Modified Navier-Stokes Equation for Incompressible Fluid Impacts

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Abstract: This study has come up with a numerical momentum-impulse scheme that models fluid impact on seas, off-shore structures using the Modified Navier–Stokes Equation. Finite volume method discretized Modified Navier-Stokes Equation. Non-slip boundary conditions at Reynolds number of about 500 000 were formulated. Modified Navier-Stokes Equation in the x-z axis was coupled with continuity equation to obtain the pressure field. Corrected velocities were computed using SIMPLER algorithm approach. Using the corrected velocities fluid impulses were determined in each control volume interfaces at constant temperature of 298K, 1 atmosphere.

Keywords: Modified Navier-Stokes Equation, Reynolds number, Fluid Impulse, Pressure-Impulse Theory, Pressure Field, Control Volume, Wall Shear Stress

1. Introduction

Owing to climate change, moving icebergs can be big enough to cause damage to offshore structures or a breaking sea wave can cause large and sudden fluid forces to be exerted on structures or sea going vessels. This study came up with momentum-impulse theory that models fluid impact on offshore structures or sea going vessels. It formulated a numerical solution for a Modified Navier-Stokes Equation in the x and z directions of the Cartesian plane using Finite Volume Method (FVM). Studies and experiments conducted by researchers like Lamb (1932), Bagnold (1939), Chan (1994), Hattori (1994), Chan and Melville (1988), Wood and Peregrine (1998) Zenit and Hunt (1998), Cox and Cocker (2000), it was found that there was no analytical justification of pressure-impulse theory during short time impacts. Secondly pressure-impulse theory neglected non-linear terms in Navier-Stokes governing differential equation, therefore pressure-impulse theory did not adequately model impacts in incompressible fluids. Pressure–impulse theory neglected even convective term in Euler differential equation. However pressure-impulse theory models potential flow which satisfy Laplace equation at low Reynolds number. It’s therefore necessary to come up with Modified Navier-Stokes Equation which can model incompressible fluid impacts at high Reynolds number characterized by turbulence. Modified Navier-Stokes Equation can describe fluid impacts at the boundary layer where most of the terms in Navier–Stokes Equation were neglected by pressure-impulse theory.

2. Governing Equations

Modified Navier-Stokes Equation in and z directions coupled with Continuity Equation in a Cartesian plane are given as follows:

\[ \frac{\partial \rho u}{\partial t} + \nabla (\rho uv) = - \nabla P + \nabla (\rho \nabla \tau) \]  
(1)

\[ \rho \frac{\partial v}{\partial t} + \nabla (\rho vw) = - \nabla P + \nabla (\rho \nabla \tau) \]  
(2)

Continuity Equation

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]  
(3)

\[ u \] is velocity component of the fluid in the x-direction, \[ v \] is velocity component of the fluid in the z-direction, \[ \rho \] density of the fluid flowing, \[ \tau \] wall shear stress, \[ \gamma \] wall shear stress diffusion coefficient.

2.1 Discretized governing differential equations using Finite Volume Method (FVM)

Discretized Modified Navier-Stokes Equation in the x-direction:

\[ u_{e} = \bar{u} + d_{e}(P_{e} - P_{b})^{n+1} \]

where

\[ d_{e} = \frac{\Delta x}{a} \]

\[ \bar{u} = \frac{\sum D_{e}(\tau_{e})_{eb} + b}{a} \]

\[ u_{e} \] gives the velocity in the x-direction at (n+1) control volume face value.

Discretized ModifiedNavier-Stokes Equation in the z-direction:

\[ w_{e} = \bar{w} + d_{e}(P_{e} - P_{b})^{n+1} \]

where

\[ d_{e} = \frac{\Delta x}{a} \]

\[ \bar{w} = \frac{\sum D_{e}(\tau_{e})_{eb} + b}{a} \]

\[ w_{e} \] is the velocity in the z-direction at (n+1) control volume face value.

Discretized continuity equation:

\[ (u_{i+1} - u_{i}) \Delta x = -(w_{j+1} - w_{j}) \Delta x \]

(6)

On a staggered grid the horizontal and vertical velocities can be shown on the diagram below.
In staggered grid discretization the scalars are stored at the centre of the control volume and velocity are centred at the faces of control volumes.

