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Rotational Light Clocks and Visualization of Spins

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Abstract: ***Rotational light clocks are invented, and their behaviors are investigated. A light clock is abstracted as a system composed of two distinct points that represent the top and bottom mirrors. For a two-dimensional rotational light clock, the top and bottom mirrors rotate around a common point that is not between the two mirrors. For a three-dimensional rotational light clock, it is always perpendicular to two planes on which the two mirrors conduct circular motions. The three-dimensional rotational light clock is used to do visualization of the spin of particles. It is possible for the three-dimensional rotational light clock to cause no time dilation if the angular velocity takes some quantized values.***

Keywords: rotational light clock, mirror, angular velocity, quantized value, visualization, spin of particles

1. Introduction

In Newtonian mechanics, speed is born from space and time; this means that space and time preexist and then speed is generated later by dividing space by time. However, in Einstein's relativity, a new concept named space-time [1-2] is provided, and space-time (which is a combination of space and time) is born from speed with finite magnitude; this means that the existence of space-time is rooted on the finitude of speed. If an object moved with an infinitely large speed, the object would not need to spend time moving from one place to another place, and this means that the object does not 'feel' the existence of time; time is vanished. If an object is stationary (its speed is zero), the object does not 'feel' the existence of space; space is vanished.

Relativity [1-2] emphasizes the equality of different frames. Nature does not give privilege to any kind of frame including inertial and non-inertial frames. Special relativity is on inertial frame and general relativity is on non-inertial frame. In this paper it will discuss the situation on rotational frames. The behaviors of both two and three dimensional rotational light clocks are studied.

2. Two-dimensional rotational light clock

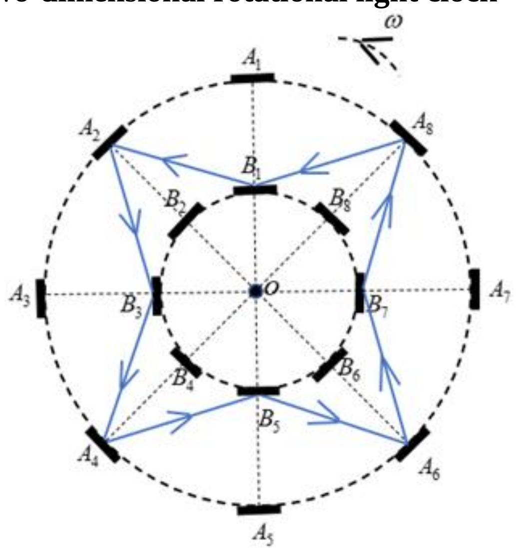


Figure 1: A two-dimensional rotational light clock

In fig.1, it shows a two-dimensional rotational light clock. The top mirror of the light clock is represented by point *A*, and bottom mirror *B*. The rectangular filled with black color is the mirror, and the blue straight line segment represents the light ray. Initially, the top and bottom mirrors are at the positions A_1 and B_1 respectively, and the light ray starts from B_1. In the second step, the top mirror moves to A_2, and the light ray hits the position A_2 at the same time. In the third step, the bottom mirror and the light ray simultaneously arrives at the position B_3. The repeated pattern continues to construct a rotational light clock. The angular velocity ω of the light clock is a constant. The trajectory of the light ray depends on the angular velocity, and the trajectory is not always closed. In fig.1 it is a closed case where the light ray travel from A_8 to B_1 to complete a revolution, and then it travels from B_1 to start the next revolution. In fig. 1, the condition $\omega.\Delta t_2 = \frac{\pi}{4}$ is satisfied; Δt_2 is the time interval for the light ray to travel from one mirror to the other mirror in the rotational frame.

Now we establish a general relationship between the time interval Δt_1 in the stationary frame and the time interval Δt_2 in the rotational frame. Denote the two radiuses of the two circles that are trajectories of the top and bottom mirrors as R_1 and R_2 respectively. In fig.1, $|OA_1| = R_1$ and $|OB_1| = R_2$. Denote the ratio of R_1 to R_2 as k.

$$\frac{R_1}{R_2} = k \ \left(k > 1\right) \tag{1}$$

For the light ray that travels from one mirror to the other mirror in the stationary frame, the following hold:

$$R_1 - R_2 = c \cdot \Delta t_1 \tag{2}$$

Combining (1) and (2), we know:

$$R_1 = \frac{k}{k-1} \cdot c \cdot \Delta t_1 \quad (3)$$

$$R_2 = \frac{1}{k-1} \cdot c \cdot \Delta t_1 \quad (4)$$

where Δt_1 is the time interval in the stationary frame for the light ray to travel from one mirror to the other mirror.

