}{\Delta X} U\Psi\left(1 - e^{-i\bar{\alpha}\Delta X}\right) + \frac{\Delta\eta}{\Delta Y} V\Psi\left(e^{i\bar{\beta}\Delta Y} - 1\right) = 2\frac{\Delta\eta}{(\Delta Y)^2}\Psi(\Delta Y\cos\beta - 1) + Gr\Delta\eta\,\theta' - M\Psi$$

$$\Rightarrow \Psi' = \Psi\left\{1 - \frac{\Delta\eta}{\Delta X} U\left(1 - e^{-\bar{\alpha} i\Delta X}\right) - \frac{\Delta\eta}{\Delta Y} V\left(e^{i\bar{\beta}\Delta Y} - 1\right) + \frac{2\Delta\eta}{(\Delta Y)^2}(\Delta Y\cos\beta - 1) - M\Delta\eta\right\} + Gr\,\Delta\eta\,\theta'$$

$\Psi' = A\Psi + B\theta'$ (15)

where

$$A = 1 - \frac{\Delta\eta}{\Delta x} U\left(1 - e^{-i\bar{\alpha}\Delta X}\right) - \frac{\Delta\eta}{\Delta Y} V\left(e^{i\bar{\beta}\Delta Y} - 1\right) + \frac{2\Delta\eta}{(\Delta Y)^2}(\cos\beta.\Delta Y - 1) - M\Delta\eta$$

$$B = Gr\Delta\eta$$

and

$$\frac{\theta'(\eta) - \theta(\eta)}{\Delta\eta} + U\theta(\eta)\frac{(1 - e^{-i\bar{\alpha}\Delta X})}{\Delta X} + V\theta(\eta)\frac{(e^{-i\bar{\beta}\Delta Y} - 1)}{\Delta Y}$$

$$= \frac{1}{\Pr}\frac{2\theta(\eta)(\cos\beta.\Delta Y - 1)}{(\Delta Y)^2} + Ec\left(\frac{U\Psi(\eta)\left(e^{i\bar{\beta}\Delta Y} - 1\right)}{(\Delta Y)^2}\right) + H\theta'$$

$$\Rightarrow \theta'(\eta) = \theta(\eta)\left\{1 - \frac{\Delta\eta}{\Delta x} U\left(1 - e^{-i\bar{\alpha}\Delta X}\right) - \frac{\Delta\eta}{\Delta Y} V\left(e^{i\bar{\beta}\Delta Y} - 1\right) + \frac{1}{\Pr}\frac{2\Delta\eta}{(\Delta Y)^2}(\cos\beta.\Delta Y - 1)\right.$$
$$\left. + H\theta'\right\} + EcU.\frac{\Delta\eta}{(\Delta Y)^2}\Psi(\eta)\left(e^{i\bar{\beta}\Delta Y} - 1\right)$$

$$\Rightarrow \theta'(\eta) = G\theta + L\Psi \qquad (16)$$

where, $G = 1 - \frac{\Delta\eta}{\Delta X} U\left(1 - e^{-i\bar{\alpha}\Delta X}\right) - \frac{\Delta\eta}{\Delta Y} V\left(e^{i\bar{\beta}\Delta Y} - 1\right) + \frac{1}{\Pr}\frac{2\Delta\eta(\cos\beta.\Delta Y - 1)}{(\Delta Y)^2} + H\vartheta'$

and $L = EcU.\frac{\Delta\eta}{(\Delta Y)^2}\left(e^{i\bar{\beta}\Delta Y} - 1\right)$

Equations (15), (16) can be written as:

$$\Psi' = A\Psi. + B(G\theta + L\Psi)$$
$$= (A + L)\Psi + BG\theta$$
$$\Rightarrow \Psi' = A_1\Psi + B_1\theta \qquad (17)$$

where, $A_1 = A + L$

$B_1 = BG$

and $\theta' = G\theta + L\Psi$ (18)

Equations (17), (18) can be conveyed in matrix form.

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$$\begin{pmatrix}\psi' \\ \theta'\end{pmatrix} = \begin{pmatrix}A_1 & B_1 \\ L & G\end{pmatrix}\begin{pmatrix}\psi \\ \theta\end{pmatrix}$$

i.e. $\eta' = T\eta$

where, $T = \begin{pmatrix}A_1 & B_1 \\ L & G\end{pmatrix}$ and $\eta = \begin{pmatrix}\psi \\ \theta\end{pmatrix}$

As eigenvalues of the augmentation matrix T are animated for attaining the stability condition, as a result, let $B_1 \to 0, \quad L \to 0.$

Hence, matrix T is as follows:

$$T = \begin{pmatrix}A_1 & 0 \\ 0 & G\end{pmatrix}$$

Thus, the Eigenvalues of T are

$$\lambda_1 = A_1, \quad \lambda_2 = G.$$

Here, the values λ_1, λ_2 must not surpass in modulus.