From discretized continuity equation \( u_{j+1} - u_j \) are velocities in the x-direction, \( w_{j+1} - w_j \) are velocities in the z-direction of the control volume and at the centre is Pressure(P) which acts as a driving force for fluid velocities.

\[
b^+ = -\Delta z \frac{\partial}{\partial z} \left( \Gamma \frac{\partial}{\partial z} \right) + u \Delta z + w \Delta x - \Delta x \frac{\partial}{\partial x} \left( \Gamma \frac{\partial}{\partial x} \right) - \Delta x b^-
\]

Consider a fluid flow domain \( \Omega \) below which has been divided into 16 rectangular control volumes.

Reynolds number of 500 000, the speed of fluid flux in the x-axis and z-axis is given as:

\[
Re = \frac{Lu}{v} \nabla
\]

which gives \( u \approx 1m/s \) as the speed of the fluid in the boundary of x-direction.

\[
500000 = \frac{0.4 \times w}{9.368 \times 10^{-7}}
\]

which gives \( w \approx 1m/s \) as the speed of the fluid in the boundary of z-direction.

\[
a = \rho \Delta x \Delta z = 1023.6 \times 0.124 \times 0.1 = 12.795
\]

where \( \rho = 1023.6 kg/m^3 \) is the density of sea water

\[
d_e = \frac{\Delta x}{a} = 0.125
\]

\[
d_n = \frac{\Delta x}{a} = 0.125
\]

\[
a_n = d_e \Delta x = 0.00977 \times 0.125 = 0.0122
\]

\[
a_e = d_e \Delta z = 0.007816 \times 1.0 = 0.007816
\]

\[
a_p = d_e \Delta z + d_n \Delta x = 0.002001
\]

\[
D_n = \frac{\Gamma \Delta x}{\partial \xi} = \frac{15.4 \times 0.125}{0.1} = 19.25 m^2/s
\]

3. Pressure Equation

Substituting the horizontal velocity Eqn (4) and vertical velocity Eqn (5) into Continuity Equation gives the pressure equation in the form:

\[
a_p P_p = a_e P_e + a_n P_n + b^p
\]

where \( a_n = d_e \Delta z, a_e = d_e \Delta z, a_p = d_e \Delta z + d_n \Delta x \)

and

\[
\Delta x = 0.125 m \Delta x = 0.125 m \Delta x = 0.125 m \Delta x = 0.125 m
\]

Fluid flow field in x-axis (u m/s)

\[
D_e = \frac{\Gamma \Delta z}{\partial \xi} = \frac{15.4 \times 0.125}{0.1} = 12.32 m^2/s
\]

\[
D_w = \frac{\Gamma \Delta z}{\partial \xi} = \frac{15.4 \times 0.125}{0.125} = 12.32 m^2/s
\]

where \( \Gamma = 15.4 m^2/s \) which is the diffusion coefficient of sea water at 25°C and 1 atmosphere Richardson et al(1965)).

Approximation of pressure field in the 16 control volumes gives the following 16 algebraic equations:

\[
P_1 = 0.6095 P_1 + 0.3905 P_2 + 620.5
\]

\[
P_2 = 0.6095 P_2 + 0.3905 P_3 + 609.7
\]

\[
P_e = 0.6095 P_e + 0.3905 P_a + 613
\]

\[
P_s = 0.6095 P_s + 607.4
\]

\[
P_a = 0.6095 P_a + 611.4
\]

\[
P_h = 0.6095 P_h + 0.3905 P_e + 617
\]

\[
P_h = 0.6095 P_h + 0.3905 P_p + 611.7
\]

\[
P_e = 0.6095 P_e + 0.3905 P_h + 625
\]

\[
P_s = 0.6095 P_s + 0.3905 P_p + 610.2
\]

\[
P_s = 0.6095 P_s + 0.3905 P_s + 608.6
\]

\[
P_p = 0.6095 P_p + 0.3905 P_p + 611.39
\]

\[
P_t = 0.6095 P_t + 0.3905 P_t + 611.485
\]

\[
P_t = 0.6095 P_t + 614.86
\]
System of algebraic equations Eqn (8) was solved using Gauss-Seidel iteration method. Partial derivative of the equations \( \frac{\partial P}{\partial P_1} < 1, \frac{\partial P}{\partial P_5} < 1, \frac{\partial P}{\partial P_a} < 1 \), this shows that unique solutions for the system of equations exists. Eqn (8) also satisfy the Scarborough criterion for convergence.