In the triangle ΔOB_1A_2, $|OA_2| = R_1$, $|OB_1| = R_2$, and

$$|B_1A_2| = c \cdot \Delta t_2 \quad (5)$$

where Δt_2 is the time interval in the rotational frame for the light ray to travel from one mirror to the other mirror.

In the triangle ΔOB_1A_2, the following relation holds according to the law of cosines.

$$|B_1A_2|^2 = |OB_1|^2 + |OA_2|^2 - 2|OB_1| \cdot |OA_2| \cdot \cos(\angle A_2OB_1) \quad (6)$$

According to $|OA_2| = R_1$, $|OB_1| = R_2$, (3), (4), (5), and $\angle A_2OB_1 = \omega \cdot \Delta t_2$, eq(6) can be written as the following eq(7):

$$(c \cdot \Delta t_2)^2 = \left(\frac{1}{k-1} \cdot c \cdot \Delta t_1\right)^2 + \left(\frac{k}{k-1} \cdot c \cdot \Delta t_1\right)^2 - 2\left(\frac{1}{k-1} \cdot c \cdot \Delta t_1\right) \cdot \left(\frac{k}{k-1} \cdot c \cdot \Delta t_1\right) \cdot \cos(\omega \cdot \Delta t_2) \quad (7)$$

Eq (7) results in the following:

$$(\Delta t_2)^2 \cdot \frac{(k-1)^2}{k^2+1-2k \cdot \cos(\omega \cdot \Delta t_2)} = (\Delta t_1)^2 \quad (8)$$

From (8) we know:

$$\frac{\Delta t_2}{\Delta t_1} = \sqrt{\frac{k^2+1-2k \cdot \cos(\omega \cdot \Delta t_2)}{(k-1)^2}} \quad (9)$$

The following will prove $\Delta t_2 > \Delta t_1$ if the angular velocity is not equal to zero; this means time dilation in the rotational frame.

In the right side of eq(9), we know $-1 \le \cos(\omega \cdot \Delta t_2) \le 1$.

If $-1 < \cos(\omega \cdot \Delta t_2) < 1$, we multiply $2k$ on both sides of this inequality, then we have:

$$-2k < 2k \cdot \cos(\omega \cdot \Delta t_2) < 2k \quad (10)$$

multiplying -1 on both ends of the inequality (10), then we have:

$$-2k < -2k \cdot \cos(\omega \cdot \Delta t_2) < 2k \quad (11)$$

We add k^2+1 on both sides of (11), the new have:

$$k^2+1-2k < k^2+1-2k \cdot \cos(\omega \cdot \Delta t_2) < k^2+1+2k \quad (12)$$

Remind $k>1$, so $k^2+1-2k>0$. Then based on (12) again, we know:

$$\frac{k^2+1-2k \cdot \cos(\omega \cdot \Delta t_2)}{k^2+1-2k} > 1 \quad (13)$$

Combing (9) and (13) we know $\Delta t_2 > \Delta t_1$.

If $|\cos(\omega \cdot \Delta t_2)| = 1$, then we know: $\omega \cdot \Delta t_2 = 2n\pi\, (n = 0, \pm 1, \pm 2, \pm 3, ...)$ and $\Delta t_2 = \Delta t_1$. These two equations and eq (2) that is $R_1 - R_2 = c \cdot \Delta t_1$ result in the following:

$$R_1 - R_2 = c \cdot \frac{2n\pi}{\omega} \quad (14)$$

Multiplying ω on the both sides of (14), we have:

$$\omega R_1 - \omega R_2 = 2n\pi c \quad (15)$$

If $n = 0$, then it results in $R_1 = R_2$, and this means the light clock does not exists, so it is not possible.