Therefore, the stability conditions are as follows

$$|A_1| \le 1, \quad |G| \le 1.$$

Let, $a = \dfrac{u\Delta\eta}{\Delta X}, \quad b = \dfrac{\Delta\eta}{\Delta Y}.V, \quad c = \dfrac{\Delta\eta}{(\Delta Y)^2}$

Hence,

$$A = 1 - a\left(1 - e^{-i\bar{\alpha}\Delta X}\right) - b\left(e^{i\bar{\beta}\Delta Y} - 1\right) + 2c(\cos\beta\Delta Y - 1) - M$$

$$L = EcUc\left(C^{i\bar{\beta}\Delta Y} - 1\right).$$

The coefficients a, b, c are real and non-negative. Therefore, the maximum modulus of A_1, G arises when $\bar{\alpha}\Delta X = m\pi$ and $\bar{\beta}\Delta Y = n\pi$ where m and n are integers and hence A_1, G are real. The values of $|A_1|, |G|$ are greatest when m and n are odd integers, then

$$A_1 = A + L$$
$$= 1 - 2a - 2b - 4c - 2cEc.$$

To satisfy $|A_1| \le 1, \; |G| \le 1$ the most negative approved values are,

$$A_1 = -1, \quad G = -1.$$

Thus, the stability conditions of the problem are assumed below.

$$1 - 2a - 2b - 4c - 2cEc \le -1.$$
$$2(a + b + 2c + cEc) \le 2.$$
$$a + b + 2c + cEc \le 1.$$
$$\frac{U\Delta\eta}{\Delta X} + \frac{|V|\Delta\eta}{\Delta Y} + \frac{2\Delta\eta}{(\Delta Y)^2} + \frac{\Delta\eta}{(\Delta Y)^2}Ec \le 1.$$

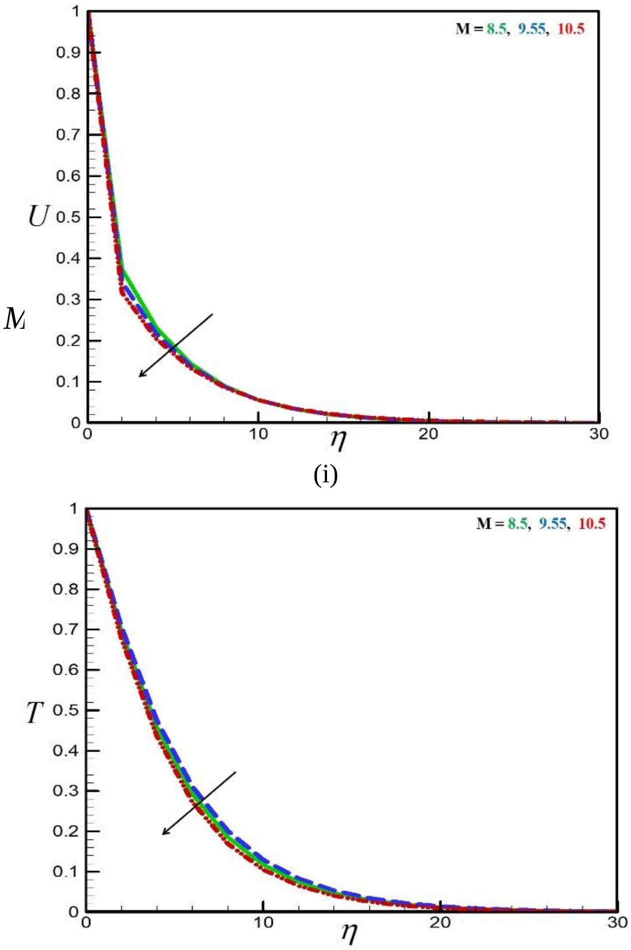
Analogously, the 2nd conditions are as follows

$$\frac{U\Delta\eta}{\Delta X} + \frac{|V|\Delta\eta}{\Delta Y} + \frac{1}{\Pr}\frac{2\Delta\eta}{(\Delta Y)^2}$$

and convergence criteria of the technique are $\Pr \ge 1$.