### 4. Results

Iteration of equation (8) by Gauss-Seidel method gave the following solutions:

<table>
<thead>
<tr>
<th>( n )</th>
<th>( P_n )</th>
<th>( P_n )</th>
<th>( P_n )</th>
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<th>( P_n )</th>
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<th>( P_n )</th>
<th>( P_n )</th>
<th>( P_n )</th>
<th>( P_n )</th>
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<tr>
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<td>600</td>
<td>600</td>
<td>600</td>
<td>600</td>
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<td>600</td>
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<td>600</td>
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<tr>
<td>1</td>
<td>1221</td>
<td>1210</td>
<td>1213</td>
<td>973.1</td>
<td>977.1</td>
<td>1212</td>
<td>1213</td>
<td>1210</td>
<td>1209</td>
<td>1211</td>
<td>1211</td>
<td>981.6</td>
<td>845.5</td>
<td>845.5</td>
</tr>
<tr>
<td>2</td>
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<td>1822</td>
<td>1732</td>
<td>1203</td>
<td>1209</td>
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<td>1600</td>
<td>1516</td>
<td>989.0</td>
<td>613.0</td>
<td>850.6</td>
</tr>
<tr>
<td>3</td>
<td>2442</td>
<td>2398</td>
<td>2138</td>
<td>1203</td>
<td>1209</td>
<td>2004</td>
<td>2027</td>
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<td>1516</td>
<td>989.0</td>
<td>613.0</td>
<td>850.6</td>
</tr>
<tr>
<td>4</td>
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<td>2360</td>
<td>1203</td>
<td>1209</td>
<td>2477</td>
<td>2502</td>
<td>1897</td>
<td>1778</td>
<td>1516</td>
<td>989.0</td>
<td>613.0</td>
<td>850.6</td>
<td>977.7</td>
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<td>1778</td>
<td>1516</td>
<td>989.0</td>
<td>613.0</td>
<td>850.6</td>
</tr>
<tr>
<td>6</td>
<td>3474</td>
<td>3042</td>
<td>2357</td>
<td>1347</td>
<td>1214</td>
<td>2010</td>
<td>2481</td>
<td>2736</td>
<td>1899</td>
<td>1778</td>
<td>1516</td>
<td>989.0</td>
<td>613.0</td>
<td>850.6</td>
</tr>
<tr>
<td>7</td>
<td>3476</td>
<td>3042</td>
<td>2357</td>
<td>1347</td>
<td>1214</td>
<td>2010</td>
<td>2481</td>
<td>2736</td>
<td>1899</td>
<td>1778</td>
<td>1516</td>
<td>989.0</td>
<td>613.0</td>
<td>850.6</td>
</tr>
<tr>
<td>8</td>
<td>3476</td>
<td>3042</td>
<td>2357</td>
<td>1347</td>
<td>1214</td>
<td>2010</td>
<td>2481</td>
<td>2736</td>
<td>1899</td>
<td>1778</td>
<td>1516</td>
<td>989.0</td>
<td>613.0</td>
<td>850.6</td>
</tr>
</tbody>
</table>

The approximated pressure field in each control volume is shown in the fig 28 below:

The pressure field in figure 3 can be found in the control volume using the SIMPLER algorithm formulae from Pantanker S.V (1980) given by:

\[
\begin{align*}
\tilde{u} &= \sum_{a} \left( \frac{\partial P_s}{\partial P_r} \right)_{a} + b^u \\
\tilde{w} &= \sum_{a} \left( \frac{\partial P_s}{\partial P_r} \right)_{a} + b^w
\end{align*}
\]

Using the formulae in Eqn (9) and pressure field in figure 3 corrected velocities across the control volumes are computed as below:

Using the pressure field in figure 3 we can find velocities between interfaces of the control volumes using the SIMPLER algorithm formulae from Pantanker S.V (1980) given by:

\[
\begin{align*}
\tilde{u} &= \sum_{a} \left( \frac{\partial P_s}{\partial P_r} \right)_{a} + b^u \\
\tilde{w} &= \sum_{a} \left( \frac{\partial P_s}{\partial P_r} \right)_{a} + b^w
\end{align*}
\]
\[
\begin{align*}
\mu_s &= \tilde{u} + d_s(P_s - P_i) = -0.2808 + 1.0946 + 0.007816(3476 - 3042), \quad 4.206\text{ m/s} \\
\nu_s &= \tilde{w} + d_s(P_s - P_i) = -0.2808 - 8.7246 + 0.00977(3476 - 2736), \quad 1.7756\text{ m/s} \\
\mu_i &= \tilde{u} + d_i(P_i - P_s) = -0.048806 + 1.098525 + 0.007816(3042 - 2357), \quad 6.4037\text{ m/s} \\
\nu_i &= \tilde{w} + d_i(P_i - P_s) = -0.2808 - 8.7747 + 0.00977(3042 - 2481), \quad 3.5745\text{ m/s} \\
\mu_{is} &= \tilde{u} + d_{is}(P_s - P_i) = -0.046714 + 1.0985 + 0.007816(2357 - 1347), \quad 8.9460\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_s - P_i) = -0.0062528 - 8.7736 + 0.00977(1347 - 1214), \quad 7.4804\text{ m/s} \\
\mu_{is} &= \tilde{u} + d_{is}(P_i - P_s) = -0.01172 - 8.8152 + 0.00977(2357 - 2010), \quad 5.4367\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.0408 + 1.0315 + 0.007816(2736 - 2481), \quad 2.9834\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.05112 - 8.712 + 0.00977(2736 - 1899), \quad 0.5857\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.04693 + 1.0315 + 0.007816(2481 - 2010), \quad 4.6631\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.0368 - 8.7592 + 0.00977(2481 - 1778), \quad 1.9277\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.047098 + 1.03517 + 0.007816(2010 - 1214), \quad 7.2096\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.016272 - 8.7696 + 0.00977(2010 - 1516), \quad 3.9595\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.001584 - 8.7736 + 0.00977(1214 - 989), \quad 6.5769\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.0174 + 1.0189 + 0.007816(1899 - 1778), \quad 1.9472\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.051096 - 8.696 + 0.00977(1899 - 977), \quad 0.2608\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.0211 + 1.0189 + 0.007816(1778 - 1516), \quad 3.0456\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.03312 - 8.7552 + 0.00977(1778 - 943), \quad 0.6304\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.0211 + 1.033 + 0.007816(1516 - 898), \quad 5.8422\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.00952 - 8.792 + 0.00977(898 - 613), \quad 6.0171\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.0084 + 1.0135 + 0.007816(977 - 943), \quad 1.2708\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.0110 + 1.0276 + 0.007816(943 - 851), \quad 1.7357\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.0110 + 1.014 + 0.007816(851 - 613), \quad 2.8632\text{ m/s} \\
\nu_{is} &= \tilde{w} + d_{is}(P_i - P_s) = -0.01632 - 8.7848 + 0.00977(1516 - 851), \quad 2.3041\text{ m/s} \\

\text{(11)}
\end{align*}
\]