And if $n \ne 0\, (n = \pm 1, \pm 2, \pm 3, \ldots)$, then it results in $|\omega R_1 - \omega R_2| = 2|n|\pi c > c$, and this leads to $|\omega R_1| > c$ or $|\omega R_2| > c$. Since $|\omega R_1|$ and $|\omega R_2|$ are the absolute values of the tangential velocities of the top and bottom mirrors respectively, it is impossible for either of the magnitude of the two velocities is larger than the light speed.

Now we know that the situation that $n = 0, \pm 1, \pm 2, \pm 3 ...$ is not possible; this means $\left|\cos(\omega \cdot \Delta t_2)\right| = 1$ is not possible to happen.

Now we know $\Delta t_2 > \Delta t_1$ according to eq (9).

3. Three-Dimensional Rotational Light Clock

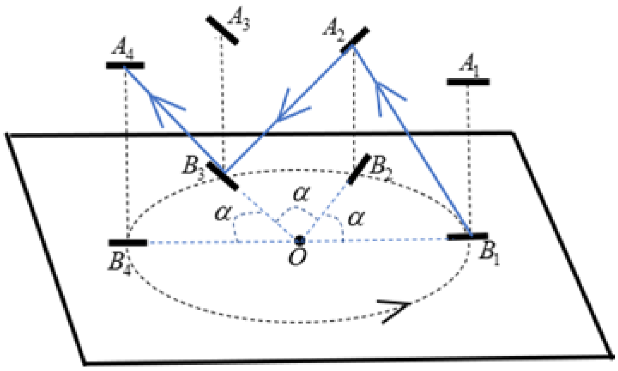


Figure 2: A three-dimensional light clock

In fig.2, it shows that the light clock makes a counterclockwise rotation with a constant angular velocity. The point *A* and *B* represent the top and bottom mirrors respectively. In the beginning, the top and bottom mirrors are at the point A_1 and B_1 respectively, then the two mirrors move to A_2 and B_2 respectively, and then to A_3 and B_3, and so on. The point $B_i \left(i = 1, 2, 3 \cdots\right)$ are on a circle with center at the point O, and the straight line segment $A_i B_i \left(i = 1, 2, 3 \cdots\right)$ is always perpendicular to the plane. The light ray (marked by the blue color) travels from B_1, and then to A_2, and then to B_3, and so on.

In the triangle $\Delta B_1 B_2 A_2$, $\left|A_2 B_2\right| = c \cdot \Delta t_1$ where Δt_1 is the time interval for the light ray to travel from one mirror to the other mirror in the stationary frame, $\left|A_2 B_1\right| = c \cdot \Delta t_2$ and $\left|B_1 B_2\right| = R \cdot (\omega \cdot \Delta t_2)$ where Δt_2 is the time interval for the light ray to travel from one mirror to the other mirror in the rotational frame.

Let $R = k \cdot (c \cdot \Delta t_1)$ where $k > 0$ and k is a real number, and then we have the following according to the law of cosines:

$$
\begin{aligned}
\left|B_1 B_2\right|^2 &= \left|OB_1\right|^2 + \left|OB_2\right|^2 - 2\left|OB_1\right| \cdot \left|OB_2\right| \cdot \cos(\angle B_1 O B_2) \\
&= R^2 + R^2 - 2R^2 \cdot \cos(\omega \cdot \Delta t_2) \\
&= \left(k \cdot (c \cdot \Delta t_1)\right)^2 + \left(k \cdot (c \cdot \Delta t_1)\right)^2 - 2\left(k \cdot (c \cdot \Delta t_1)\right)^2 \cdot \cos(\omega \cdot \Delta t_2) \\
&= 2\left(k \cdot (c \cdot \Delta t_1)\right)^2 \cdot \left(1 - \cos(\omega \cdot \Delta t_2)\right) \\
&= 2k^2 c^2 (\Delta t_1)^2 \cdot \left(1 - \cos(\omega \cdot \Delta t_2)\right)
\end{aligned}
\tag{16}
$$

According to Pythagorean theorem applied to triangle $\Delta B_1 B_2 A_2$, the following relation holds:

$$\left|A_2 B_1\right|^2 = \left|A_2 B_2\right|^2 + \left|B_1 B_2\right|^2 \tag{17}$$