6. Results and Discussion

The present study is emphasized to scrutinize the problem under discussion, the results of numerical values of non-dimensional velocity and temperature profiles within the boundary conditions are computed for different values of magnetic parameter M, Grashof number Gr, Prandtl number Pr, Eckert number Ec, heat generation parameter H respectively. Also, computed results of velocity and temperature profiles are represented and physical explanation is explained here.



(i)

(ii)

Figure-a: Effect of Magnetic parameter (*M*) on velocity and temperature profile whereGr = 4.5, Pr = 1.38, Ec = 0.1, $H = 0.1$

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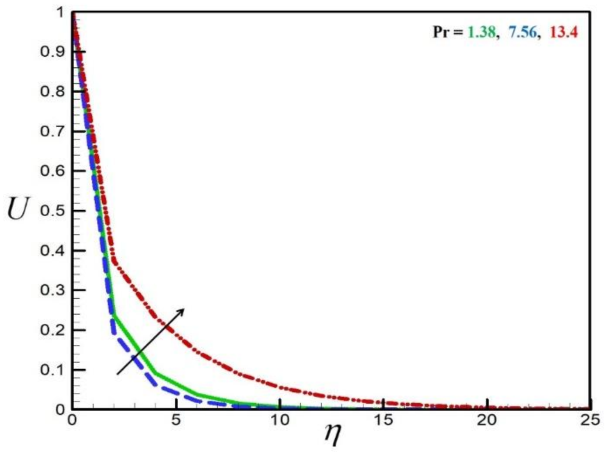

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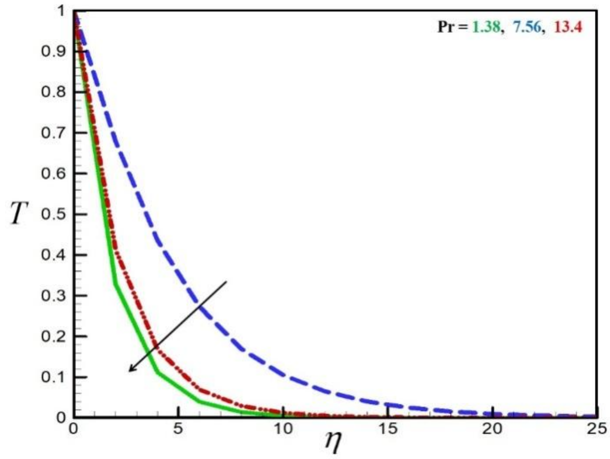
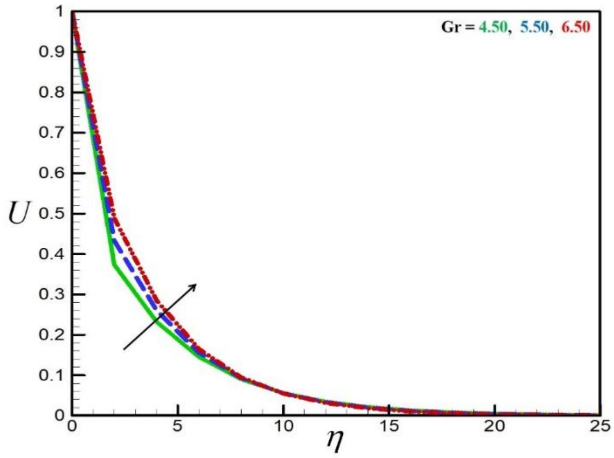


(ii)

Figure-b: Effect of Prandtl number (Pr) on velocity and temperature profile where M = 8.5, Gr = 4.5, Ec = 0.1, $H = 0.1$