### Table 2: Fluid impulses in the x-direction of the control volumes

<table>
<thead>
<tr>
<th>Control Volume</th>
<th>Δk (metres) (Control Volume Side length)</th>
<th>IMPULSE = \int \left[ (\rho u - \rho u_i) \Delta z \right] Ns</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 - 0.125</td>
<td>\rho \Delta z(u_i - u_s) = 1023.6 \times 0.1(4.206 - 1) = 328.1662</td>
</tr>
<tr>
<td>2</td>
<td>0.125 - 0.250</td>
<td>\rho \Delta z(u_i - u_s) = 1023.6 \times 0.1(6.4037 - 4.206) = 224.9566</td>
</tr>
<tr>
<td>3</td>
<td>0.250 - 0.375</td>
<td>\rho \Delta z(u_i - u_s) = 1023.6 \times 0.1(8.9460 - 6.4037) = 260.2298</td>
</tr>
<tr>
<td>4</td>
<td>0.375 - 0.500</td>
<td>\rho \Delta z(u_i - u_s) = 1023.6 \times 0.1(4.206 - 1) = 328.1662</td>
</tr>
<tr>
<td>5</td>
<td>0.250 - 0.375</td>
<td>\rho \Delta z(u_i - u_s) = 1023.6 \times 0.1(7.2096 - 4.6631) = 260.6597</td>
</tr>
<tr>
<td>6</td>
<td>0.125 - 0.250</td>
<td>\rho \Delta z(u_i - u_s) = 1023.6 \times 0.1(4.6631 - 2.9834) = 171.9341</td>
</tr>
<tr>
<td>7</td>
<td>0 - 0.125</td>
<td>\rho \Delta z(u_i - u_s) = 1023.6 \times 0.1(2.9834 - 1) = 203.0208</td>
</tr>
<tr>
<td>8</td>
<td>0 - 0.125</td>
<td>\rho \Delta z(u_i - u_s) = 1023.6 \times 0.1(1.9472 - 1) = 96.9554</td>
</tr>
<tr>
<td>9</td>
<td>0.125 - 0.250</td>
<td>\rho \Delta z(u_i - u_s) = 1023.6 \times 0.1(3.0456 - 1.9472) = 112.4322</td>
</tr>
<tr>
<td>10</td>
<td>0.250 - 0.375</td>
<td>\rho \Delta z(u_i - u_s) = 1023.6 \times 0.1(5.8422 - 3.0456) = 286.2599</td>
</tr>
</tbody>
</table>

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Table 3: Fluid Impulses in the z-direction of the control volumes

<table>
<thead>
<tr>
<th>Control volume</th>
<th>$\Delta z$ (metres)</th>
<th>Control volume side width</th>
<th>$IMPULSE = I = {\rho w}^x - {\rho w}^y \Delta z$</th>
<th>$N_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 – 0.1</td>
<td>$\rho \Delta z(w_{z1} - w_{z2}) = 1023.6 \times 0.125(1.7756 - 1) = 99.2380$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0 – 0.1</td>
<td>$\rho \Delta z(w_{z1} - w_{z2}) = 1023.6 \times 0.125(3.5745 - 1) = 329.4073$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0 – 0.1</td>
<td>$\rho \Delta z(w_{z1} - w_{z2}) = 1023.6 \times 0.125(5.4367 - 1) = 567.6758$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0 – 0.1</td>
<td>$\rho \Delta z(w_{z1} - w_{z2}) = 1023.6 \times 0.125(7.4804 - 1) = 829.1672$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.1 – 0.2</td>
<td>$\rho \Delta z(w_{z1} - w_{z2}) = 1023.6 \times 0.125(6.5769 - 7.4804) = -115.6028$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.1 – 0.2</td>
<td>$\rho \Delta z(w_{z1} - w_{z2}) = 1023.6 \times 0.125(3.9595 - 5.4367) = -189.7081$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.1 – 0.2</td>
<td>$\rho \Delta z(w_{z1} - w_{z2}) = 1023.6 \times 0.125(0.5857 - 1.7756) = -152.2477$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.1 – 0.2</td>
<td>$\rho \Delta z(w_{z1} - w_{z2}) = 1023.6 \times 0.125(0.57879 - 1.5229) = -120.789$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.2 – 0.3</td>
<td>$\rho \Delta z(w_{z1} - w_{z2}) = 1023.6 \times 0.125(0.25566 - 0.57879) = -41.345$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.2 – 0.3</td>
<td>$\rho \Delta z(w_{z1} - w_{z2}) = 1023.6 \times 0.125(0.63066 - 1.92622) = -165.767$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.2 – 0.3</td>
<td>$\rho \Delta z(w_{z1} - w_{z2}) = 1023.6 \times 0.125(2.3041 - 3.9595) = -211.8084$</td>
<td></td>
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</tr>
<tr>
<td>12</td>
<td>0.2 – 0.3</td>
<td>$\rho \Delta z(w_{z1} - w_{z2}) = 1023.6 \times 0.125(6.0171 - 6.5769) = -71.6264$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0.3 – 0.4</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>0.3 – 0.4</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.3 – 0.4</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>0.3 – 0.4</td>
<td>-</td>
<td></td>
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</tr>
</tbody>
</table>


Figure 4: Impulses in the x-direction of the control volumes (Graphical output)
5. Conclusion

The change in flux velocities in the control volume interfaces causes impulse. Impulsive effects are higher at deeper levels of sea water than at shallow levels as shown by the graphical outputs.

References