Expressing each item in (17) in terms of time interval, then we have:

$$(c \cdot \Delta t_2)^2 = (c \cdot \Delta t_1)^2 + 2k^2 c^2 (\Delta t_1)^2 \cdot \left(1 - \cos(\omega \cdot \Delta t_2)\right) \tag{18}$$

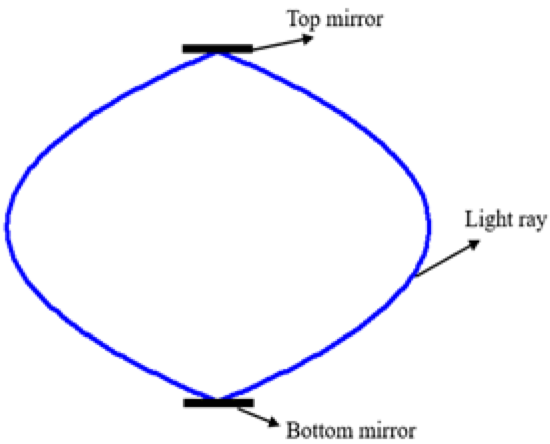
From (18) we know:

$$\frac{(\Delta t_2)^2}{1 + 2k^2\left(1 - \cos(\omega \cdot \Delta t_2)\right)} = (\Delta t_1)^2 \tag{19}$$

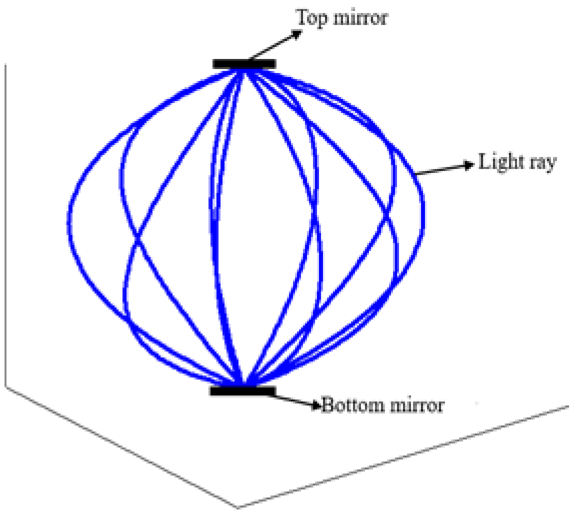
Consider the case $\omega \cdot \Delta t_2 = 2\pi$, the light clock looks same after making two resolutions, and this visualizes half spin that means an object looks same after making two resolutions. In this case, we can know $\Delta t_2 = \Delta t_1$ according to (19); that means no time dilation happens. And for the case $\omega \cdot \Delta t_2 = \frac{\pi}{2}$, the light clock looks same after making half resolution, and this corresponds to spin 2, that means an object looks same after making half resolution.

4. Light Bending

In fig.3 it shows the light bending 'observed' by the mirrors (or the observer moving with the mirrors) in the rotational light clocks. The light ray travels between the top and bottom mirrors along the trajectories of curved line segments. In the numerical experiments to obtain the trajectory, the light speed is set as 1, and the distance between the two mirrors set as 1.



(a) Two-dimensional rotational light clock



(b) Three-dimensional rotational light clock

Figure 3: Light bending 'seen' by mirrors of the rotational light clocks

5. Conclusions

Two and three dimensional rotational light clocks are investigated and the relationships between the time intervals of the stationary and rotational frames are given. It is possible for the three-dimensional light clock to cause no time dilation, and the three-dimensional light clock can be used to visualize the spin of particles. It can be extended to higher dimensions, such as the case in which the top mirror moves in the fourth dimension, and the bottom mirror moves on the surface of a 2-sphere (which is three dimensional; this can also build up a rotational light clock. Generally, a rotational light clock can be built up by the arrangement that the top mirror moves in the $(N+1)^{th}$ dimension, and the bottom mirror moves on the surface of a N-sphere.

References

[1] G. F. R. Ellis, R. M. Williams, Flat and Curved Space Times, Oxford University Press, 2000.
[2] R. Ferraro, Einstein's Space-Time: An Introduction to Special and General Relativity, Springer Science, 2007