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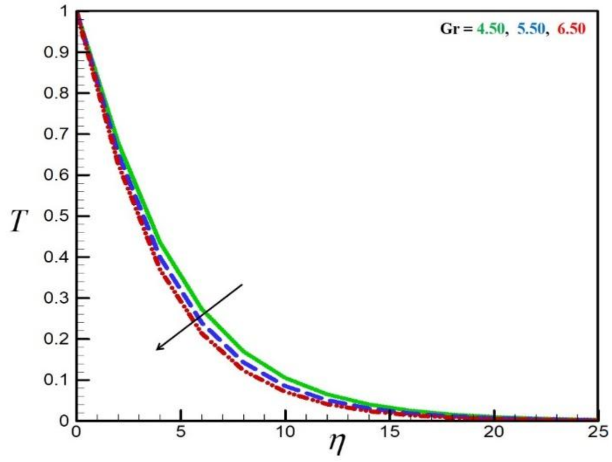
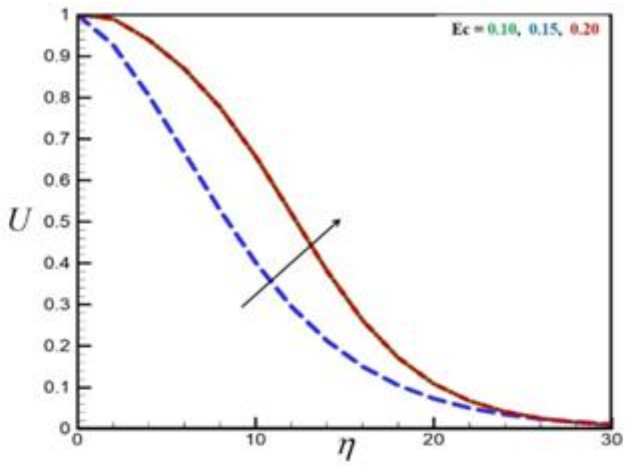


(ii)

Figure-c: Effect of Grashof number (Gr) on velocity and temperature profile where M = 8.5, Pr = 1.38, Ec = 0.1, $H = 0.1$



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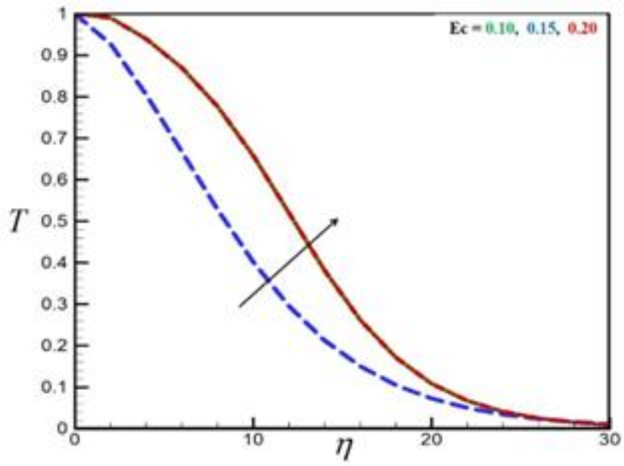
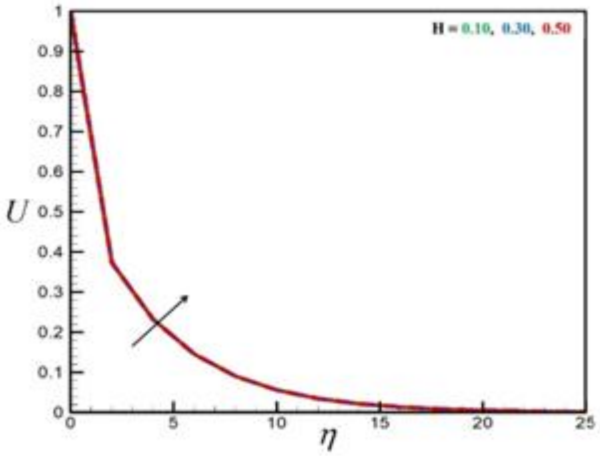


(ii)

Figure-d: Effect of Eckert number (*Ec*) on velocity and temperature profile where *M = 8.5, Gr = 4.5, Pr = 1.38,* $H = 0.1$



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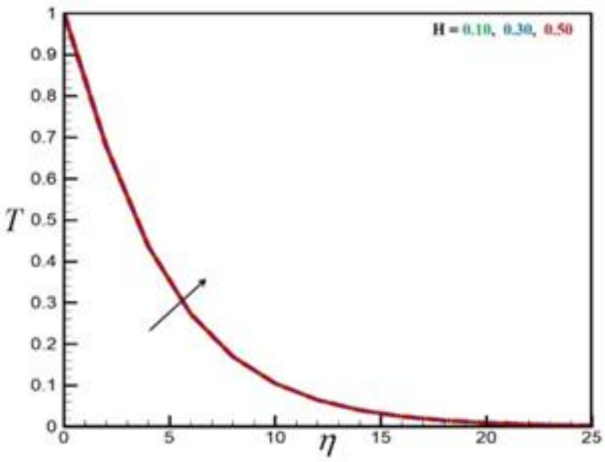

(ii)

Figure-e: The effect of heat generation parameter (H) on velocity and temperature profile where *M = 8.5, Gr = 4.5, Pr = 1.38, Ec = 0.1*

Effect of Magnetic parameter (M)

Figure a-i displays that for intensifying values of magnetic parameter M, the velocity of the flow field falls. It is because the application of transverse magnetic field to electrically conducting fluids gives to accelerate a body force attributed as Lorentz force. So it is created a tendency to reduce the speed of the motion of the fluid in the boundary layer and hence the fluid velocity decreases. Figure a- ii, it is seen that the temperature profiles decrease due to the increasing effect of M . This implies that higher values of magnetic field parameters produce lower heat transfer. So,the temperature gradient decreases for reducing the rate of heat convection in the flow.

Effect of Prandtl number (Pr)

Velocity and temperature profiles for some accurate values of Prandtl number Pr =1.38, 7.56, 13.4 which are significant in the sense that they physically resemble ammonia gas, water (at 18°), seawater (0°), are shown in Figures b-i, b-ii respectively

Figure b-i arrivals velocity profile is an upsurge for increasing values of $\Pr$.Because a higher Prandtl number creates high viscosity. Furthermore, heat is diffused away from the heated surface more rapidly for higher values of *Pr*. Figure b-ii, it is grasped that the temperature profiles decrease due to the increasing effect of Pr. It is due to the fact, increasing values $\Pr$ is proportional to reduced thermal conductivity.

Effect of Grash of number (Gr)

The velocity profile increases for increasing values of Gr which are shown in figure c-i. Because rising in the buoyancy force hindrances the fluid flow and enhances the velocity. Figure c-ii elucidates that the temperature profile decrease for increasing values of Gr. It means that increasing values Gr, decreases the rate of flow of temperature within the boundary layer.

Effect of Eckert number (Ec)

Figure d-iexhibits that the velocity profile upsurges for increasing values of Ec. Since the kinetic energy is higher than the heat transfer of the flow and increases the thermal buoyancy force within the thermal boundary layer. Thus accelerates the linear flow rate. Since frictional heating, as well as heat energy, is deposited in liquid, therefore the temperature profile rises for increasing values of Ec which are shown in figure d-ii. .

Effect of Heat generation (H)

Figure e-iarrivals that the velocity profile upsurges for increasing values of H. It is because the heat is produced, the buoyancy force rises, which makes the flow rate increase. Figure e- ii arrivals that with the increasing value of the heat generation parameter, the temperature delivery also increases purposely.

7. Conclusions

The numerical solution of the resulting momentum and the thermal equations are conveyed for characteristic values of the thermophysical parameters embedded in the fluid convection process. The effects of the magnetic parameter *M*, the Prandtl number Pr, Grashof number Gr, Eckert number Ec and the internal heat generation parameter H, on velocity and temperature profiles are analyzed and interpreted in physical terms and brings out the following conclusions.

- The velocity profiles rise for rising values of Pr, Gr, Ec, and H however it declines with increasing values of M
- The temperature i.e. thermal boundary layer increases owing to the various values of Ec, H. whereas it decreases with increasing values of M, Pr, and Gr.

Nomenclature

B : magnetic field intensity (JT^{-1});

B_0 : applied uniform magnetic field (JT^{-1})

C_p : specific heat at constant pressure ($kJkg^{-1}.^0C$);

Ec : Eckert number ;

g : acceleration due to gravity(ms^{-2});

Gr :Grashof number;

M : magnetic parameter;

MHD: magnetohydrodynamics ;

$\Pr$: Prandtl number ;

U_0 : constant velocity (ms^{-1});

$\overline{T}$: dimensionless temperature of the flow fluid ; T_w : the temperature at the fluid (K);

T_∞ : the temperature outside the boundary layers (K):

Greek Symbols

α : thermal diffusivity ($m^2 s^{-1}$); β : co-efficient of volumetric expansion(K^{-1});

ρ : density of the fluid (km^{-3});

θ : dimensionless temperature ;

η : dimensionless time ; υ: kinematic viscosity of the fluid ($m^2s{-}1$);

References

[1] Molla, M. M., Taher, M. A., Chowdhury, M. M. K., and Hossain, M. A..Magnetohydrodynamic natural convection flow on a sphere in the presence of heat generation, Non-linear Anal.Model.Control. 10(4),2005,pp. 349-363.

[2] Rahman, M. M., Sattar, M. A., Magnetohydrodynamic convective flow and heat transfer of a micropolar fluid past a continuously moving vertical porous flat plate in the presence of heat generation or absorption, ASME. J. Heat Trans. 128, 2006, pp.142-152.

[3] Alam, M. S., Rahman, M. M., Samad, M. A., Numerical study of the combined free-forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion, Non-linear Anal. Model.Control,11, 2006, pp.331-343.

[4] Ali, M., Alim, M., Nasrin, R., &Alam, M..Numerical analysis of heat and mass transfer along a stretching wedge surface. J. Naval Arch. Marine Engin. 14(2),2017, pp.135-144. https://doi.org/10.3329/jname.v14i2.30633

[5] Alam, M.S., Haque, M. M., Uddin, M. J., Unsteady MHD free convective heat transfer flow along with a vertical porous flat plate with internal heat generation. Int. J. Advan. Appl. Math. Mech. 2(2), 2014, pp.52 – 61.

[6] Singha, G.K., Deka, P.N., Skin-friction for unsteady free convection MHD flow between to heated vertical plates. J.Theor. Appl. Mech. 33(4), 2006, pp. 259–280.

[7] Sharma, P.R., Singh, G., Unsteady MHD free convective flow and heat transfer along with a vertical porous plate with variable suction and internal heat generation. Int. J. Appl. Math. Mech., 4(5), 2008, pp.1–8

[8] Jha, B.K., Ajibade, A.O.,Free convective flow of heat generating/absorbing fluid between vertical porous plates with periodic heat input, Int.Commun. Heat Mass Transf. 36, 2009, pp.624–631.

[9] Khan, A., Ellahi, R., Gulzar, M.M., Effect of heat transfer on peristaltic motion of Oldroyd fluid in the presence of the inclined magnetic field, J. Magn. Magn.Mater.372,2014, pp.97–106.

[10] Sheikholeslami, M., Bandpy, M.G., Free convection of ferrofluid in a cavity heated from below in the presence of an external magnetic field.Power Technol.256, 2014, pp.490–498.

[11] Rao B. M., Reddy G. V., Raju M. C. Varma S. V. K., Unsteady MHD Free Convective Heat and Mass Transfer Flow Past a Semi-Infinite Vertical Permeable Moving Plate with Heat Absorption, Radiation, Chemical Reaction and Soret Effects, Int. J. Sci. Emer. Tech. 6,2013,pp.241 -252.

[12] Vajravelu, K., Hadjinicolaou, A., Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation, Int. Comm. Heat Mass Transf. ,20, 1993, pp.417-430.

[13] Hossain, M. A., Das, S.K. and Pop, I., Heat transfer response of MHD free convective flow along a vertical plate to surface temperature oscillation, Int. J. Non-Linear Mech., 33(3), 1998,pp. 541-553.

[14] Kim, Y. J., Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction, Int. J. Eng. Sci., 38, 2000, pp.833-841.

[15] Chandrakala, P. and Antony Raj, S., Radiation effects on MHD flow past an impulsively started vertical plate with uniform heat flux, Indian J. Math, 50(3), 2008, pp.519-532.

[16] Tarammim, A., Ullah, M. and Uddin, M., A Study of Two Dimensional Unsteady MHD Free Convection Flow over a Vertical Plate. Open J. Fluid Dynamics, 10, 2020 pp.342-355. doi: 10.4236/ojfd.2020.104020.

[17] Chandrakala, P. and Bhaskar, P.N., Thermal radiation effects on MHD flow past a vertical oscillating plate. Int. J. Appl. Mech. Engg., 14(2), 2009, pp.349-358.

[18] Senapati, N., Das, T.K. and Dhal, R.K., Radiation effects of MHD free convective heat transfer past an oscillating hot vertical surface in porous, J. of the Orissa math, 31(2), 2012, pp.77-90.

[19] Ahmad, F., Hussain, S., Alanbari, A. M .and Alharbi, R. S., MHD Flow and Heat Transfer Through a Porous Medium Over a Stretching//Shrinking Surface With Suction., Sci. Int. (Lahore), 27(2), 2015, pp.931-935,

[20] Ullah, M., Tarammim, A. and Uddin, M.A Study of Two Dimensional Unsteady MHD Free Convection Flow over a Vertical Plate in the Presence of Radiation. Open J. Fluid Dynamics, 11, 2021, pp .20-33. doi: 10.4236/ojfd.2021.111002

[21] Uddin M. J., Trammim A. "A study of unsteady MHD free convection, internal heat generation, and bouncy force for boundary layer flow over a vertical plate in the presence of radiation." The International Journal of Engineering and Science (IJES), 10(03), 2021: pp. 34-45. DOI:10.9790/1813-1003013445